Interpolation theory for differential forms

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Timetable: 16 hrs. First lecture on December 11, 2024, 15:00 (dates already fixed, see on Calendar of Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Credits: 3

Course requirements (at least recommended): Numerical Analysis I, suitable for students with background in analysis or differential geometry and/or approximation theory.

Examination and grading: Seminar

SSD: MAT/08

Aim: the principal aim of this course is to understand the main challenges of interpolation of differential forms. To do so, techniques based on integration on simplices (i.e. on the construction of *weights*) are studied. This involves the identification of unisolvent sets and their comparison in terms of appropriate functionals, the k-Lebesgue constants, that are discussed and characterised. At the end of the course, the student is in a position to deal with the existing literature and aware of modern challenges in the field.

Course contents:

- Introduction and preliminaries:
  1. quick review of discrete dierential geometry (from differential forms to polynomial differential forms);
  2. quick review of simplices and triangulations (simplices, subsimplices, complexes);
  3. quick review of interpolation theory (projectors, degrees of freedom).
- One-dimensional framework:
  1. interpolation by point evaluations on the line;
  2. interpolation by integral segments on the line;
  3. Lebesgue constant vs generalised Lebesgue constant;
  4. Fekete problem.
- Multi-dimensional framework:
  1. Whitney forms;
  2. simplicial elements and small simplices;
  3. weights: uniform vs non uniform small simplices;
  4. the generalised Lebesgue constant for k-forms;
  5. application to other shapes (n-balls).
- Applications:

1. computational aspects;
2. the problem of conditioning in FEM;
3. preconditioning by weights.

Bibliography:

1. L. Bruni Bruno, Weights for high order Whitney finite elements, PhD thesis (2022)
2. F. Rapetti and A. Bossavit, Whitney forms of higher degree, SIAM J. Numer. Anal., 47 (2009)
3. F. Rapetti, High order edge elements on simplicial meshes, ESAIM: M2AN, 41 (2007)
4. R. A. Nicolaides, On a class of
   nite elements generated by Lagrange interpolation, SIAM J. Numer. Anal., 9 (1972)
5. A. Alonso Rodríguez and F. Rapetti, On a generalization of the Lebesgues constant, Journal of Computational Physics, 428 (2021)
6. D. N. Arnold, R. S. Falk, and R. Winther, Finite element exterior calculus, homological techniques, and applications, Acta Numer., 15 (2006)