Set separation and necessary conditions for minima (with applications, in particular, to Optimal Control Theory)

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Timetable: 24 hrs. First lecture on February 28, 2023 14:00, (dates already fixed, see Calendar of Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: Basic Calculus, Basic Lebesgue measure theory. (Other prerequisites – for instance fixed point theorems, absolutely continuous maps, – will be recalled during the course).

Examination and grading: An oral exam based on lectures contents and/or a scientific article preventively chosen.

SSD: MAT/05

Aim: This course, which does not require any particular prerequisite, aims primarily to frame the general notion of constrained minimum (in finite and infinite dimension) within the elementary concept of separation of sets: roughly speaking, a point is of local minimum if it locally separates the set of profitable states from the set of reachable states. Most of classical and more recent minimum problems can be seen under this perspective.

Through open mappings arguments (which will give us the occasion to see some notions of generalized differentiation and of approximating cone), necessary conditions for set separation can be translated into necessary conditions for minima: after immediately recognizing some of the more standard results in Calculus, we will apply the set-separation approach to Optimal controls of ODE’s (so, in particular, to Calculus of Variations).

Time permitting, some connections with Differential Geometric Controllability or Hamilton-Jacobi equations will be treated as well.

Course contents:
1. Brower fixed point theorem and a parameterized version of Banach fixed point theorem.
   A directional ‘open mapping’ theorem with low regularity. Set separation and cone separability
2. Review of ODE’s with vector fields measurable in time: local and global existence, uniqueness, continuity and differentiability with respect to initial conditions.
3. An abstract constrained minimum problem.
4. The Pontryagin Maximum Principle (PMP) with end-point constraints, with applications.
5. Controllability of control systems, at the first or higher order (Lie brackets).
6. If time permits: basic elements of Hamilton-Jacobi PDE’s.