Doctoral Program in Mathematical Sciences
Department of Mathematics "Tullio Levi-Civita"
University of Padova

Doctoral Program in Mathematical Sciences

Catalogue of the courses 2023-2024

Updated April 9th, 2024

INTRODUCTION

This Catalogue contains the list of activities offered to the Graduate Students in Mathematical Sciences for the year 2023-2024.

The activities in this Catalogue are of three types.

1. Courses offered by the Graduate School (= Courses of the Doctoral Program)
2. Courses offered by one of its curricula.
3. Other activities:
   a) selected courses offered by the PhD school in Information Engineering;
   b) selected courses offered by other PhD schools or other Institutions;
   c) reading courses

(This offer includes courses taught by internationally recognized external researchers. Since these courses might not be offered again in the near future, we emphasize the importance for all graduate students to attend them.)

Taking a course from the Catalogue gives an automatic acquisition of credits, while crediting of courses not included in the Catalogue (such as courses offered by the Scuola Galileiana di Studi Superiori, Summer or Winter schools, Series of lectures devoted to young researchers, courses offered by other PhD Schools) is possible but it is subject to the approval of the Executive Board. **Moreover, at most one course of this type may be credited.**

We underline the importance for all students to follow courses, with the goal of **broadening their culture in Mathematics**, as well as developing their knowledge in their own area of interest.

REQUIREMENTS FOR GRADUATE STUDENTS

Within the **first two years of enrollment** all students are required to

- pass the exam of at least four courses from the catalogue, among which at least two must be taken from the list of “Courses of the Doctoral Program”, while at most one can be taken among the list of “reading courses”
- participate in at least one activity among the “soft skills”
- attend at least two more courses (for such activities the PhD student must produce a brief summary on what she/he learned. These summaries should be attached to the annual report)

Students are warmly encouraged to take more courses than the minimum required by these rules, and to commit themselves to follow regularly these courses. It is also recommended that one half of the exams are taken during the first year. At the end of each course the instructor will inform the Coordinator and the Secretary on the activities of the course and of the registered students.

Students **must register** to all courses of the Graduate School that they want to attend, independently of their intention to take the exam or not. We recommend to register as early as possible: the Graduate School may cancel a course if the number of registered students is too low. If necessary, the registration to a Course may be canceled.

<u>Courses for Master of Science in “Mathematics”</u>

Students have the possibility to attend some courses of the Master of Science in Mathematics and get credits. The recommendation that a student takes one of these courses must be made by the supervisor and the type of exam must be agreed between the instructor and the supervisor.

Courses attended in other Institutions and not included in the catalogue.

Students activities within Summer or Winter schools, series of lectures devoted to young researchers, courses offered by the Scuola Galileiana di Studi Superiori, by other PhD Schools or by PhD Programs of other Universities may also be credited, according to whether an exam is passed or not; the student must apply to the Coordinator and crediting is subject to approval by the supervisor and the Executive board. We recall that **at most one course** not included in the Catalogue may be credited.

Seminars

a) All students, during the three years of the program, must attend the **Colloquia of the Department** and participate regularly in the Graduate Seminar ("**Seminario Dottorato**"), whithin which they are also required to deliver a talk and write an abstract.
b) Students are also strongly encouraged to attend the seminars of the research groups that are relevant for their work.

HOW TO REGISTER AND UNREGISTER TO COURSES

The registration to a Course must be done online.
Students can access the **online registration form** in the dedicated page of the Doctoral Course website at https://dottorato.math.unipd.it/current-activity/FutureActivities clicking on "click to enroll" of the chosen courses. The registration lists can be reached also via the website of the Department of Mathematics at https://prev-www.math.unipd.it/userlist/
In order to register, fill the registration form with all required data, and validate with the command "Subscribe". The system will send a confirmation email message to the address indicated in the registration form; please save this message, as it will be needed in case of cancellation.

Registration to a course implies the commitment to follow the course.

Requests of **cancellation** to a course must be submitted in a timely manner, and **at least one month before the course** (except for courses that begin in October and November) using the link indicated in the confirmation email message.

REQUIREMENTS FOR PARTICIPANTS NOT ENROLLED IN THE GRADUATE SCHOOL OF MATHEMATICS

The courses in this catalogue, although part of activities of the Graduate School in Mathematics, are open to all students, graduate students, researchers of this and other Universities.
For organization reasons, external participants are required to **communicate their intention** (loretta.dallacosta@unipd.it) to take a course at least two months before its starting date if the course is scheduled in January 2024 or later, and as soon as possible for courses that take place until December 2023.
In order to **register,** follow the procedure described in the preceding section.
Possible **cancellation** to courses must also be notified.

List of Courses

Courses of the Doctoral Program

1. Prof.ssa Cristiana Bertolin
   Elliptic curves and Periods **DP-1**
2. Prof.ssa Laura Caravenna
   Introduction to Optimal Transport **DP-2**
3. Prof. Ramon Codina
   Mixed and stabilised finite element methods **DP-3,4**
4. Prof. Christos Efthymiopoulos
   Perturbative methods in dynamical systems **DP-5,6**
5. Prof. Francesco Esposito
   Introduction to Harmonic Analysis on Semisimple Groups **DP-7,8**
6. Prof. Massimo Lanza de Cristoforis,
   Integral operators in Hölder spaces **DP-9**

Courses of the "Mathematics" area

1. Prof. Alexandr Buryak
   Integrable Systems of PDEs and their infinite dimensional algebra of symmetries **M-1,2**
2. Dott. Alessandro Goffi, Giulio Tralli
   Nonlinear methods for linear equations: the low-regularity theory **M-3,4**
3. Dott. Elio Marconi
   Flows of Sobolev vector fields **M-5**
4. Prof.ssa Gabriella Pinzari
   Introduction to Kolmogorov-Arnold-Moser theory **M-6,7**
5. Prof. Sergiy Plaksa
   Monogenic functions and basic elliptic equations of mathematical physics **M-8,9**
6. Prof. Fulvio Ricci
   Harmonic analysis on nilpotent groups **M-10**
7. Prof. Eric Sommers
   Introduction to Hessenberg Varieties **M-11,12**

Courses of the "Computational Mathematics" area

1. Dr. Manuel Francesco Aprile,
   Linear and non-linear formulations for Combinatorial Optimization **MC-1**

2. Prof. Martin Buhmann
   Kernels and Partitions of Regular Domains and Compact Sets **MC-2**

3. Dott.Alekos Cecchin
   Stochastic and mean field optimal control **MC-3**

4. Dr. Alberto Chiarini, Prof. Giovanni Conforti
   A renormalisation group approach to log-Sobolev inequalities **MC-4**

5. Prof. Andrea Roncoroni
   Interface of Finance, Operations and Risk Management **MC-5,6**

6. Prof. Simone Scotti
   Hawkes processes: from theory to (financial) practice **MC-7,8**

Soft Skills

1. Maths information: retrieving, managing, evaluating, publishing **SS-1**
2. Introduction to the use of "Mathematica" in Mathematics and Science **SS-2**
3. Our experience in writing a successful post-doctoral application **SS-3**

Courses in collaboration with the Doctoral School in "Information Engineering"

Please check regularly the website of the Doctoral Course in Information Engineering at the URL https://phd.dei.unipd.it/course-catalogues/

To be confirmed

Calendar of activities on
`https://calendar.google.com/calendar/u/0/embed?src=fvsl9bgkbnhhkqp5mmqpiurn6c@group.calendar.google.com&ctz=Europe/Rome`

1. Prof. Subhrakanti Dey
   Distributed Machine Learning and Optimization: from ADMM to Federated and multiagent Reinforcement Learning **DEI-1,2**

2. Prof. Giorgio Maria Di Nunzio
   Bayesian Machine Learning **DEI-3,4**

3. Prof. Marco Fabris
   Analysis and Control of Multi-agent Systems **DEI-5,6**

4. Prof. Gianluigi Pillonetto
   Applied Functional Analysis and Machine Learning **DEI-7,8**

5. Prof. Domenico Salvagnin
   Heuristics for Mathematical Optimization **DEI-9**

6. Prof. Gian Antonio Susto
   Elements of Deep Learning **DEI-10,11**

Courses in collaboration with the Doctoral School on "Economics and Finance" University of Verona

for complete Catalogue and class schedule see on
https://www.dse.univr.it/?ent=oi&ava=&cs=1008&id=746&lang=en

1. Prof.ssa Sara Svaluto-Ferro
   Stochastic Processes in Finance **VR-1**

Courses of the Doctoral Program

Elliptic curves and Periods

Cristiana Bertolin[1]

[1] Dipartimento di Matematica "Tullio Levi-Civita"
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Timetable: 24 hrs. First lecture on Thursday October 12th, 2023, 10:30 (dates already fixed, see on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: Algebra, Calculus, Geometry, of the first level degree in mathematics

Examination and grading: Please contact the teacher of the course by e-mail

SSD:

Aim:
To discover the geometrical origin of some transcendental conjectures

Course contents:

- Introduction to algebraic curves
- Elliptic curves over the field of complex numbers
- Geometrical description of the law group on elliptic curve
- The Weierstras $"p"$-function, the Weierstrass ζ- function, and the Serre f_q-function with their doubly periodicity
- The differential forms of the first, the second and the third kind
- The periods of an elliptic curves and their transcendence properties

eventual: Seminar of Prof. Michel Waldschmidt

Bibliography:

- Silverman, J.H. The Arithmetic of Elliptic Curves. Graduate Texts in Mathematics. Vol. 106. Springer-Verlag (1986). ISBN 0-387-96203-4.
- Silverman, J.H. An Introduction to the Theory of Elliptic Curves (PDF) (2006). Summer School on Computational Number Theory and Applications to Cryptography. University of Wyoming.
- Waldschmidt, M. Algebraic independence of periods of elliptic functions. (PDF)(2016). Winter School on modular functions in one and several variables, Goa University, December 2014. From notes by R. Thangadurai.

Introduction to Optimal Transport

Laura Caravenna[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita"*
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Timetable: 24 hrs. First lecture on Monday October 23rd, 2023, 10:30 (dates already fixed, see on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: Some functional analysis functional analysis, and some notions of basic PDEs. The essential required notions will be recalled in the course. Contact me for doubts.

Examination and grading: Oral examination on the content of the course, or presentation of a related research paper or research topic, according to the preferences of each student.

SSD: MAT/05

Aim: With the first part of the course, of about 16 hours, students will learn the main features of the theory of optimal transport. The last part will discuss selected applications to PDEs.

Course contents:

Monge formulation of Optimal Transport Problems and its limits. Kantorovich Formulation of Optimal Transport Problem, existence of optimal plans, Kantorovich- Rubinstein duality for general cost functions. Necessary and sufficient optimality conditions for transport plans, c-cyclical monotonicity, c-concavity, c-transforms in special cases. Optimal transport plans versus optimal transport maps and existence of optimal maps, with a special focus on Brenier's theorem for the quadratic cost function. Connection with the Monge- Amp'ere equation. Wasserstein distances and basic properties. Selected applications to be sorted. Curves in the Wasserstein spaces and relation with the continuity equation, geodesics, Benamou-Brenier dynamical formulation, AC curves in theWasserstein spaces. Introduction to gradient flows in metric spaces and the JKO minimization scheme for some evolution equation.

eventual The selection of which applications we focus on will be fixed during the first week of the course, depending on the interests of the students.

Bibliography:

- L. Ambrosio, E. Brué and D. Semola: Lectures on Optimal Transport, Springer, 2022
- A. Figalli, F. Glaudo: An Invitation to Optimal Transport, Wasserstein Distances and & Gradient Flows, 2022
- F. Santambrogio: Optimal Transport for Applied Mathematicians, Birkhauser (2015)
- C. Villani: Topics in Optimal Transportation, American Mathematical Society (2003)
- L. Ambrosio, N. Gigli, G. Savaré: Gradient Flows in Metric Spaces and in the Space of Probability Measures, Birkhauser (2005)
- F. Santambrogio: Euclidean, Metric, and Wasserstein Gradient Flows: an overview, Bulletin of Mathematical Sciences, available online (2017).

Mixed and stabilised finite element methods

Ramon Codina

Universitat Politècnica de Catalunya
Email: ramon.codina@upc.edu

Timetable: 24 hrs. First lecture on November 6, 2023 (dates already fixed see calendar on: https://dottorato.math.unipd.it/calendar) (6 hours a week for 4 weeks: 3 hours on Monday afternoon; 3 hours on Tuesday morning), Torre Archimede, 2BC30.

Course requirements: Analysis and Linear Algebra of any degree in Mathematics. An introduction to Functional Analysis.

Examination and grading: 50% homeworks, 50% written exam or, alternatively if the student wishes, presentation of a recent research paper (to be agreed with the instructor)

Aim: To introduce several mixed problems in linear partial differential equations and to explain how to approximate them using the finite element method.

Course contents:

- **Introduction to the finite element method - 6 hrs**. Introduction to the theory and practice of conforming Galerkin FEM methods for elliptic equations.
- **Introduction to mixed methods - 2 hrs**. Some examples: Darcy's problem, Stokes' problem, Maxwell's problem, Elasticity, Reissner-Mindlin plates. Finite element approximation. Matrix structure.
- **Finite element approximation of mixed problems: general theory - 2 hrs**. Babuška-Lax-Milgram's theorem. Generalised Céa's lemma. Ladyzhenskaya-Babuška-Brezzi's theorem. de Rham's complex.
- **Stabilised finite element methods - 2 hrs**. Basic concept. The variational multi-scale approach. Application to mixed problems.
- **Darcy's problem - 2 hrs**. Primal form, dual form. Inf-sup conditions and examples of compatible approximations. Stabilised finite element approximation.
- **Stokes' problem - 2 hrs**. Two-field formulation. Three-field formulation. Inf-sup conditions and examples of compatible approximations. Stabilised finite element approximation.
- **Maxwell's problem - 2 hrs**. Kikuchi formulation. An example of inf-sup stable approximation. Augmented stabilised finite element approximation.
- **Elasticity - 2 hrs**. Stress-displacement formulation. Stress-strain-displacement formulation. Inf-sup conditions. Stabilised finite element approximation.
- **Reissner-Mindlin plates - 2 hrs**. Derivation of the problem. Inf-sup conditions and examples of compatible approximations. Stabilised finite element approximation.
- **Introduction to hybrid methods - 2 hrs**. Hybridisation of Poisson's problem. Inf-sup conditions. Stabilised finite element approximation.

Bibliography:

- S.C. Brenner and L.R. Scott.
  The mathematical theory of finite element methods (Springer–Verlag, 1994).
- D. Boffi, F. Brezzi and M. Fortin.
  Mixed Finite Element Methods and Applications (Springer-Verlag, 2013).
- A. Ern and J.-L. Guermond.
  Theory and Practice of Finite Elements (Springer-Verlag, 2004)

Perturbative methods in dynamical systems

Christos Efthymiopoulos[1]

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Timetable: 24 hrs. First lecture on Thursday November 2nd, 2023, 14:30 (dates already fixed, see on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements:

Examination and grading: After the fourth lecture, students will be asked to choose a project to develop using mathematica.
An oral exam will take place at the end of the course, including presentation of the project results and questions on the course material.

SSD: MAT/07

Aim:
This ourse aims to provide a self-contained introduction to the use of the methods of perturbation theory in the study of both regular and chaotic motions in dynamical systems. After a short review of basic definitions pertinent to dynamical systems' theory, the course will present two types of perturbative methods, both of use in the study of the characterization of the solutions and of the local structure of the phase space in the neighborhood of a basic solution of a dynamical system such as a fixed point or a periodic orbit:
i) direct methods (for example, Lindstedt), which aim to construct the solutions directly under the form of a power series in a suitably defined small parameter, and
ii) indirect or normal form methods (for example, the Poincaré normal form), which aim at introducing a transformation of the variables in the form of series in the small parameter.
The presentation will be example-driven, starting from a simple dynamical system representing a nonlinear oscillator with dissipation and external driving. The students will be motivated to make computations in perturbation theory using mathematica and solve some project problems. Some rigorous estimates on the dependence of the size of the perturbative terms as a function of the order of the theory, based on suitable norm definitions in functional spaces related to the dynamical system under study, will be given in the last part of the course.

Lectures plan:

Lectures 1-2: Introduction to dynamical systems Basic definitions. Integrability. Equilibria. The characterization of linear stability. Periodic orbits. Floquet stability. Nonlinear stability. Invariant manifolds in (partially) hyperbolic equilibria or periodic orbits. Chaos, Lyapunov exponents.

Lectures 3-5: Introduction to basic methods of perturbation theory part I: direct methods The example of the Duffing oscillator with dissipation and external driving. Perturbative (series) representation of the solutions in the neighborhood of the stable fixed point using Lindstedt series. Perturbative (series) representation of the invariant manifolds emanating from the unstable fixed point using the parametrization method. Study of the intersections of the stable and unstable manifolds by the Poincaré - Melnikov method.

Lectures 6-7: Introduction to basic methods of perturbation theory part II: indirect methods Linear normal forms and their classification. The Poincaré normal form in the neighborhood of a stable fixed point. The Moser normal form in the neighborhood of an unstable fixed point.

Lectures 8-9: Normal Forms in Hamiltonian dynamical systems Basic review of Hamiltonian mechanics. Symplectic transformations and Poincaré invariants. The method of generating functions. Near-to-identity canonical transformations with the method of Lie series. The normal form of Birkhoff. A review of stability in nearly-integrable Hamiltonian systems. Lecture 10: Rigorous estimates in perturbation theory Norms for polynomial and for real-analytic functions. Divisors. Norm estimates on the basis of iterative lemmas.

Lectures 11-12: The passage to systems with many (or infinitely many) degrees of freedom. Perturbation theory in the example of a system with N non-linearly coupled oscillators, with N large. The limit of 1+1 (space and time) field equations. Perturbative computation of the spectrum of the Schrōdinger equation in a one-dimensional nonlinear oscillator model, and in the perturbed hydrogen atom.

Bibliography:

1. J.A. Sanders, F. Verhulst and J. Murdock, Averaging Methods in Nonlinear Dynamical systems 2d ed., Springer-Verlag, Appl. Math. Sciences vol. 59, 2007, 451 pp

2. S. Wiggins: Introduction to Applied Nonlinear Dynamical Systems and Chaos, Appl. Math. Sciences vol. 2, Springer-Verlag, 2003.

Additional bibliography: Parts of openly available lecture notes by i) G. Benettin, ii) F. Fasso‘, iii) M. Guzzo. Some lecture notes tailored to the needs of the course will be provided by the insegnant.

Introduction to Harmonic Analysis on Semisimple Groups

Francesco Esposito[1]

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Timetable: 24 hrs. First lecture on Tuesday May 7th, 2024, 14:00 (dates already fixed, see on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: The prerequisites are reduced to the minimum:
- standard notions from first and second year courses in analysis
- elementary linear algebra
All other needed concepts will be illustrated in the course.

Examination and grading: Lectures will be complemented with exercise sheets which may be handed in for grading. Alternatively, the exam may consist in an oral examination where the the student is supposed to deliver a lecture on a chosen argument.

SSD: MAT02/03/05

Aim:
Classically, harmonic analysis deals with the expansion of functions of one or more real variables as series or integrals of simple harmonics. A natural setting of the theory is that of locally compact commutative topological groups. Applications range from number theory to differential equations. Noncommutative harmonic analysis on Lie groups is more recent and was initially forged for the needs of invariant theory and quantum mechanics. It studies possibly infinite-dimensional representations of a Lie group, the special functions on the group afforded by the matrix coefficients, and the expansion of functions on the group in terms of these. The course is meant as an introduction to the representation theory and harmonic analysis on semisimple Lie groups. The introduction will succintly survey the commutative theory and some of its applications. Next, theory for compact groups will be dealt in greater detail, up to the Peter-Weyl theorem. Finally, the course will concentrate on infinite-dimensional representations of semisimple Lie groups, representations of its Lie algebra, and characters of these. The basic example will be the group $SL(2, \mathbb{R})$.

Course contents:

1. Introduction
2. Compact groups
3. Unitary representations of locally compact groups
4. Parabolic induction, principal series and characters
5. Representations of the Lie algebra
6. Plancherel formula

7. Invariant eigendistributions
8. Harmonic analysis on the Schwartz space

Bibliography:

1. An introduction to harmonic analysis on semisimple Lie groups" V.S. Varadarajan.
2. "Representation theory of semisimple Lie groups" A.W. Knapp.

Integral operators in Hölder spaces

Massimo Lanza de Cristoforis[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita"*
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Timetable: 24 hrs. First lecture on October 5th, 2023, 16:45 (date already fixed, see calendar on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30

Course requirements: Calculus, basics of real and functional analysis.

Examination and grading: Written/oral questions.

SSD: MAT/05

Aim: Develop basic skills in the theory of integral operators and their applications to potential theory and partial differential equations.

Course contents: Weakly singular integral operators in spaces on measured spaces. A conditions of Ahlfors regularity. Weakly singular potential operators. Conditions of action into generalized Hölder spaces for weakly singular potential operators. Singular integral operators on subsets of $\mathbb{R}^n$ in spaces of Hölder continuous functions. Applications to differential equations.

Bibliography:

1 M. Dalla Riva, M. Lanza de Cristoforis, and PaoloMusolino, Singularly Perturbed Boundary Value Problems. A Functional Analytic Approach, Springer, Cham, 2021.

2 G.B. Folland, Real analysis. Modern techniques and their applications, Second edition. John Wiley & Sons, Inc., New York, 1999.

3 A. E. Gatto. Boundedness on inhomogeneous Lipschitz spaces of fractional integrals singular integrals and hypersingular integrals associated to non-doubling measures. Collect. Math. 60, 1 (2009), 101–114.

4 S. Mikhlin, Multidimensional singular integrals and integral equations. Translated from the Russian by W. J. A. Whyte. Translation edited by I. N. Sneddon Pergamon Press, Oxford-New York-Paris 1965.

5 M. Lanza de Cristoforis, Student hand-outs, Academic Year 2022/23.

Courses of the “Mathematics” area

Integrable Systems of PDEs and their infinite dimensional algebra of symmetries

Alexandr Buryak[1]

[1] Faculty of Mathematics National Research University Higher School of Economics, Moscow
Email: aburyak@hse.ru

Timetable: 24 hrs. First lecture on June 2024,... , Torre Archimede, Room 2BC30.

Aim: The goal of this course is to introduce the students to the notion of integrable system of evolutionary PDEs, study their properties and illustrate several examples and some of their applications in very diverse branches of mathematics.

Classical integrable systems find their origin in analytical mechanics and can be formalized as Hamiltonian systems of ODEs (i.e. Hamiltonian vector fields on a symplectic manifold) possessing sufficiently many (globally defined) conserved quantities in involution to give rise to a Lagrangian torus foliation in their phase space. This geometry makes it possible to solve these systems in a remarkably explicit form. One can generalize the notion of integrable system to a non-Hamiltonian setting, considering dynamical systems (i.e. vector fields on any manifold) with a rich algebra of infinitesimal symmetries.

The modern theory of integrable systems, which is the object of this course, deals with analogous concepts when transported to the context PDEs. Started in the second half of the 20th century mostly with motivations from mathematical physics, it had a resurgence in the last 30 years in light of the discovery of several surprising connections with entirely different branches of mathematics, with the notable example of algebraic geometry (in particular the theory of algebraic curves and the intersection theory of their moduli spaces).

This course will mainly concentrate on the notion of integrable system as a system of partial differential equations possessing an infinite dimensional algebra of infinitesimal symmetries. Our approach will be mostly formal, with next to no prerequisites, from analysis or other fields. Regarding the class of partial differential equations, we will consider systems of evolutionary PDEs with one spatial variable. We will discuss classical examples of such integrable systems, like the Korteweg-de Vries (KdV) equation, and we will prove general theorems on their behaviour and properties, focusing in the second half of the course on the theory of the famous Kadomtsev–Petviashvili (KP) hierarchy. In the final lectures we will prove Okounkov's theorem, which states that the generating function of simple Hurwitz numbers (the number of coverings of the Riemann sphere, of given genus and degree, with simple ramification points) solves the KP hierarchy

Course contents: The course will be structured in five modules, each of approximately 4/5 hours.

Motivations and the KdV equation: The Korteweg-de Vries equation is the main and historically most relevant example of integrable evolutionary PDE. It describes surface waves in shallow water. We will derive it from Euler equations and the continuity equation. It will serve as running example and motivation for the first part of the course.

Algebraic formlism for evolutionary PDEs: Here we start developing the algebraic tools for studying evolutionary PDEs in the language of differential polynomials and local functionals, for one and several independent space variables.

The KP hierarchy: The Kodomtsev-Petviashvili hierarchy of integrable PDEs is a system in one space variable and infinite unknown functions. It is defined through a Lax representation using pseudodifferential operators. Its importance lies in the fact that in contains, as its reductions, an infinite famility of other integrable hierarchies (including KdV).

Tau functions of KP and Sato Grassmannian: We introduce technical tools to study the KP hierarchy and its solutions, namely dressing operators, tau functions, the Fock space and the Sato Grassmannian.

Okounkov theorem on KP and Hurwitz numbers: After a very quick reminder of Hurwitz theory, we prove Okounkov famous result that the generating series of simple Hurwitz numbers (the number of covers of given degree of the Riemann sphere by a Riemann surface of given genus with simple ramification) is a tau function of the KP hierarchy.

Reference for the Course:

Detailed notes of the course, based on the notes [Bur22] for a similar PhD course delivered at the Faculty of Mathematics of the HSE University in Spring 2022, will be made available for participants. Material will be drawn from several research articles, including, but not restricted to [BRZ21, DKJM83, Dic03, Dor78, LL87, Ok00, SG69].

Reference:

Bur22 A. Buryak. Integrable systems as systems of PDEs with an infinite dimensional algebra of symmetries. Notes for a PhD course delivered at the Faculty of Mathematics of the HSE University in Spring 2022.
Available at https://sites.google.com/site/alexandrburyakhomepage/home.

BRZ21 A. Buryak, P. Rossi, D. Zvonkine. Moduli spaces of residueless meromorphic differentials and the KP hierarchy. arXiv:2110.01419.

DKJM83 E. Date, M. Kashiwara, M. Jimbo, T. Miwa. Transformation groups for soliton equations. Nonlin- ear integrable systems – classical theory and quantum theory (Kyoto, 1981), 39–119, World Sci. Publishing, Singapore, 1983.

Dic03 L. A. Dickey. Soliton equations and Hamiltonian systems. Second edition. Advanced Series in Mathematical Physics, 26. World Scientific Publishing Co., Inc., River Edge, NJ, 2003.

Dor78 I. Ya. Dorfman. Formal variational calculus in the algebra of smooth cylindrical functions. Functional Analysis and Its Applications 12 (1978), 101–107.

LL87 L. D. Landau, E. M. Lifshitz. Fluid Mechanics. Second edition. Course of Theoretical Physics, Volume 6, Butterworth-Heinemann Ltd, 1987.

Ok00 A. Okounkov. Toda equations for Hurwitz numbers. Mathematical Research Letters (2000), 7 (4).

SG69 C. H. Su, C. S. Gardner. Korteweg–de Vries equation and generalizations. III. Derivation of the Korteweg–de Vries equation and Burgers equation. Journal of Mathematical Physics 10 (1969), 536–539.

Nonlinear methods for linear equations: the low-regularity theory

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[2] *Università degli Studi di Padova*
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Timetable: 16 hrs First lecture on Tuesday March 5th, 2023, 10:30 (dates already fixed, see on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30..

Course requirements: Basic knowledge of classical functional spaces, without PDE requirements.

Examination and grading: The exam will be oral and tailored on the students' interests.

SSD: MAT/05

Aim: Introduce some classical and modern methods to study regularity properties of solutions to the Laplace equation, focusing on nonvariational techniques based mostly on the maximum principle.

Course contents:

- Introduction and motivations: the importance of the regularity theory for elliptic equations;
- Review of maximum principles and applications;
- Weak-Harnack inequalities via Aleksandrov-Bakel'man-Pucci techniques;
- Harnack inequalities and Hölder a priori estimates;
- The notion of viscosity solution;
- The Bernstein technique to obtain a priori gradient estimates;
- Hölder/Lipschitz regularity estimates via doubling variables: the Ishii-Lions method;
- Lipschitz regularity estimates via doubling variables: the weak Bernstein method.

Bibliography:

1. L. Caffarelli and X. Cabré, Fully nonlinear elliptic equations, American Mathematical Society, Providence, RI, 1995.
2. X. Fernández-Real and X. Ros-Oton, Regularity theory for elliptic PDE, Zurich Lectures Notes in Advanced Mathematics, European Mathematical Society, 2022.
3. L. C. Evans, Partial Differential Equations, American Mathematical Society, Providence, RI, 2010.

4. D. Gilbarg and N.S. Trudinger, Elliptic partial differential equations of second order, Springer-Verlag, Berlin, 2001.
5. Q. Han and F. Lin, Elliptic partial differential equations, second edition, Courant Lecture Notes in Mathematics, American Mathematical Society, Providence, RI, 2011.

Flows of Sobolev vector fields

Elio Marconi[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita"*
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Timetable: 16 hrs.; First lecture on Wednesday November 29th, 2023, 14:30 (dates already fixed, see on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: Sobolev spaces, measure theory, weak formulation of PDEs.

Examination and grading: seminar about a research paper on the subject.

Aim: The aim of this course is to provide an introduction to the theory of the ODE and the associated continuity equation for weakly differentiable vector fields, and to illustrate some research directions in this domain.

Course contents:

1. Preliminaries on ODEs and the continuity equation (PDE) in the classical setting: regularity estimates of the flow and the method of characteristics.
2. Duality ODE-PDE for irregular vector fields: Ambrosio superposition principle.
3. The Eulerian point of view: the uniqueness theorem by Di Perna and Lions.
4. The Lagrangian point of view: the a priori regularity estimate by Crippa and De Lellis.

Bibliography:

L. Ambrosio & G. Crippa: Continuity equations and ODE flows with non-smooth velocity. Proceedings of the Royal Society of Edinburgh: Section A, 144 (2014), 1191–1244.

R.J.DiPerna & P.L.Lions: Ordinary differential equations, transport theory and Sobolev spaces. Invent. Math., 98 (1989), 511–547.

G. Crippa & C. De Lellis: Estimates and regularity results for the DiPerna-Lions flow. J. Reine Angew. Math. 616 (2008), 15–46.

Introduction to Kolmogorov-Arnold-Moser theory

Gabriella Pinzari[1]

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Timetable: 16 hours. First lecture on April 3rd, 2023, 11:00 (dates already fixed, see on: https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: minimal knowledge of Hamiltonian systems (canonical coordinates; symplectic transformations; Liouville-Arnold Theorem)

Examination and grading: 45'-1 hr seminar by the candidate

SSD: MAT/07 and MAT/05.

Aim: to present the ideas of Kolmogorov-Arnold-Moser theory

Course contents:

- Recap on Hamiltonian systems; canonical coordinates; canonical transformations; Liouville-Arnold Theorem. Focus on holomorphic Hamiltonians; Cauchy inequalities; decay of Fourier coefficients of holomorphic periodic functions.
- Diophantine inequalities.
- Perturbative schemes: norms; Normal Form Theory; KAM algorithm and convergence; measure of the Kolmogorov set.
- Generalization to properly—degenerate hamiltonians;

  if there is time enough

- Lower-dimensional quasi-periodic motions (whiskered tori).
- If there is time: generalization to vector-fields; some application

References:

1) V.I. Arnold. Small denominators and problems of stability of motion in classical and celestial mechanics. Russian Math. Surveys, 18(6):85–191, 1963.

2) Benettin G., Galgani L., Giorgilli A., Strelcyn, J. M. (1984), A proof of Kolmogorov's theorem on invariant tori using canonical transformations defined by the Lie method. Nuovo Cimento B 79(11):201–223.

3) L. Chierchia, A. N. Kolmogorov's 1954 paper on nearly-integrable Hamiltonian systems. A comment on: "On conservation of conditionally periodic motions for a small change in Hamilton's function" [Dokl. Akad. Nauk SSSR (N.S.) 98 (1954), 527–530; MR0068687], Regul. Chaotic Dyn., 13 (2008), no. 2, 130–139.

4) L. Chierchia and G. Pinzari. Properly–degenerate KAM theory (following V.I. Arnold). Discrete Contin. Dyn. Syst. Ser. S, 3(4):545–578, 2010.

5) A. N. Kolmogorov, On the conservation of conditionally periodic motions under small perturbation of the Hamiltonian, Dokl. Akad. Nauk. SSR 98 (1954), 527-530

6) J. K. Moser, On invariant curves of area-preserving mappings of an annulus, Nach. Akad. Wiss. Göttingen, Math. Phys. Kl. II 1 (1962), 1-20

7) J. Pp̈schel. Nekhoroshev estimates for quasi-convex Hamiltonian systems. Math. Z., 213(2): 187–216, 1993.

Monogenic functions and basic elliptic equations of mathematical physics

Prof. Sergiy Plaksa

[1] *Institute of Mathematics of the National Academy of Sciences of Ukraine*
Email: plaksa62@gmail.com

Timetable: 12 hrs. First lecture on November 6th, 2023, 12:30 (dates already fixed see on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: Basic notions on holomorphic functions in the complex plane and of elementary functional analysis.

Examination and grading: exam

Aim: studying properties of monogenic functions of a hypercomplex variable and their applications for constructing solutions of equations of mathematical physics

Course contents:

Determination of hypercomplex algebras associated with the three-dimensional Laplace equation and the biharmonic equation. Commutative harmonic algebras. A biharmonic algebra.

Differentiation in Banach algebras. The Lorch derivative and the Gâteaux derivative. The principal extension of analytic functions of a complex variable into a commutative Banach algebra.

Monogenic functions in a three-dimensional commutative algebra: constructive description by means of analytic functions of a complex variable. Analogues of Cauchy–Riemann conditions.

Integral theorems in a three-dimensional commutative algebra. Gauss–Ostrogradsky formula and Cauchy theorem for a surface integral. Stokes formula and Cauchy theorem for a curvilinear integral. Morera theorem. Cauchy integral formula. Power series. Equivalent definitions of monogenic functions

and possibly also:

Monogenic functions in infinite-dimensional vector spaces associated with the three-dimensional Laplace equation.

Bibliography:

Main:

1. Hille, E., Phillips, R.S.: Functional analysis and semi-groups. Amer. Math. Soc., Providence, R.I. (1957)

2. Lorch, E.R.: The theory of analytic function in normed abelin vector rings. Trans. Amer. Math. Soc. **54**, 414–425 (1943)

3. Blum, E.K.: A theory of analytic functions in Banach algebras. Trans. Amer. Math. Soc. **78**, 343–370 (1955)

4. Plaksa, S.A.: Commutative algebras associated with classic equations of mathematical physics. In: Rogosin, S.V., Koroleva, A.A. (eds) Advances in Applied Analysis, Trends in Mathematics, pp. 177–223. Springer, Basel (2012)

5. Plaksa, S.A., Gryshchuk, S.V., Shpakivskyi, V.S.: Commutative algebras of monogenic functions associated with classic equations of mathematical physic. In: Agranovsky, M., Ben-Artzi, M., Galloway, G., Karp, L., Reich, S., Shoikhet, D., Weinstein, G., Zalcman, L. (eds.) Complex Analysis and Dynamical Systems IV, pp. 245–258. Contemporary Mathematics, **553**, Amer. Math. Soc., Providence, RI (2011)

6. Plaksa, S.A., Shpakovskii, V.S.: Constructive description of monogenic functions in a harmonic algebra of the third rank. Ukr. Math. J., **62** (8), 1251–1266 (2011)

7. Grishchuk, S.V., Plaksa, S.A.: Monogenic functions in a biharmonic algebra. Ukr. Math. J., **61** (12), 1865–1876 (2009)

8. Grishchuk, S.V., Plaksa, S.A.: Basic properties of monogenic functions in a biharmonic plane. In: Agranovsky, M., Ben-Artzi, M., Galloway, G., Karp, L., Maz'ya, V., Reich, S., Shoikhet, D., Weinstein, G., Zalcman, L. (eds.) Complex Analysis and Dynamical Systems V, pp. 127–134. Contemporary Mathematics, **591**, Amer. Math. Soc., Providence, RI (2013)

Additional:

1. Shpakivskyi, V.S.: Constructive description of monogenic functions in a finite-dimensional commutative associative algebra. Adv. Pure Appl. Math., **7** (1), 63—75 (2016)

2. Plaksa, S.A., Shpakivskyi, V.S.: Cauchy theorem for a surface integral in commutative algebras. Complex Variables and Elliptic Equations. **59** (1), 110–119 (2014)

3. Plaksa, S.A., Shpakovskii, V.S.: On the logarithmic residues of monogenic functions in a three-dimensional harmonic algebra with two-dimensional radical. Ukr. Math. J., **65** (7), 1079–1086 (2013)

4. Tkachuk, M.V., Plaksa, S.A.: An analog of the Menchov–Trokhimchuk theorem for monogenic functions in a three-dimensional commutative algebra. Ukr. Math. J. **73** (8), 1299–1308 (2022)

5. Plaksa, S.A., Shpakivskyi, V.S.: Limiting values of the Cauchy type integral in a three-dimensional harmonic algebra. Eurasian Math. J.. **3** (2), 120–128 (2012)

6. Gryshchuk, S.V., Plaksa, S.A.: Monogenic functions in the biharmonic boundary value problem. Mathematical Methods in the Applied Sciences. **39** (11), 2939–2952 (2016)

Harmonic analysis on nilpotent groups

Fulvio Ricci[1]

[1]Scuola Normale Superiore di Pisa, Piazza dei Cavalieri 7, 56126 Pisa
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Timetable: 16 hrs. First lecture on January 29, 2024, 15:30 (dates already fixed, see on https://dottorato.math.unipd.it/calendar/), Torre Archimede, Room 2BC30.

Course requirements: Functional Analysis, Fourier series and Fourier transform in $\mathbb{R}^n$

Examination and grading: Seminar and interview

Aim: Fundamental notions and properties of analysis on Lie groups. Analysis of sublaplacians on nilpotent groups.

Course contents:
Vector fields on manifolds and their flows. Lie groups and Lie algebras. Convolution on Lie groups. Dilations and homogeneous groups. Hypoelliptic operators on manifolds and Lie groups. Sub-elliptic estimates on homogeneous groups. Hörmander's systems of vector fields. Hypoellipticity of sublaplacians: preliminaries. Fundamental solutions and subelliptic estimates.

Bibliography: Notes to be distributed during the course.

Introduction to Hessenberg Varieties

Eric Sommers

Department of Mathematics and Statistics, University of Massachusetts Amherst, Amherst, MA 01003, USA
Email address: esommers@umass.edu

Timetable: 12 hours. First lecture on Monday, October 9, 2023, 14:00 (dates already fixed, see on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course Requirements: A year of abstract algebra, some exposure to algebraic varieties.

Examination and Grading: Students will give a talk on an aspect of the subject using a research article as a basis for the talk.

SSD: MAT/02 and MAT/03

Aim: Hessenberg varieties are projective subvarieties of the flag variety, which appear as fibers of proper maps to affine subvarieties of the Lie algebra. They are generalizations of the Springer varieties that play a central role in the representation theory of finite Chevalley groups, as well as the infinite-dimensional representation theory of real Lie algebras. Hessenberg varieties are smooth in the regular semisimple case and their cohomology carries a still mysterious representation of the Weyl group, but it is the geometry of the regular nilpotent Hessenberg varieties that play a more direct role in the singularities of the nilpotent cone and their local intersection cohomology groups. Hessenberg varieties are also interesting due to their connections to algebraic combinatorics via chromatic quasisymmetric functions. The aim of this course is to supply some of the background in Lie theory and combinatorics to read the current literature on Hessenberg varieties and their applications to representation theory and algebraic combinatorics.

Course contents:

1. Roots systems, Weyl groups, and related invariant theory.
2. The symmetric group setting. Symmetric functions.
3. The geometry of the flag variety. Bruhat decomposition and Schubert varieties. Bruhat order.
4. Springer varieties and statement of the Springer correspondence. Examples for the symmetric group and in rank 2.
5. Hessenberg varieties and their geometry. Vanishing of cohomology in odd degree. Poincaré polynomials.
6. Connections to combinatorics: chromatic quasisymmetric functions, the dot-action representation, and the Stanley-Stembridge conjecture.
7. Computational methods. Example: computing the dimensions of irreducible representations of the Weyl groups via the Springer correspondence.

Bibliography

[1] Walter Borho and Robert MacPherson. *Partial resolutions of nilpotent varieties.* In *Analysis and topology on singular spaces, II, III (Luminy, 1981)*, volume 101 of *Astérisque*, pages 23–74. Soc. Math. France, Paris, 1983.

[2] Patrick Brosnan and Timothy Y. Chow. Unit interval orders and the dot action on the cohomology of regular semisimple Hessenberg varieties. *Adv. Math.*, 329:955–1001, 2018.

[3] Roger W. Carter. *Finite groups of Lie type*. Wiley Classics Library. John Wiley & Sons, Ltd., Chichester, 1993.

[4] Paola Cellini and Paolo Papi. ad-nilpotent ideals of a Borel subalgebra. *J. Algebra*, 225(1):130–141, 2000.

[5] Corrado De Concini, George Lusztig, and Claudio Procesi. Homology of the zero-set of a nilpotent vector field on a flag manifold. *J. Amer. Math. Soc.*, 1(1):15–34, 1988.

[6] Jens Carsten Jantzen. Nilpotent orbits in representation theory. In *Lie theory*, volume 228 of *Progr. Math.*, pages 1–211. Birkhäuser Boston, Boston, MA, 2004.

[7] Filippo de Mari, Claudio Procesi, and Mark A. Shayman. Hessenberg varieties *Trans. Amer. Math. Soc.*, 332(2):529–534, 1992.

[8] Marth Precup and Eric Sommers. Perverse sheaves, nilpotent Hessenberg varieties, and the modular law Accepted in *Pure and Applied Mathematics Quarterly*, arXiv:2201.13346. 27 pages.

[9] John Shareshian and Michelle L. Wachs. Chromatic quasisymmetric functions. *Adv. Math.*, 295:497–551, 2016.

[10] Eric Sommers and Julianna Tymoczko. Exponents for B-stable ideals. *Trans. Amer. Math. Soc.*, 358(8):3493–3509, 2006.

Courses of the “Computational Mathematics” area

Linear and non-linear formulations for Combinatorial Optimization

Manuel Francesco Aprile[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova*
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Timetable: 16 hrs. First lecture on Tuesday April 9th, 2024, 10:30 (dates already fixed, see on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: None. Familiarity with basic concepts in geometry of polyhedra and discrete optimization will be helpful.

Examination and grading: Seminar.

SSD: MAT/09

Aim: Combinatorial optimization problems are ubiquitous in many fields, spanning from logistics and artificial intelligence to computational biology. The crux of solving such problems lies in efficiently selecting an optimal solution from a finite but very large set of objects. This course aims to equip students with a toolkit of techniques to construct effective mathematical programming formulations for these combinatorial problems. Special emphasis will be placed on exploring "hot" topics that have undergone substantial advancements in recent years, such as extended formulations. Moreover, the course will delve into intriguing connections between combinatorial optimization and other fields such as computational complexity.

Course contents:

- Classical linear formulations from the literature.
- Extended formulations: Yannakakis' theorem, connection with communication protocols.
- Positive and negative results on the existence of small extended formulations.
- Semidefinite formulations.
- Introduction to hierarchies: from Sherali-Adams to Sum of Squares.

Bibliography:

Relevant material and research papers will be provided during the course. Most of the topics covered can be found in: Conforti, Cornuéjols, Zambelli (2014).
Integer programming. Springer International Publishing.

Kernels and Partitions of Regular Domains and Compact Sets

Martin Buhmann[1]

[1] *Justus-Liebig-Universität Giessen, Germany*
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Timetable: 13 hrs. First lecture on November 21, 2023, 12:30 (date already fixed, see calendar on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: Recommended: Numerical Analysis I and Analysis I and II or Approximation Theory

Examination and grading:

Aim: Understanding the particulars of the approximtion theory of many variables, namely kernel-based methods for regular and scattered data, interpolation *vs.* quasi-interpolation including polynomial reproduction, positive and strictly positive interpolation matrices and kernel functions, partitions of compact spaces and polynomial precision, topology of regular domains (mostly conic sections) and compact metric spaces.

Course contents: 13 hours, one introduction, four parts with three hours each

- 0. Part: Introduction
- I. Part: Basics on Kernels and Quasi-Interpolation.
  1. Interpolation in several variables by polynomials and otherwise.
  2. Kernel functions for interpolation; radial basis functions and main examples. Complete and multiple monotonicity.
  3. Concept of Quasi-Interpolation and compariion with interpolation.
- II. Part: Positive Definite Functions on Regular Domains, especially Spheres.
  1. Positive definiteness, strictly and semi positive definiteness of functions and interpolation matrices.
  2. Positive definite functions on spheres in many dimensions.
  3. Positive definite functions on other conic sections and on simplices.
- III. Part: Polynomial Reproduction with Kernels.
  1. Concept of polynomial reproduction especially with quasi-interpolation.
  2. Relation of this to approximation orders; examples.
  3. Conditions for polynomial precision, examples especially with respect to partitions of unity.
- IV. Part: Partitions of Compact Sets.
  1. Quadrature methods.
  2. Partitions of compact sets for cubature.
  3. Generalisations to compact metric spaces.

Stochastic and mean field optimal control

Alekos Cecchin[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova*
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Timetable: 16 hours; first lecture on November 6th, 2023, 10:30 (date already fixed, see calendar on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: Basic knowledge of stochastic calculus (Brownian motion, stochastic differential equations, filtrations, martingales, ...), as presented, for example, in the course on stochastic analysis of the master degree. Some concepts will be recalled during the course.

Examination and grading: Oral presentation of a research paper related to the topics covered in the course, based on student's interest.

SSD: MAT/06 and MAT/05.

Aim: Introduce the classical tools to analyze stochastic optimal control problems (dynamic programming, viscosity solutions, backward SDEs, relaxed controls) and then use these methods to study the recent theory of mean field control problems.

Course contents: Introduction to the classical theory of stochastic control problems with some motivating example. These problems consist in minimizing a cost in which the state variable is given by a controlled stochastic differential equation driven by a Brownian motion. The course will then cover the following topics:

- Equivalence of weak and strong formulation, existence of optimal relaxed controls via weak convergence methods;
- Dynamic programming principle: value function, Hamilton-Jacobi-Bellman equation, verification theorem, viscosity solutions of second order PDEs;
- Backward stochastic differential equations: representation of the value function for the weak formulation, necessary conditions for optimality given by the stochastic Pontryagin's maximum principle, relation with dynamic programming equation.

In the second part, we introduce the recent thoery of mean field control problems, also called optimal control of McKean-Vlasov dynamics. In these problems, the cost and the coefficients of the state equation depend also on the law of the state process, and can be reformulated as optimal control of the Fokker-Planck equation. We show how to extend the results established for the classical problem to the mean field case. In particular, the Hamilton-Jacobi-Bellman equation is stated in the Wasserstein space of probability measures, which is infinite dimensional. Thus we introduce a notion of differentiability of functions defined on the Wasserstein space.

A renormalisation group approach to log-Sobolev inequalities

Alberto Chiarini[1], Giovanni Conforti[2]

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[2] *Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova*
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Timetable: 16 hrs. First lecture on Monday March 4, 2024, 14:30 (dates already fixed, see on https://dottorato.math.unipd.it/calendar/), Torre Archimede, Room 2BC30

Course requirements: Elements of Stochastic Analysis and Partial Differential Equations;

Examination and grading: Oral exam;

SSD:

Aim: In this course we survey a novel renormalization group approach to log-Sobolev inequalities and to related properties of Glauber-Langevin dynamics. Interestingly, this approach is related to more or less recent theories, such as Eldan's stochastic localization, optimal transport and stochastic control. In particular, the course will elucidate the link between the Polchinski flow and Hamilton-Jacobi-Bellman equations. The course is mainly based on the survey article [3] by Bauerschmidt, Bodineau and Dagallier.

Course contents:
The course is divided in 8 Lectures of 2 hours each to be spread in two/three weeks. The rough plan of the lectures is the following:

1. Introduction to Glauber-Langevin dynamics and convergence to equilibrium.
2. Log-Sobolev inequality, Hypercontractivty and Bakry-Émery Theorem.
3. Renormalized potential and the Polchinski equation.
4. Log-Sobolev inequality via a multiscale Bakry-Émery method.
5. Pathwise Polchinski flow and stochastic localisation perspective.
6. Stochastic control and transport perspective on the Polchinski flow.
7. Application to a spin glass model.
8. Application to entropic optimal transport.

Bibliography:

[1] Dominique Bakry, Ivan Gentil, and Michel Ledoux. Analysis and geometry of Markov diffusion operators, volume 103. Springer, 2014.

[2] Roland Bauerschmidt and Thierry Bodineau. Log-sobolev inequality for the continuum sine-gordon model. Communications on Pure and Applied Mathematics, 74(10):2064–2113, 2021.

Interface of Finance, Operations and Risk Management

Andrea Roncoroni[1]

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Timetable: 16 hrs. First lecture on October 5th, 2023, 12:30, (date already fixed, see calendar on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: Introductory financial derivatives and arbitrage pricing theory

Examination and grading: Project work

SSD:

Aim: This course offers an introduction to the Interfaces of Finance, Operations, and Risk Management (iFORM) with a focus on Integrated Risk Management (IRM). This is a relatively new research area dealing with timely, complex, and boundary-spanning issues in a variety of commercial and industrial setups. iFORM research work addresses ways to better integrate physical, financial, and informational flows by combining the operational choices of the firm with its financial decisions and merging information flows between the firm and its customers and suppliers with informational flows between the firm and its investors. We highlight the main standing, emerging, and forthcoming contributions in IRM.

Course contents:

1. iFORM and IRM (3h)
   - A closed-loop view of operations-finance interfaces.
   - A framework for integrated risk management.
   - Risk identification, integration conditions, and operational vs. financial flexibility.
   - IRM optimization: relationship analysis and approach choice.
2. Static hedging (3h)
   - Contingent claim design: linear, piecewise linear, parametric, custom.
   - Business exposure. Examples: Primary commodity production, Stochastic clearance price model, Generalized newsvendor model, Multinational production capacity allocation.
   - Direct hedging, cross hedging, and combined hedging.
   - Mathematical formulations of optimal custom static hedging. Operational handling integration.
3. Sample models (4h)
   - Claim design models: Brennan-Solanki (1981), Carr-Madan (2001).
   - Static hedging models with nonclaimable risk: McKinnon (1967), Rolfo (1980), Brown-Toft (2002).

- IRM models: Ritchken-Tapiero (1986), Chowdhry-Howe (1999), Gaur-Seshadri (2005), Ding-Dong-Kouvelis (2007), Chen et al. (2015).
- The simplest IRM model with combined custom hedging.

4. Combined custom hedging (6h)
   - Problem statement and solution existence and uniqueness. Examples.
   - The design integral equation system.
   - Corporate value assessment.
   - Newsvendor IRM with combined custom hedging: solution and analysis.

Bibliography:

1. Birge, J.R. (2015). OM Forum-Operations and Finance Interactions. Manufacturing & Service Operations Management 17(1), 4-15.
2. Brennan, M.J., Solanki, R. (1981). Optimal Portfolio Insurance. Journal of Financial and Quantitative Analysis 16(3), 279-300.
3. Brown, G.W. and Toft, K.B. (2002). How Firms Should Hedge. Review of Financial Studies 14, 1283-1324.
4. Chen, L., Li, S., Wang, L. (2014). Capacity Planning with Financial and Operational Hedging in Low-Cost Countries. Production and Operations Management 23, 1495-1510.
5. Chowdhry, B., Howe, J. T. B. (1999). Corporate Risk Management for Multinational Corporations: Financial and Operational Hedging Policies. European Finance Review 2, 229-246.
6. Ding, Q., Dong, L., Kouvelis, P. (2007). On the Integration of Production and Financial Hedging Decisions in Global Markets. Operations Research 55, 470-489.
7. Gaur, V., Seshadri, S. (2005). Hedging Inventory Risk Through Market Instruments. Manufacturing & Service Operations Management 7(2), 103-120.
8. Guiotto, P., Roncoroni, A. (2022). Combined Custom Hedging. Operations Research 70(1), 38-54.
9. Roncoroni, A. (2022): Lecture notes.
10. McKinnon, R. (1967). Futures Markets, Buffer Stocks, and Income Stability for Primary Producers. Journal of Political Economy 75, 844-861.
11. Ritchken, P.H., Tapiero, C.S. (1986). Contingent Claims Contracting for Purchasing Decisions in Inventory Management. Operations Research 34(6), 864-870.
12. Rolfo, J. (1980). Optimal Hedging under Price and Quantity Uncertainty: The Case of a Cocoa Producer. Journal of Political Economy 88, 100-116.
13. Zhao, L., Huchzermeier, A. (2015). Operations–Finance Interface Models: A Literature Review and Framework. European Journal of Operational Research 244, 905-917.
14. Zhao, L., Huchzermeier, A. (2017). Integrated Operational and Financial Hedging with Capacity Reshoring. European Journal of Operational Research 260, 557-570.

Hawkes processes: from theory to (financial) practice

Prof. Simone Scotti[1]

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Email: simone.scotti@unipi.it

Timetable: 16 hrs. First lecture on April 4th, 2024, 10:30 (dates already fixed, see on - https://dottorato.math.unipd.it/calendar/), Torre Archimede, Room 2BC30

Course requirements: Probability and Stochastic Calculus (basic)

Examination and grading: Oral presentation (seminar) of a research paper related to the topic of the course.

SSD: MAT/06, SECS-S/06

Aim: Events that are observed over time naturally show clustering phenomena: an earthquake happening increases the probability of so-called aftershocks, namely minor readjustments along the portion of a fault that slipped during the mainshock. Similar clustering patterns are observed, e.g., in criminology, when dealing with certain types of crime data, such as burglary and gang violence, due to crime specific patterns of criminal behaviour. As a last example, in financial markets, selling a huge amount of a stock could induce successive selling activity with relative jumps clustering in the price or, on a larger scale, the collapse of an investment bank could create a financial turmoil and shock-waves through the world's financial centres.

Hawkes processes were introduced for the first time by A. Hawkes in 1971 to model the occurrence of seismic events. They are stochastic point processes particularly suitable to describe these self-exciting phenomena, in which the occurrence of an event increases the probability of a future arrival of another event. For this reason, they are also known under the name of "self-exciting point processes" and they have been applied in numerous fields throughout science, computer science, engineering, and human sciences.

More precisely, on a filtered probability space, denoting by $\mathbb{N}$ the counting process, i.e., $\mathbb{N}_t$ is the random variable counting the number of relevant events on the time interval $(0;t)$, we define the conditional intensity

$$\lambda_t := \lim_{\Delta\to 0} \mathbb{E}\left[N_{t+\Delta} - N_t \mathcal{F}_\sqcup\right]$$

where $\mathcal{F}_\sqcup$ denotes the information available up to time t. The original self-exciting process is defined via an intensity that is a function of past events:

$$\lambda_t := \mu + \int_0^t \gamma(t-u)dN_u = \mu + \sum_{T_i<t} \gamma(t-T_i)$$

where $0 < T_1 < T_2 < \cdots < T_n < \dots$ are the time instants at which the relevant events occur, μ is a base level for the intensity and $\gamma(u) \geq, u > 0, u > 0$ is the exciting kernel. So, each event makes the intensity jump upwardly and then, between successive events, it decays according to the function γ. A typical kernel is $\gamma(u) = \alpha\beta e^{-\beta u}$, for $\alpha, \beta > 0$ and in this case the pair (N, λ) is a Markov process. The aim of this PhD course at the University of Padova is to provide a rigorous Mathematical introduction to point processes and a solid basis on the necessary

Stochastic Calculus tools needed to handle models based on Hawkes processes. Markovianity, which represents a desired theoretical property of typical stochastic models, will also be discussed, as opposed to modeling with memory, hence leading to non-Markovian settings. This dilemma Markov/non-Markov opens the door to technical challenges, some of which are still the object of an active debate and a rich stream of research. A Statistical overview, with many Mathematical hints, will be also given on simulation of Hawkes processes and estimation of Hawkes-based models. The connection with branching processes, alpha stable processes and Volterra processes (with and without jumps) will also be discussed. To conclude, recent applications to Finance will be covered, together with possible other uses, at the frontier of research, in different research fields.

Course contents: the 16 hours course will cover the following topics, in 8 lectures:

1 & 2 Probability and Statistics background: counting, Poisson and Cox processes; the definition and the role of the intensity and examples from Economics and Finance. Basic Statistics: graphical analysis, Kolmogorov Smirnov test.

3 & 4 Hawkes Processes: definition, properties, the self exciting feature and dynamic contagion. Markovian and non-Markovian Hawkes processes: related challenges.

5 Simulation: the intensity-based and the cluster-based approaches. Estimation: parametric or non parametric?

6 Possible extensions: branching processes, alpha stable processes and Volterra processes with jumps.

7 Application to Finance: stochastic volatility nowadays, memory or not? Jumps cluster analysis.

8 Other applications: insurance and market microstructure

References:

[1] Bernis, G., Scotti, S. Clustering effects via Hawkes processes. In: From probability to finance, 2020, Springer, pp. 145–181.

[2] Dassios, A., Zhao, H. A dynamic contagion process. Advances in Applied Probability, 43(3), 2011, pp. 814–846.

[3] Hawkes, A. Point Spectra of Some Mutually Exciting Point Processes. Journal of the Royal Statistical Society: Series B (Methodological), 33(3), 1971, pp. 438–443.

[4] Hawkes, A. Hawkes processes and their applications to finance: a review. Quantitative Finance, 18(2), 2018, pp. 193–198.

[5] Jaisson, T., Rosenbaum, M. Limit theorems for nearly unstable Hawkes processes. Annals of Applied Probability, 25(2), 2015, pp. 600–631.

[6] Jang, J., Oh, R. A review on Poisson, Cox, Hawkes, shot-noise Poisson and dynamic contagion process and their compound processes. Annals of actuarial Science, 15(3), 2021,pp. 623–644.

[7] Jiao, Y., Ma, C., Scotti, S., Zhou, C. The Alpha-Heston stochastic volatility model. Mathematical finance, 31(3), 2021, pp. 943–978.

[8] Raffaelli, I., Scotti, S., Toscano, G. Hawkes-Driven Stochastic Volatility Models: Goodness-of-Fit Testing of Alternative Intensity Specifications with S&P500 Data, preprint 2022.

Soft Skills

Doctoral Program in Mathematical Sciences

a.a. 2023/2024

SOFT SKILLS

MATHS INFORMATION: RETRIEVING, MANAGING, EVALUATING, PUBLISHING

(Information Literacy for Math Phd Students)

Abstract: This course deals with the bibliographic databases and the resources provided by the University of Padova; citation databases and metrics for research evaluation; open access publishing and the submission of PhD theses and research data in UniPd institutional repositories.

Course Program: 5 Seminars (missing details will be communicated soon)

1. **The advanced services of "*Math Library*" and of the "*University Library System*" (hrs 01:30)**
   Digital Library and GalileoDiscovery: the use of the physical and electronic resources of the University libraries, in particular e-books and e-journals.
   The advanced services of the library: remote access, document delivery and interlibrary loan.

   The Seminar is aimed at all Doctoral Students

2. **The Bibliographic research and advanced features of MathSciNet (hrs 02:00)**
   MathSciNet: research strategies with examples.
   Advanced features: full-text retrieval in MSN, references export, the bibliometric index MCQ.
   A view of the multidisciplinary databases Web of Science and Scopus.

   The Seminar is aimed at all Doctoral Students

3. **Scientific communication and open access (hrs 02:00)**
   Traditional publishing market and open access publishing, the way to Open Access and CC licences.
   Unipd policy and regulations for OA, the institutional and disciplinary repositories.

   The Seminar is aimed at all Doctoral Students

4. **A reference manager for the management of bibliographies: Zotero (hrs 01:00)**
5. **Where to publish and the evaluation of academic research (hrs 01:30)**

Language: The Course will be held in Italian or in English according to the participants

Timetable:

Seminar 1	November 17th, 2023 – 10:30-12:30, room 2AB45
Seminar 2	December 5th, 2023 – 10:00-12:30, room 2BC30
Seminar 3	May 6th, 2024 – 09:00-11:00, Meeting Room 7B1
Seminar 4	Scheduled in May (more details soon)
Seminar 5	

Doctoral Program in Mathematical Sciences

a.a. 2023/2024

SOFT SKILLS

Introduction to the use of “Mathematica” in Mathematics and Science

Prof. Francesco Fassò

Timetable: 12 hours. First lecture on Friday October 6th, 2023, at 12:30, Room 1C150.

Pratical infos:

All PhD students may have a license of Mathematica (provided by the campus Unipd license): https://asit.unipd.it/servizi/contratti-software-licenze/mathematica, installed on a personal machine

Course content:

The aim of this soft skill course is to provide the basic competences to use the symbolic, numerical and graphical capabilities of Mathematica, with a focus on the needs of mathematicians and scientists. The course is a hands-on course, which takes place entirely in a computer lab. A first part of the course, for a total of about 5-6 hours in 2-3 sessions, will assume no previous knowledge of Mathematica and provides the capabilities to use it at a basic ("everyday") level.

For interested students, a second and more advanced part of the course will provide an introduction to (functional) programming with Mathematica (with an eye on the needs of a mathematician, of course).

Doctoral Program in Mathematical Sciences

a.a. 2023/2024

SOFT SKILLS

Our experience in writing a successful post doctoral application

(Proff. Annalisa Massaccesi and Elio Marconi)

Timetable: 2 hours. Thursday, May 16th, from 10.00 to (at most) 12.00, room 2BC30, in dual form (in presence and via Zoom)

Pratical infos: registration list at https://servizi-esterno.math.unipd.it/userlist/lista/view?id=89

Courses in collaboration with the Doctoral School on “Information Engineering”

for complete Catalogue and class schedule see on

https://phd.dei.unipd.it/course-catalogues/

Please check regularly the website of the Doctoral Course

Calendar of activities on

`https://calendar.google.com/calendar/u/0/embed?src=fvsl9bgkbnhhkqp5mmqpiurn6c@group.calendar.google.com&ctz=Europe/Rome`

Distributed Machine Learning and Optimization: from ADMM to Federated and multiegent Reinforcement Learning

Prof. Subhrakanti Dey[1]

[1] *Signals and Systems, Uppsala University, Sweden*
Email: Subhra.Dey@signal.uu.se

Timetable: 20 hrs (see Class Schedule on https://phd.dei.unipd.it/course-catalogues/)

Course requirements: Advanced calculus, and probability theory and random processes.

Examination and grading: A project A project assignment for students in groups of 2 requiring about 20 hours of work.

SSD:

Aim: The aim of this course is to introduce postgraduate students to the topical area of Distributed Machine Learning and Optimization. As we enter the era of Big Data, engineers and computer scientists face the unenviable task of dealing with massive amounts of data to analyse and run their algorithms on. Often such data reside in many different computing nodes which communicate over a network, and the availability and processing of the entire data set at one central place is simply infeasible. One needs to thus implement distributed optimization techniques with communicationefficient message passing amongst the computing nodes. The objective remains to achieve a solution that can be as close as possible to the solution to the centralized optimization problem. In this course, we will start with distributed optimization algorithms such as the Alternating Direction Method of Multipliers (ADMM), and discuss its applications to both convex and non-convex problems. We will then explore distributed statistical machine learning methods, such as Federated Learning as well as consensus based fully distributed algorithms. The final topic will be based on multi-agent reinforcement learning and its applications to safe (constrained) data-driven (model free) control in a multi-agent setting. This course will provide a glimpse into this fascinating subject, and will be of relevance to graduate students in Electrical, Mechanical and Computer Engineering, Computer Science students, as well as graduate students in Applied Mathematics and Statistics, along with students dealing with large data sets and machine learning applications to Bioinformatics.

Course contents:

- Lectures 1-4: Precursors to distributed optimization algorithms: parallelization and decomposition of optimization algorithms (dual de- composition, proximal minimization algorithms, augmented Lagrangian and method of multipliers), The Alternating Direction Method of Multipliers (ADMM): (Algorithm, convergence, optimality conditions, applications to machine learning problems)
- Lectures 5-7: Applications of distributed optimization to distributed machine learning, Federated Learning, fully distributed, consensus based methods under communication constraints

- Lectures 8-10: Multiagent reinforcement learning, safe (constrained) reinforcement learning and its applications to data-driven multiagent control, inverse reinforcement learning

References:

1. S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers, Foundations and Trends in Machine Learning, 3(1):1122, 2011.
2. Dimitri Bertsekas and John N. Tsitsiklis, Parallel and Distributed Computation: Numerical Methods, Athena Scientific, 1997.
3. S. Boyd and L. Vandenverghe, Convex Optimization, Cambridge University Press.
4. R. Sutton and A. G. Barto, Reinforcement Learning, 2nd Edition, Bradford Books.
5. D. Bertsekas, Rollout, Policy Iteration and Distributed Reinforcement Learning, Athena Scientific, 2020.

Relevant recent papers will be referred to and distributed during the lectures.

Bayesian Machine Learning

Giorgio Maria Di Nunzio[1]

[1] *Department of Information Engineering*
Email: dinunzio@dei.unipd.it

Timetable: 20 hrs (see Class Schedule on https://phd.dei.unipd.it/course-catalogues/)

Course requirements: Basics of Probability Theory. Basics of R Programming.

Examination and grading: Homework assignments and final project.

SSD: Information Engineering

Aim: The course will introduce fundamental topics in Bayesian reasoning and how they apply to machine learning problems. In this course, we will present pros and cons of Bayesian approaches and we will develop a graphical tool to analyse the assumptions of these approaches in classical machine learning problems such as classification and regression.

Course contents:

1. Introduction of classical machine learning problems.
   - Mathematical framework
   - Supervised and unsupervised learning
2. Bayesian decision theory
   - Two-category classification
   - Minimum-error-rate classification
   - Bayes decision theory
   - Decision surfaces
3. Estimation
   - Maximum Likelihood Estimation
   - Expectation Maximization
   - Maximum A Posteriori
   - Bayesian approach
4. Graphical models
   - Bayesian networks
   - Two-dimensional visualization
5. Evaluation
   - Measures of accuracy

References:

1. J. Kruschke, Doing Bayesian Data Analysis: A Tutorial Introduction With R and Bugs, Academic Press 2010

2. Christopher M. Bishop, Pattern Recognition and Machine Learning (Information Science and Statistics), Springer 2007
3. Richard O. Duda, Peter E. Hart, David G. Stork, Pattern Classification (2nd Edition), Wiley-Interscience, 2000
4. Yaser S. Abu-Mostafa, Malik Magdon-Ismail, Hsuan-Tien Lin, Learning from Data, AML-Book, 2012 (supporting material available at http://amlbook.com/support.html)
5. David J. C. MacKay, Information Theory, Inference and Learning Algorithms, Cambridge University Press, 2003 (freely available and supporting material at http://www.inference.phy.cam.ac.uk/mackay/
6. David Barber, Bayesian Reasoning and Machine Learning, Cambridge University Press, 2012 (freely available at http://web4.cs.ucl.ac.uk/staff/D.Barber/pmwiki/pmwiki.php?n=
7. Kevin P. Murphy, Machine Learning: A Probabilistic Perspective, MIT Press, 2012 (supporting material http://www.cs.ubc.ca/ murphyk/MLbook/)
8. Richard McElreath, Statistical Rethinking, CRC Presso, 2015 (supporting material https://xcelab.net/rm/statistical-rethinking/)

Analysis and Control of Multi-agent Systems

Marco Fabris[1]

[1] *Department of Information Engineering*
Email: marco.fabris.1@unipd.it

Timetable: 20 hrs (see Class Schedule on https://phd.dei.unipd.it/course-catalogues/)

Course requirements: Linear Algebra and basic Calculus

Examination and grading: oral presentation of either any topic contained in the references [2], [3], [5], [6], [9], [10] or any other related work in the scientific literature that may also include the own student's research

SSD: Information Engineering

Aim: Multi-agent systems (MASs), or networked dynamic systems (NDS), are systems composed of dynamic agents that interact with each other over an information exchange network. These systems can be used to perform team objectives with applications ranging from formation flying to distributed computation. Challenges associated with these systems are their analysis and synthesis, arising due to their decoupled, distributed, large-scale nature, and due to limited interagent sensing and communication capabilities. This course provides an introduction to these systems via tools from graph theory, dynamic systems and control theory. The course will cover a variety of modeling techniques for different types of networked systems and proceed to show how their properties, such as stability, performance and security, can be analyzed. The course will also explore techniques for designing these systems. The course will also cover novel applications by presenting recent results obtained in the secure-by-design consensus and optimal time-invariant formation tracking.

Course contents:

- Lecture 1. Introduction to MASs, synchronization and coordination, illustration of the course goals. Modeling NDSs and related examples such as opinion dynamics, wireless sensing networks, robot rendezvous, cyclic pursuit.
- Lecture 2. Elements of graph theory: basic notation and algebraic graph theory.
- Lecture 3. Consensus theory: the linear agreement protocol both in continuous and discrete time, firstly for unweighted graphs and then for weighted digraphs.
- Lecture 4. Secure-by-design linear agreement protocol against edge-weight perturbations seen as an application of the small-gain theorem.
- Lecture 5. The nonlinear agreement protocol along with examples such as coupled oscillators and the Kuramoto model. Passivity as a tool to analyze stability of the nonlinear agreement protocol.
- Lectures 6-7. Formation control: gradient dynamics and potential-based control. Rigidity theory. A distance-based formation controller and its stability analysis.
- Lecture 8. The optimal time-invariant formation tracking (OIFT) problem as an application of the Pontryagin's Maximum Principle. Distributed OIFT.

- Lectures 9-10. Bearing-based formation control. Bearing rigidity. A bearing-only formation controller. Bearing-based formation maneuvering.

References:

1 D. Zelazo's Ph.D. course "Analysis and Control of Multi-agent systems", held at the Department of Information Engineering (UniPD), 2019.

2 F. Bullo with the contribution of Jorge Cortés, Florian Dörfler, and Sonia Martínez, "Lectures on Networked Systems", Vol. 1. No. 3. Seattle, DC, USA: Kindle Direct Publishing, 2020.

3 M. Mehran and M. Egerstedt, "Graph theoretic methods in multiagent networks", Princeton University Press, 2010.

4 R. A. Horn and C. R. Johnson, "Matrix Analysis", Cambridge University Press, 1990.

5 C. Godsil and G. Royle, "Algebraic Graph Theory", Springer, 2009.

6 F. R. K. Chung, "Spectral graph theory", Vol. 92. American Mathematical Soc., 1997.

7 M. Fabris and D. Zelazo, "Secure consensus via objective coding: Robustness analysis to channel tampering", IEEE Transactions on Systems, Man, and Cybernetics: Systems 52.12 (2022): 7885-7897.

8 M. Fabris and D. Zelazo, "A Robustness Analysis to Structured Channel Tampering over Secureby- design Consensus Networks", IEEE Control Systems Letters, 2023.

9 W. Ren and R. Beard, "Distributed Consensus in Multi-Vehicle Cooperative Control", Springer, 2008.

10 H. S. Ahn, "Formation control", Springer International Publishing, 2020.

11 M. Fabris, A. Cenedese and J. Hauser, "Optimal time-invariant formation tracking for a secondorder multi-agent system", 18th European Control Conference (ECC). IEEE, 2019.

12 M. Fabris and A. Cenedese, "Optimal Time-Invariant Distributed Formation Tracking for Second- Order Multi-Agent Systems", arXiv preprint arXiv:2307.12235 (2023).

13 S. Zhao and D. Zelazo, "Bearing rigidity and almost global bearing-only formation stabilization" IEEE Transactions on Automatic Control 61.5 (2015): 1255-1268.

Further potentially relevant recent papers will be referred to and distributed during the lectures.

Applied Functional Analysis and Machine Learning

Prof. Gianluigi Pillonetto[1]

[1] *Department of Information Engineering, Univ. Padova*
e-mail: giapi@dei.unipd.it

Timetable: 28 hrs (see Class Schedule on https://phd.dei.unipd.it/course-catalogues/)

Enrollment: add the course to the list of courses you plan to attend using the Course Enrollment Form (requires SSO authentication) and, if you are taking the course for credits, to the Study and Research Plan.

Course requirements: The classical theory of functions of real variable: limits and continuity, differentiation and Riemann integration, infinite series and uniform convergence. The arithmetic of complex numbers and the basic properties of the complex exponential function. Some elementary set theory. A bit of linear algebra.

Examination and grading: Homework assignments and final test.

SSD: Information Engineering

Aim: The course is intended to give a survey of the basic aspects of functional analysis, machine learning, regularization theory and inverse problems.

Course contents:

Review of some notions on metric spaces and Lebesgue integration: Metric spaces. Open sets, closed sets, neighborhoods. Convergence, Cauchy sequences, completeness. Completion of metric spaces. Review of the Lebesgue integration theory. Lebesgue spaces.

Banach and Hilbert spaces: Finite dimensional normed spaces and subspaces. Compactness and finite dimension. Bounded linear operators. Linear functionals. The finite dimensional case. Normed spaces of operators and the dual space. Weak topologies. Inner product spaces and Hilbert spaces. Orthogonal complements and direct sums. Orthonormal sets and sequences. Representation of functionals on Hilbert spaces.

Reproducing kernel Hilbert spaces, inverse problems and regularization theory: Representer theorem. Reproducing Kernel Hilbert Spaces (RKHS): definition and basic properties. Examples of RKHS. Function estimation problems in RKHS. Tikhonov regularization. Support vector regression and classification. Extensions of the theory to deep kernel-based networks: multi-valued RKHSs and the concatenated Representer Theorem.

References:

1. G. Pillonetto, T. Chen, A. Chiuso, G. De Nicolao, L. Ljung. Regularized System Identification – learning dynamic models from data, Springer Nature 2022
2. W. Rudin. Real and Complex Analysis, McGraw Hill, 2006
3. C.E. Rasmussen and C.K.I. Williams. Gaussian Processes for Machine Learning. The MIT Press, 2006

4. H. Brezis, Functional analysis, Sobolev spaces and partial differential equations, Springer 2010
5. G. Pillonetto, A. Aravkin, D. Gedon, L. Ljung, A.H. Ribeiro and T.B. Schön, Deep networks for system identification: a Survey, eprint 2301.12832 arXiv, 2023

Heuristics for Mathematical Optimization

Prof. Domenico Salvagnin[1]

[1] *Department of Information Engineering, Padova*
email: dominiqs@gmail.com - domenico.salvagnin@unipd.it

Timetable: 20 hrs (see Class Schedule on https://phd.dei.unipd.it/course-catalogues/)

Course requirements:
- Moderate programming skills (on a language of choice)
- Basics in linear/integer programming.

Examination and grading: Final programming project.

SSD: Information Engineering

Aim: Make the students familiar with the most common mathematical heuristic approaches to solve mathematical/combinatorial optimization problems. This includes general strategies like local search, genetic algorithms and heuristics based on mathematical models.

Course contents:

- Mathematical optimization problems (intro)
- Heuristics vs exact methods for optimization (intro)
- General principle of heuristic design (diversification, intensification, randomization)
- Local search-based approaches
- Genetic/population based approaches
- The subMIP paradigm
- Applications to selected combinatorial optimization problems: TSP, QAP, facility location, scheduling.

References:

1. Gendreau, Potvin "Handbook of Metaheuristics", 2010
2. Marti, Pardalos, Resende "Handbook of Heuristics", 2018

Elements of Deep Learning

Prof. Gian Antonio Susto[1]

[1] Department of Information Engineering, Univ. Padova
e-mail: gianantonio.susto@dei.unipd.it

Timetable: 24 hrs (see Class Schedule on https://phd.dei.unipd.it/course-catalogues/)

Course requirements: Basics of Machine Learning and Python Programming.

Examination and grading: Final project.

SSD: Information Engineering

Aim: The course will serve as an introduction to Deep Learning (DL) for students who already have a basic knowledge of Machine Learning. The course will move from the fundamental architectures (e.g. CNN and RNN) to hot topics in Deep Learning research.

Course contents:

- Introduction to Deep Learning: context, historical perspective, differences with respect to classic Machine Learning.
- Feedforward Neural Networks (stochastic gradient descent and optimization).
- Convolutional Neural Networks.
- Neural Networks for Sequence Learning.
- Elements of Deep Natural Language Processing.
- Elements of Deep Reinforcement Learning.
- Unsupervised Learning: Generative Adversarial Neural Networks and Autoencoders.
- Laboratory sessions in Colab.
- Hot topics in current research.

References:

1. Arjovsky, M., Chintala, S., Bottou, L. (2017). Wasserstein GAN. CoRR, abs/1701.07875.
2. Bahdanau, D., Cho, K., Bengio, Y. (2014). Neural Machine Translation by Jointly Learning to Align and Translate. CoRR, abs/1409.0473.
3. I. Goodfellow, Y. Bengio, A. Courville ‘Deep Learning’, MIT Press, 2016
4. Goodfellow, I.J., Pouget-Abadie, J., Mirza, M., Xu, B., Warde-Farley, D., Ozair, S., Courville, A.C., Bengio, Y. (2014). Generative Adversarial Nets. NIPS.
5. Hochreiter, S., Schmidhuber, J. (1997). Long Short-Term Memory. Neural computation, 9 8, 1735-80.
6. Kalchbrenner, N., Grefenstette, E., Blunsom, P. (2014). A Convolutional Neural Network for Modelling Sentences. ACL.

7. Krizhevsky, A., Sutskever, I., Hinton, G.E. (2012). ImageNet Classification with Deep Convolutional Neural Networks. Commun. ACM, 60, 84-90.

8. LeCun, Y. (1998). Gradient-based Learning Applied to Document Recognition.

9. Mikolov, T., Sutskever, I., Chen, K. (2013). Representations of Words and Phrases and their Compositionality.

10. Vincent, P., Larochelle, H., Lajoie, I., Bengio, Y., Manzagol, P. (2010). Stacked Denoising Autoencoders: Learning Useful Representations in a Deep Network with a Local Denoising Criterion. Journal of Machine Learning Research, 11, 3371-3408.

11. Zaremba, W., Sutskever, I., Vinyals, O. (2014). Recurrent Neural Network Regularization. CoRR, abs/1409.2329.

Courses in collaboration with the Doctoral School on “Economics and Finance” of the University of Verona

for complete Catalogue and class schedule see on

https://www.dse.univr.it/?ent=oi&ava=&cs=1008&id=746&lang=en

Please check regularly the website of the Doctoral Course

Stochastic Processes in Finance

Sara Svaluto-Ferro[1]

[1] *Dipartimento di Economia, Universitá di Verona*
Email: sara.svalutoferro@univr.it

Timetable: 24 hrs. First lecture on April, 2024, University of Verona, PhD School of Economics and Finance.

Examination and grading: Individual written and reasoned report on one of the topics of the course.

SSD:

Aim: This is a graduate lecture on recent topics in stochastic processes in finance with particular attention to Markov processes.

Course contents:

Part 1. - Markov processes

1. Stochastic differential equations
2. Semigroups, generators and martingale problems
3. Kolmogorov equations
4. Affine and polynomial (jump)-diffusions

Part 2. Applications

1. Pricing methods for Markovian models
2. Interest rate theory
3. Risk management.

References:

1. Jacod, Jean, and Albert Shiryaev. Limit theorems for stochastic processes. Vol. 288. Springer Science & Business Media, 2013.
2. Ethier, Stewart N., and Thomas G. Kurtz. Markov processes: characterization and convergence. John Wiley & Sons, 2009.
3. Duffie, Darrell, Damir Filipović, and Walter Schachermayer. "Affine processes and applications in finance." The Annals of Applied Probability 13.3 (2003): 984-1053.
4. Cuchiero, Christa, Martin Keller-Ressel, and Josef Teichmann. "Polynomial processes and their applications to mathematical finance." Finance and Stochastics 16 (2012): 711-740.
5. Filipović, Damir, and Martin Larsson. "Polynomial diffusions and applications in finance." Finance and Stochastics 20.4 (2016): 931-972.
6. Filipovic, Damir. Term-Structure Models. A Graduate Course. Springer, 2009.