Perturbative methods in dynamical systems

Christos Efthymiopoulos

1 Dipartimento di Matematica “Tullio Levi-Civita”
Email: christos.efthymiopoulos@unipd.it

Timetable: 24 hrs. First lecture on Thursday November 2nd, 2023, 14:30 (dates already fixed, see on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements:

Examination and grading: After the fourth lecture, students will be asked to choose a project to develop using mathematica. An oral exam will take place at the end of the course, including presentation of the project results and questions on the course material.

SSD: MAT/07

Aim:
This course aims to provide a self-contained introduction to the use of the methods of perturbation theory in the study of both regular and chaotic motions in dynamical systems. After a short review of basic definitions pertinent to dynamical systems’ theory, the course will present two types of perturbative methods, both of use in the study of the characterization of the solutions and of the local structure of the phase space in the neighborhood of a basic solution of a dynamical system such as a fixed point or a periodic orbit:

i) direct methods (for example, Lindstedt), which aim to construct the solutions directly under the form of a power series in a suitably defined small parameter, and

ii) indirect or normal form methods (for example, the Poincaré normal form), which aim at introducing a transformation of the variables in the form of series in the small parameter.

The presentation will be example-driven, starting from a simple dynamical system representing a nonlinear oscillator with dissipation and external driving. The students will be motivated to make computations in perturbation theory using mathematica and solve some project problems. Some rigorous estimates on the dependence of the size of the perturbative terms as a function of the order of the theory, based on suitable norm definitions in functional spaces related to the dynamical system under study, will be given in the last part of the course.

Lectures plan:


Lectures 3-5: Introduction to basic methods of perturbation theory part I: direct methods The example of the Duffing oscillator with dissipation and external driving. Perturbative (series)
representation of the solutions in the neighborhood of the stable fixed point using Lindstedt series. Perturbative (series) representation of the invariant manifolds emanating from the unstable fixed point using the parametrization method. Study of the intersections of the stable and unstable manifolds by the Poincaré - Melnikov method.

**Lectures 6-7:** Introduction to basic methods of perturbation theory part II: indirect methods
Linear normal forms and their classification. The Poincaré normal form in the neighborhood of a stable fixed point. The Moser normal form in the neighborhood of an unstable fixed point.

**Lectures 8-9:** Normal Forms in Hamiltonian dynamical systems

**Lectures 11-12:** The passage to systems with many (or infinitely many) degrees of freedom.
Perturbation theory in the example of a system with N non-linearly coupled oscillators, with N large. The limit of 1+1 (space and time) field equations. Perturbative computation of the spectrum of the Schrödinger equation in a one-dimensional nonlinear oscillator model, and in the perturbed hydrogen atom.

**Bibliography:**


Additional bibliography: Parts of openly available lecture notes by i) G. Benettin, ii) F. Fasso’, iii) M. Guzzo. Some lecture notes tailored to the needs of the course will be provided by the insegnant.