Hawkes processes: from theory to (financial) practice

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Timetable: 16 hrs. First lecture on April 4th, 2024, 10:30 (dates already fixed, see on - https://dottorato.math.unipd.it/calendar/), Torre Archimede, Room 2BC30

Course requirements: Probability and Stochastic Calculus (basic)

Examination and grading: Oral presentation (seminar) of a research paper related to the topic of the course.

SSD: MAT/06, SECS-S/06

Aim: Events that are observed over time naturally show clustering phenomena: an earthquake happening increases the probability of so-called aftershocks, namely minor readjustments along the portion of a fault that slipped during the mainshock. Similar clustering patterns are observed, e.g., in criminology, when dealing with certain types of crime data, such as burglary and gang violence, due to crime specific patterns of criminal behaviour. As a last example, in financial markets, selling a huge amount of a stock could induce successive selling activity with relative jumps clustering in the price or, on a larger scale, the collapse of an investment bank could create a financial turmoil and shock-waves through the world's financial centres.

Hawkes processes were introduced for the first time by A. Hawkes in 1971 to model the occurrence of seismic events. They are stochastic point processes particularly suitable to describe these self-exciting phenomena, in which the occurrence of an event increases the probability of a future arrival of another event. For this reason, they are also known under the name of “self-exciting point processes” and they have been applied in numerous fields throughout science, computer science, engineering, and human sciences.

More precisely, on a filtered probability space, denoting by $N$ the counting process, i.e., $N_t$ is the random variable counting the number of relevant events on the time interval $(0; t)$, we define the conditional intensity

$$
\lambda_t := \lim_{\Delta \to 0} \mathbb{E} [N_{t+\Delta} - N_t | F_t]
$$

where $F_t$ denotes the information available up to time $t$. The original self-exciting process is defined via an intensity that is a function of past events:

$$
\lambda_t := \mu + \int_0^t \gamma(t-u) dN_u = \mu + \sum_{T_i < t} \gamma(t-T_i)
$$

where $0 < T_1 < T_2 < \cdots < T_n < \ldots$ are the time instants at which the relevant events occur, $\mu$ is a base level for the intensity and $\gamma(u) \geq 0$, $u > 0$ is the exciting kernel. So, each event makes the intensity jump upwardly and then, between successive events, it decays according to the function $\gamma$. A typical kernel is $\gamma(u) = \alpha \beta e^{-\beta u}$, for $\alpha, \beta > 0$ and in this case the pair $(N, \lambda)$ is a Markov process. The aim of this PhD course at the University of Padova is to provide a rigorous Mathematical introduction to point processes and a solid basis on the necessary
Stochastic Calculus tools needed to handle models based on Hawkes processes. Markovianity, which represents a desired theoretical property of typical stochastic models, will also be discussed, as opposed to modeling with memory, hence leading to non-Markovian settings. This dilemma Markov/non-Markov opens the door to technical challenges, some of which are still the object of an active debate and a rich stream of research. A Statistical overview, with many Mathematical hints, will be also given on simulation of Hawkes processes and estimation of Hawkes-based models. The connection with branching processes, alpha stable processes and Volterra processes (with and without jumps) will also be discussed. To conclude, recent applications to Finance will be covered, together with possible other uses, at the frontier of research, in different research fields.

Course contents: the 16 hours course will cover the following topics, in 8 lectures:


3 & 4 Hawkes Processes: definition, properties, the self exciting feature and dynamic contagion. Markovian and non-Markovian Hawkes processes: related challenges.

5 Simulation: the intensity-based and the cluster-based approaches. Estimation: parametric or non parametric?

6 Possible extensions: branching processes, alpha stable processes and Volterra processes with jumps.

7 Application to Finance: stochastic volatility nowadays, memory or not? Jumps cluster analysis.

8 Other applications: insurance and market microstructure

References:


