Kernels and Partitions of Regular Domains and Compact Sets

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**Timetable:** 13 hrs. First lecture on November 21, 2023, 12:30 (date already fixed, see calendar on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

**Course requirements:** Recommended: Numerical Analysis I and Analysis I and II or Approximation Theory

**Examination and grading:**

**Aim:** Understanding the particulars of the approximation theory of many variables, namely kernel-based methods for regular and scattered data, interpolation vs. quasi-interpolation including polynomial reproduction, positive and strictly positive interpolation matrices and kernel functions, partitions of compact spaces and polynomial precision, topology of regular domains (mostly conic sections) and compact metric spaces.

**Course contents:** 13 hours, one introduction, four parts with three hours each

- 0. Part: Introduction
- I. Part: Basics on Kernels and Quasi-Interpolation.
  1. Interpolation in several variables by polynomials and otherwise.
  2. Kernel functions for interpolation; radial basis functions and main examples. Complete and multiple monotonicity.
  3. Concept of Quasi-Interpolation and comparison with interpolation.
- II. Part: Positive Definite Functions on Regular Domains, especially Spheres.
  1. Positive definiteness, strictly and semi positive definiteness of functions and interpolation matrices.
  2. Positive definite functions on spheres in many dimensions.
  3. Positive definite functions on other conic sections and on simplices.
  1. Concept of polynomial reproduction especially with quasi-interpolation.
  2. Relation of this to approximation orders; examples.
  3. Conditions for polynomial precision, examples especially with respect to partitions of unity.
- IV. Part: Partitions of Compact Sets.
  1. Quadrature methods.
  2. Partitions of compact sets for cubature.