Stochastic and mean field optimal control

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Timetable: 16 hours; first lecture on November 6th, 2023, 10:30 (date already fixed, see calendar on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: Basic knowledge of stochastic calculus (Brownian motion, stochastic differential equations, filtrations, martingales, ...), as presented, for example, in the course on stochastic analysis of the master degree. Some concepts will be recalled during the course.

Examination and grading: Oral presentation of a research paper related to the topics covered in the course, based on student’s interest.

SSD: MAT/06 and MAT/05.

Aim: Introduce the classical tools to analyze stochastic optimal control problems (dynamic programming, viscosity solutions, backward SDEs, relaxed controls) and then use these methods to study the recent theory of mean field control problems.

Course contents: Introduction to the classical theory of stochastic control problems with some motivating example. These problems consist in minimizing a cost in which the state variable is given by a controlled stochastic differential equation driven by a Brownian motion. The course will then cover the following topics:

- Equivalence of weak and strong formulation, existence of optimal relaxed controls via weak convergence methods;
- Dynamic programming principle: value function, Hamilton-Jacobi-Bellman equation, verification theorem, viscosity solutions of second order PDEs;
- Backward stochastic differential equations: representation of the value function for the weak formulation, necessary conditions for optimality given by the stochastic Pontryagin’s maximum principle, relation with dynamic programming equation.

In the second part, we introduce the recent theory of mean field control problems, also called optimal control of McKean-Vlasov dynamics. In these problems, the cost and the coefficients of the state equation depend also on the law of the state process, and can be reformulated as optimal control of the Fokker-Planck equation. We show how to extend the results established for the classical problem to the mean field case. In particular, the Hamilton-Jacobi-Bellman equation is stated in the Wasserstein space of probability measures, which is infinite dimensional. Thus we introduce a notion of differentiability of functions defined on the Wasserstein space.