Doctoral Program in Mathematical Sciences
Department of Mathematics "Tullio Levi-Civita"
University of Padova

Doctoral Program in Mathematical Sciences

Catalogue of the courses 2024-2025

Updated January 23rd, 2025

INTRODUCTION

This Catalogue contains the list of activities offered to the Graduate Students in Mathematical Sciences for the year 2024-2025.

The activities in this Catalogue are of three types.

1. Courses offered by the Graduate School (= Courses of the Doctoral Program)
2. Courses offered by one of its curricula.
3. Other activities:
   a) selected courses offered by the PhD school in Information Engineering;
   b) selected courses offered by other PhD schools or other Institutions;
   c) reading courses

(This offer includes courses taught by internationally recognized external researchers. Since these courses might not be offered again in the near future, we emphasize the importance for all graduate students to attend them.)

Taking a course from the Catalogue gives an automatic acquisition of credits, while crediting of courses not included in the Catalogue (such as courses offered by the Scuola Galileiana di Studi Superiori, Summer or Winter schools, Series of lectures devoted to young researchers, courses offered by other PhD Schools) is possible but it is subject to the approval of the Executive Board. **Moreover, at most one course of this type may be credited.**

We underline the importance for all students to follow courses, with the goal of **broadening their culture in Mathematics**, as well as developing their knowledge in their own area of interest.

REQUIREMENTS FOR GRADUATE STUDENTS

Within the **first two years of enrollment** all students are required to

- pass the exam of at least four courses from the catalogue, among which at least two must be taken from the list of "Courses of the Doctoral Program", while at most one can be taken among the list of "reading courses"
- participate in at least one activity among the "soft skills"
- attend at least two more courses (for such activities the PhD student must produce a brief summary on what she/he learned. These summaries should be attached to the annual report)

Students are warmly encouraged to take more courses than the minimum required by these rules, and to commit themselves to follow regularly these courses. It is also recommended that one half of the exams are taken during the first year. At the end of each course the instructor will inform the Coordinator and the Secretary on the activities of the course and of the registered students.

Students **must register** to all courses of the Graduate School that they want to attend, independently of their intention to take the exam or not. We recommend to register as early as possible: the Graduate School may cancel a course if the number of registered students is too low. If necessary, the registration to a Course may be canceled.

<u>Courses attended in other Institutions and not included in the catalogue.</u>

Students activities within Summer or Winter schools, series of lectures devoted to young researchers, courses offered by the Scuola Galileiana di Studi Superiori, by other PhD Schools or by

PhD Programs of other Universities may also be credited, according to whether an exam is passed or not; the student must apply to the Coordinator and crediting is subject to approval by the supervisor and the Executive board. We recall that **at most one course** not included in the Catalogue may be credited.

Seminars

a) All students, during the three years of the program, must attend the **Colloquia of the Department** and participate regularly in the Graduate Seminar ("**Seminario Dottorato**"), whithin which they are also required to deliver a talk and write an abstract.
b) Students are also strongly encouraged to attend the seminars of the research groups that are relevant for their work.

HOW TO REGISTER AND UNREGISTER TO COURSES

The registration to a Course must be done online.
Students can access the **online registration form** in the dedicated page of the Doctoral Course website at https://dottorato.math.unipd.it/current-activity/FutureActivities clicking on "click to enroll" of the chosen courses. The registration lists can be reached also via the website of the Department of Mathematics at https://prev-www.math.unipd.it/userlist/
In order to register, fill the registration form with all required data, and validate with the command "Subscribe". The system will send a confirmation email message to the address indicated in the registration form; please save this message, as it will be needed in case of cancellation.

Registration to a course implies the commitment to follow the course.

Requests of **cancellation** to a course must be submitted in a timely manner, and **at least one month before the course** (except for courses that begin in October and November) using the link indicated in the confirmation email message.

REQUIREMENTS FOR PARTICIPANTS NOT ENROLLED IN THE GRADUATE SCHOOL OF MATHEMATICS

The courses in this catalogue, although part of activities of the Graduate School in Mathematics, are open to all students, graduate students, researchers of this and other Universities.
For organization reasons, external participants are required to **communicate their intention** (loretta.dallacosta@unipd.it) to take a course at least two months before its starting date if the course is scheduled in January 2025 or later, and as soon as possible for courses that take place until December 2024.
In order to **register,** follow the procedure described in the preceding section.
Possible **cancellation** to courses must also be notified.

List of Courses

Courses of the Doctoral Program

1. Proff. Luca Baracco, Olga Bernardi
   The Minimal Action Principle and applications **DP-1,2**
2. Prof. Marco Di Summa
   Convex polyhedra and Their Diameter **DP-3,4**
3. Prof. Giambattista Giacomin
   Products of random matrices: theory and applications **DP-5,6**
4. Prof. Giulio G. Giusteri
   Special Functions and Applications **DP-7**
5. Prof. Daniel Labardini Fragoso
   Hyperbolic Geometry, Continued Fractions and Cluster Algebras **DP-8,9**
6. Prof.ssa Orsola Tommasi
   Introduction to Moduli Spaces **DP-10,11**

Courses of the "Mathematics" area

1. Dott. Gabriele Bogo
   Curves in Hilbert Modular Surfaces **M-1**
2. Dott. Paolo Bonicatto, Elio Marconi
   Introduction to scalar conservation laws **M-3**
3. Dott. Ludovico Bruni Bruno
   Interpolation theory for differential forms **M-4,5**
4. Dott. Mattia Fogagnolo, Prof.ssa Valentina Franceschi
   The Isoperimetric Problem: Techniques and Applications **M-6,7**
5. Prof. Martino Garonzi
   The Maximal Subgroups of the Symmetric Group **M-8**
6. Prof. Andrea Spiro, Dott.ssa Marta Zoppello
   Topics of Control Theory from a Differential Geometric point of view **M-9,10**
7. Prof. Stefano Urbinati
   Polyhedral structures in algebraic geometry **M-11**

Courses of the “Computational Mathematics” area

1. Prof. Bernardo D’Auria.
   Stability of Queueing Networks **MC-1**

2. Prof. Andrea Macrina
   Mathematical Climate Finance **MC-2,3,4**

3. Prof.ssa Yuliya Mishura
   Bessel, Cox-Ingersoll-Ross, Ornstein-Uhlenbeck and Gaussian-Volterra processes with Wiener and fractional drivers **MC-5,6**

4. Prof. Alvise Sommariva
   Numerical cubature and its applications **MC-7**

Courses in collaboration with the Doctoral School in “Information Engineering”

Please check regularly the website of the Doctoral Course in Information Engineering at the URL https://phd.dei.unipd.it/course-catalogues/

Calendar of activities on
`https://calendar.google.com/calendar/u/0/embed?src=fvsl9bgkbnhhkqp5mmqpiurn6c@group.calendar.google.com&ctz=Europe/Rome`

1. Prof. Gianluigi Pillonetto
   Applied Functional Analysis and Machine Learning **DEI-1,2**

2. Prof. Gian Antonio Susto
   Elements of Deep Learning **DEI-3**

Courses of the Doctoral Program

The Minimal Action Principle and applications

Luca Baracco[1], Olga Bernardi[2]

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[2] *Dipartimento di Matematica "Tullio Levi-Civita"*
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Timetable: 24 hours; second semester.

Credits: 4

Course requirements: Basic notions of Mathematical Physics and Calculus of Variations.

Examination and grading: oral exam.

SSD: MAT/07-05

Aim: A sample of various settings, also beyond classical Lagrangian and Hamiltonian framework, where the "Least Action Principle" play a fundamental role. The course include basic notions and proofs of Aubry-Mather Theory and Mather-Mané Theory. Moreover, we discuss connections both with mathematical billiards and Symplectic Topology.

Course contents: For a Lagrangian $L : TM \to \mathbb{R}$ on a tangent bundle, the Euler-Lagrange equations:

$$\frac{d}{dt}\partial_{\dot{x}}L(x,\dot{x}) - \partial_x L(x,\dot{x}) = 0$$

encode the variational character of Lagrangian Mechanics. In fact, it is well-known that a curve $x : [t_0, t_1] \to X$ solves E-L if and only if the first variation of the corresponding Action

$$\int_{t_0}^{t_1} L(x,\dot{x})dt,$$

with fixed extremals, vanishes. Concerning minimality, in general it holds for short times. In fact, due to the possible presence of conjugated points, critical curves cease to be minimizing for larger times. Only under certain convexity assumptions there are "Action minimizing orbits". For such a distinguished and mechanical relevant class of Lagrangian –the so-called Tonelli Lagrangians– the Legendre transform is a global diffeomorphism and E-L equations equal to Hamilton's equations

$$\begin{cases} \dot{x} = \partial_y H(x,y) \\ \dot{y} = -\partial_x H(x,y) \end{cases}$$

for the corresponding Hamiltonian $H : T^*X \to \mathbb{R}$. For autonomous systems, H gives the conserved energy value along a solution.

Beyond Lagrangian and Hamiltonian setting, the search of dynamically relevant minimal objects is one of the central topics of modern theory of Dynamical Systems. One of the first results in this direction go back to the Eighties with the so-called Aubry-Mather theory for monotone twist map. An important application of this theory is the study of mathematical billiards, from Birkhoff to –more recent– types of billiards like symplectic and outer billiards. The generalization of such a theory from one to more degrees of freedom have been developed two decades later with Mather-Mané theory, where minimizing measures, instead of trajectories, play a crucial role. This significant theory has connections from Hamilton-Jacobi equation to Symplectic Topology.

The aim of this PhD course is to present –in a self-contained way– the "Minimal Action Principle" in different settings. This principle can be considered a sort of –largely accepted– "thriftiness" of Nature in its motions. In more detail, the plan of the course includes:

1. BASIC NOTIONS OF AUBRY-MATHER THEORY: Monotone twist maps, minimal orbits, the minimal action (or Mather's β function) for monotone twist maps.
2. THE MINIMAL ACTION FOR CONVEX LAGRANGIANS: Basic notions of Mather-Mané theory, Mané critical value, the Aubry set.
3. THE MINIMAL ACTION FOR CONVEX BILLIARDS: Birkhoff, symplectic and outer billiards and their corresponding generating functions. Length and area spectrum invariants.
4. THE MINIMAL ACTION IN SYMPLECTIC TOPOLOGY: A bit of symplectic geometry, the graph selector, boundary rigidity phenomena, the Aubry set and non-removable intersections.

Bibliography: The course follows the main lines of the book (present also in Padova math library):
– Siburg K.F. The principle of least action in geometry and dynamics. Lecture Notes in Mathematics, vol. 1844, xiii+ 128 pp. Berlin, Germany: Springer, (2004).

Moreover, additional related publications include:

– Bangert V. Mather Sets for Twist Maps and Geodesics on Tori Dynamics Reported, Volume 1 Dynamics Reported, (1988).
– Paternain G.P. Polterovich L. Siburg K.F. Boundary rigidity for Lagrangian submanifolds, non-removable intersections, and Aubry-Mather theory, Volume 3, Number 2, (2003).
– Sorrentino A. Action-minimizing methods in Hamiltonian dynamics: an introduction to Aubry-Mather theory. Monograph in the Series: Mathematical Lecture Notes Vol. 50, Princeton University Press, (2015).
– Tabachnikov S. Geometry and Billiards (Student Mathematical Library) (Student Mathematical Library, 30), (2005).

Convex polyhedra and their diameter

Marco Di Summa[1]

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Timetable: 24 hrs, First lecture on November 6, 2024, 12:30 (dates already fixed, see Calendar of Activities on https://dottorato.math.unipd.it/calendar/, Torre Archimede, Room 2BC30.

Credits: 4

Course requirements: Standard Mathematical knowledge will be sufficient

Examination and grading: Seminar

SSD: MAT/09

Aim: A convex polyhedron is defined as the set of solutions to a given system of linear inequalities, and as such, it generalizes to n dimensions the commonly known notion of 3-dimensional polyhedron, which is used, for instance, to describe the structure of molecules and chemical compounds. Besides being natural and interesting mathematical objects, general n-dimensional polyhedra are fundamental in the field of optimization.

In this course we will introduce some basic properties of polyhedra and discuss their importance and usefulness. After reviewing some fundamental structural results, we will focus on the notion of diameter of a polyhedron. In order to understand what the diameter of a polyhedron is, one can visualize the 3-dimensional case, where the notions of vertex and edge should be clear: The diameter of a polyhedron is the maximum distance between any two vertices, where, for a fixed pair of vertices, the distance is measured by counting the minimum number of steps needed to move from one vertex to the other by traversing edges. (This can be properly extended to the n-dimensional case.) We will survey classical and famous results on the diameter of polyhedra, as well as some recent achievements. We will spend some time on the Hirsch conjecture, which was posed in 1957, proved for some special but very important cases in the subsequent decades, and finally disproved by Francisco Santos in 2010 in a paper that won the Fulkerson prize, one of the most important awards in the area of Discrete Mathematics. However, a weaker version, called the polynomial Hirsch conjecture, is still open, and its correctness would be extremely important to assess the efficiency of some algorithms for the solution of linear optimization problems.

The topic of this course can be interesting for students in various fields ofMathematics, as it connects (at least) combinatorial and discrete geometry, graph theory, and optimization. Graphs and their diameter are also studied in algebra and probability (e.g., random walks are sometimes the tool to prove the existence of a short path between two vertices).

Course contents:

- Polyhedra: basic facts, inequality description, vertex/ray description, fundamental examples
- The role of polyhedra in several disciplines
- The skeleton of a polyhedron and the notion of diameter

- Lower and upper bounds on the diameter of a polyhedron
- Hirsch conjecture and its link with the efficiency of linear programming algorithms
- Correctness of the conjecture for 0/1 polytopes and for some other relevant cases
- Santos' counterexample to the Hirsch conjecture (sketch)
- The (weaker) polynomial Hirsch conjecture and some related results
- Time permitting) The recent notion of circuit diameter

Bibliography:
For the introductory part on polyhedra, students can refer to Chapter 3 of the following book, also accessible online:

- M. Conforti, G. Cornuéjols, G. Zambelli, Integer Programming, Springer, 2014.

The notion of diameter and some classical related results are discussed, e.g., in Chapter 3 of the following book:

- G. M. Ziegler, Lecture on Polytopes, Springer, 2007.

More specific references (including those needed to be prepared for the exam, which is in the form of student seminar) will be provided during the course.

Products of random matrices: theory and applications

Giambattista Giacomin[1]

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Timetable: 24 hrs, First lecture on November 12, 2024, 10:30 (dates already fixed, see Calendar of Activities on https://dottorato.math.unipd.it/calendar/, Torre Archimede, Room 2BC30.

Credits: 4

Course requirements:
A first course in probability with measure theory, some knowledge of stochastic processes with discrete time (Markov chains, martingales) is welcome, but the important concepts will be (re)introduced in some detail.

Examination and grading: Oral exam

SSD: MAT/06

Aim: Learning fundamental of products of random matrices.

Course contents:
Products of random matrices appear as a fundamental model and/or tool in a number of mathematical contexts, ranging from very abstract to very applied ones. The theory is rich already in the case in which one deals with products of independent and identically distributed (IID) random $d \times d$ matrices $(M_n)_{n=1,2,...}$, with d an integer larger than 1: one of the main issue in this context is understanding the leading asymptotic behavior, for $n \to \infty$, of $\log ||M_1 M_2 \ldots M_n||$, with $|| \cdot ||$ a matrix norm, assuming that $\log ||M_1||$ is in L^1. This is the natural generalization of the case in which $d = 1$, where $M_1 M_2 \ldots M_n$ is just a product of IID real random variable and the issue we raise is easily solved by applying the Law of Large Numbers, and the answer is very explicit. The situation is more involved when $d = 2$ or more: in fact, the issue we just raised is the identification of the top Lyapunov exponent of the product of random matrices we consider and, while the theory is very complete, the answer is somewhat involved and definitely not as explicit as for $d = 1$. It involves in particular the construction of an auxiliary process that is interesting in its own right.

In the first part of the course develops the the theory that leads to a formula, the Furstenberg formula, for the top Lyapunov exponent. While some results will be given (and proven) for general d, the full theory will be developed only for $d = 2$. We will also study the second Lyapunov exponent (i.e., for $d = 2$ we will study all the Lyapunov exponents, since there are d Lyapunov exponents): the key result that we will prove is Oseledets Theorem.

The second part of the course focuses on applications. Possible topics include:

1. Anderson localization in one dimension: the Schrödinger operator with random potentials
2. Products of random matrices and random walks on groups

3. The transfer matrix method in statistical mechanics and application to some disordered models.

In reality the two parts of the course will be to a certain extent entangled: the random matrices that are relevant for the applications will be used as examples in the theoretical part of the course.

Bibliography:

There will be lecture notes, mostly based (for the first part of the course) on

- P. Bougerol and J. Lacroix, Products of random matrices with applications to Schrödinger operators, Progress in Probability and Statistics, 8, Birkhäuser, 1985.
- M. Viana, Lectures on Lyapunov exponents, Cambridge Studies in Advanced Mathematics, 145 Cambridge University Press, 2014.

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Special Functions and Applications

Prof. Giulio G. Giusteri[1]

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Timetable: 24 hrs, First lecture on January 7, 2025, 10:30 (dates already fixed, see Calendar of Activities on https://dottorato.math.unipd.it/calendar/, Torre Archimede, Room 2BC30.

Credits: 4

Course requirements:
Basic notions of analysis and algebra, ordinary differential equations and partial differential equations

Examination and grading: Oral examination on the program and on a student's project

SSD: MAT/07

Aim: To present various families of special functions, their emergence and usefulness in applied mathematics contexts.

Course contents:

- Recap of basic facts in complex analysis: holomorphic functions, Laurent series, contour integrals, Cauchy theorem. Euler's Gamma function.
- The Probability Integral: from error estimates to heat conduction and boundary layers
- The Heat equation and Laplace transform
- Legendre and Hermite polynomials: Schrödinger equation and the quantum harmonic oscillator.
- Curvilinear coordinates. Laplace equation, separation of variables. Laguerre polynomials.
- Polar coordinates and spherical harmonics: the orbitals of the hydrogen atom.
- Cylindrical coordinates and Bessel functions: vibration of a membrane and the bi-harmonic Stokes problem.
- Further applications (or functions) can be selected based on the audience (possible topics in fluid mechanics, potential theory, stochastic analysis, numerical solution of PDEs).

Bibliography:

[1] N. N. Lebedev, Special Functions and Their Applications, Prentice–Hall, 1965.

[2] G. Arfken, Mathematical Methods for Physicists, 3rd ed., Academic Press, 1985.

[3] I. S. Gradshteyn, I. M. Ryzhik, Table of Integrals, Series, and Products, 7th ed., Elsevier, 2007

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Hyperbolic Geometry, Continued Fractions and Cluster Algebras

Prof. Daniel Labardini Fragoso[1]

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Timetable: 24 hrs. First lecture on March 12th, 2025, 12:00 (dates already fixed, see Calendar of Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Credits: 4

Course requirements:
The prerequisites are reduced to the minimum:

- differential and integral calculus of one and several variables;
- elementary linear algebra;
- elementary algebra.

Knowledge of Differential or Riemannian Geometry would be helpful, but not necessary.

Examination and grading: Please contact the teacher of the course by e-mail.

SSD: MAT/02-MAT/03

Aim: To arrive at the notion of Cluster Algebra from two distinct, albeit related, elementary starting points: Hyperbolic Geometry and Continued Fractions.

Course contents:

1. Hyperbolic Geometry
   - Möbius transformations
   - Models and isometries of the hyperbolic plane
   - The Farey diagram and its symmetries
   - Ptolemy's identity in the hyperbolic plane
2. Continued Fractions
   - Recursive formalism
   - Finite continued fractions
   - Infinite continued fractions
   - Examples: the continued fractions of $1 + \sqrt{2}$, $\frac{1+\sqrt{5}}{2}$ and e
   - Equivalent numbers
   - Continued fractions vs. perfect matchings of snake graphs
3. Cluster Algebras
   - Basic definitions and examples

- The Laurent phenomenon
- Cluster variables vs. perfect matchings of snake graphs

Bibliography:

1. James Anderson. Hyperbolic geometry. Springer-Verlag. Springer Undergraduate Mathematics Series. 2007.
2. Ilke Canakci, Ralf Schiffler. Cluster algebras and continued fractions. Compositio Mathematica 154 (2018), no. 3, 565–593. arXiv:1608.06568
3. Ilke Canakci, Ralf Schiffler. Snake graphs and continued fractions. European Journal of Combinatorics 86 (2020), 103081, 19 pp. arXiv:1711.02461
4. James W. Cannon, William J. Floyd, Richard Kenyon, Walter R. Parry. Hyperbolic Geometry. Flavors of Geometry (edited by Silvio Levy). Cambridge University Press, MSRI Publications, Volume 31. 1997.
5. Sergey Fomin, Andrei Zelevinsky. Cluster algebras IV: Coefficients. Compos. Math. 143 (2007), no. 1, 112–164. arXiv:math/0602259
6. Allen Hatcher. Topology of numbers. American Mathematical Society, 2022.
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9. Toshihiro Nakanishi. An application of Penner's coordinates of Teichmüller space of punctured surfaces. RIMS Kokyuroku Bessatsu, Kyoto, 2010. B17: Infinite dimensional Teichmüller spaces and moduli spaces, 105–114.
10. Robert Penner. Lambda lengths. http://www.ctqm.au.dk/research/MCS/lambdalengths.pdf
11. Robert Penner. Decorated Teichmüller Theory. European Mathematical Society, the QGM Master Class Series. 2012. DOI 10.4171/075
12. Boris Springborn. The hyperbolic geometry of Markov's theorem on Diophantine approximation and quadratic forms. L'Enseignement Mathématique (2) 63 (2017), 333–373. arXiv:1702.05061
13. Lauren K. Williams. Cluster algebras: an introduction. Bull. Amer. Math. Soc. (N.S.) 51 (2014), no. 1, 1–26. arXiv:1212.6263
14. P.M.H. Wilson. Curved spaces, from Classical Geometries to Elementary Differential Geometry. Cambridge University Press. 2008.

Introduction to moduli spaces

Orsola Tommasi[1]

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Timetable: 24 hrs, First lecture on February 3, 2025, 10:30 (dates already fixed, see on https://dottorato.math.unipd.it/calendar/) Torre Archimede, Room 2BC30

Credits: 4

Course requirements: Basic notions of geometry, as provided by a Mathematics degree. This course is open to any PhD student. Depending on the students' background, I will include a review of useful notions from geometry and topology.

Examination and grading:

SSD: MAT/03

Aim: Learning the fundamentals of moduli spaces.

Course contents:

A moduli space is a space parametrizing all possible objects of a certain fixed type. A classical example is the following: let us fix a compact oriented surface $\sum$. Then the space of all possible complex structures on $\sum$ is the moduli space $\mathcal{M}_g$ of genus g Riemann surfaces, where g is the topological genus of $\sum$. By construction, the points of $\mathcal{M}_g$ correspond to the isomorphism classes of Riemann surfaces of genus g. This kind of construction generalizes to many other classification problems. In good cases, moduli spaces will turn out to be complex manifolds or varieties. However, in most cases the moduli space will not be truly a manifold, but rather a mild generalization of it, called an orbifold. Although the construction of moduli spaces originates in algebraic geometry, moduli spaces themselves are of interest also in other areas of mathematics, such as other areas of geometry, topology, group theory, analysis and mathematical physics. In this course, I would like to present the basic ideas and formalism underlying the concept of moduli space, along with some main examples of interdisciplinary interest. Besides the moduli space of Riemann surfaces, interesting examples with applications in different fields include:

- mirror symmetry, a construction from theoretical physics that predicts that there exist pairs of topological spaces (X,X*) such that the moduli space of complex structures on X is isomorphic to the moduli space of symplectic structures on X*, and vice versa;
- modular curves in number theory, which are moduli spaces parametrizing elliptic curves with additional structures.

Bibliography:

• Kock, Joachim; Vainsencher, Israel. An invitation to quantum cohomology. Kontsevich's formula for rational plane curves. Progress in Mathematics, 249. Birkhäuser Boston, Inc., Boston, MA, 2007. xiv+159 pp.

- Newstead, P. E. Introduction to moduli problems and orbit spaces. Tata Institute of Fundamental Research Lectures on Mathematics and Physics, 51. Tata Institute of Fundamental Research, Bombay; by the Narosa Publishing House, New Delhi, 1978. vi+183 pp.
- Cox, David A.; Katz, Sheldon. Mirror symmetry and algebraic geometry. Mathematical Surveys and Monographs, 68. American Mathematical Society, Providence, RI, 1999. xxii+469 pp.
- Harris, Joe; Morrison, Ian. Moduli of curves. Graduate Texts in Mathematics, 187. Springer-Verlag, New York, 1998. xiv+366 pp.

Courses of the “Mathematics” area

Curves in Hilbert Modular Surfaces

Dr. Gabriele Bogo[1]

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Timetable: 16 hrs. First lecture on March 3, 2025, 14:00 (dates already fixed see on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30

Credits: 3

Course requirements: Basic algebraic number theory and algebraic geometry (algebraic varieties). Previous exposure to the concept of moduli spaces is helpful but not required.

Examination and grading: seminar about a research paper on the subject

SSD: MAT/03

Aim: Belyi theorem states that every algebraic curve defined over $\overline{\mathbb{Q}}$ is a branched cover of the projective line minus three points. Such a curve is, by Poincaré's unifomization theorem, a (possibly compactified) quotient $\mathbb{H}/\Gamma$ of the upper half-plane by a discrete group $\Gamma \subset SL_2(\mathbb{R})$. In some cases, the group Γ is *arithmetic*, i.e., it comes from a algebra (the curve is then a modular or Shimura curve), but in most cases it is not. An important example of non-arithmetic curves is provided by Teichmüller curves, a class of curves first discovered in relation with the dynamics of rational billiard tables. The first aim of this course is to provide a unified treatment of algebraic curves over $\mathbb{Q}$ as totally geodesic curves in the moduli space of abelian surfaces, with respect to the Kobayashi metric. The second aim is to discuss in detail two classes of such curves, the Hirzebruch-Zagier curves, and the Teichmüller curves in genus two. If time permits, we will discuss the special points on these curves.

Course contents:

1. The moduli space $\mathcal{A}_g$ of abelian varieties of dimension g and the locus of real multiplication. Hilbert modular varieties (an overview).
2. Kobayashi metric on $\mathcal{A}_g$ and totally geodesic curves (Kobayashi curves). They are defined over number fields and lie in the locus of real multiplication.
3. Further motivation: Belyi's theorem and Ellenberg-McReynolds's result ("every curve is a TeichmÏler curve").
4. Kobayashi curves as curves in Hilbert modular varieties and modular embeddings. Arithmetic Fuchsian groups.
5. Example I. Totally geodesic curves in $\mathcal{A}_2$ for the Riemannian metric: Hirzebruch-Zagier curves. Their relation with modular curves and moduli interpretation.
6. Teichmüller space and Teichmüller metric; mapping class group and moduli space of curves (an overview).
7. Example II. Totally geodesic curves in $\mathcal{A}_2$ for the Teichmüller metric: Teichmüller curves. Their relation with rational billiards (flat surfaces) and moduli interpretation.

8. If time permits, special (complex multiplication) points in the arithmetic and nonarithmetic cases.

Bibliography:

1. Hirzebruch, F. , Zagier, D. Intersection numbers of curves on Hilbert modular surfaces and modular forms of Nebentypus, Invent. Math.36(1976), 57–113.

2. McMullen, C. Billiards and Teichmüller curves on Hilbert modular surfaces, J. Amer.Math. Soc.16(2003), no.4, 857–885.

3. Möller, M. , Viehweg, E. Kobayashi geodesics in Ag , J. Differential Geom.86(2010), no.2, 355–379.

Introduction to scalar conservation laws

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Timetable: 16 hrs. First lecture on February 3, 2025, 14:30 (dates already fixed, see on Calendar of Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Credits: 3

Course requirements: calculus, measure theory.

Examination and grading: oral presentation of a research paper

SSD: MAT/05

Aim: introduce the theory of entropy solutions of scalar conservation laws. Both the classical theory initiated by Kruzkov and the kinetic formulation will be presented.

Course contents:

- Well-posedness of scalar conservation laws in the class of bounded entropy solutions: approximation schemes, Kruzkov a priori estimate
- Kinetic formulation of scalar conservation laws
- Lagrangian description of entropy solutions

Bibliography:

KRUŽKOV, S. N., First order quasilinear equations with several independent variables, Matematicheskiĭ Sbornik, 81 (123), 1970.
PERTHAME, B., Kinetic Formulation of Conservation Laws, Oxford Lecture Series in Mathematics and Its Applications.

Interpolation theory for differential forms

Ludovico Bruni Bruno[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova and Istituto Nazionale di Alta Matematica*
Email:ludovico.brunibruno@unipd.it, brunibruno@altamatematica.it

Timetable: 16 hrs. First lecture on December 11, 2024, 15:00 (dates already fixed, see on Calendar of Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Credits: 3

Course requirements (at least recommended): Numerical Analysis I, suitable for students with background in analysis or differential geometry and/or approximation theory.

Examination and grading: Seminar

SSD: MAT/08

Aim: the principal aim of this course is to understand the main challenges of interpolation of differential forms. To do so, techniques based on integration on simplices (i.e. on the construction of *weights*) are studied. This involves the identification of unisolvent sets and their comparison in terms of appropriate functionals, the k-Lebesgue constants, that are discussed and characterised. At the end of the course, the student is in a position to deal with the existing literature and aware of modern challenges in the field.

Course contents:

- Introduction and preliminaries:
  1. quick review of discrete dierential geometry (from differential forms to polynomial differential forms);
  2. quick review of simplices and triangulations (simplices, subsimplices, complexes);
  3. quick review of interpolation theory (projectors, degrees of freedom).
- One-dimensional framework:
  1. interpolation by point evaluations on the line;
  2. interpolation by integral segments on the line;
  3. Lebesgue constant vs generalised Lebesgue constant;
  4. Fekete problem.
- Multi-dimensional framework:
  1. Whitney forms;
  2. simplicial elements and small simplices;
  3. weights: uniform vs non uniform small simplices;
  4. the generalised Lebesgue constant for k-forms;
  5. application to other shapes (n-balls).
- Applications:

1. computational aspects;
2. the problem of conditioning in FEM;
3. preconditioning by weights.

Bibliography:

1. L. Bruni Bruno, Weights for high order Whitney finite elements, PhD thesis (2022)
2. F. Rapetti and A. Bossavit, Whitney forms of higher degree, SIAM J. Numer. Anal., 47 (2009)
3. F. Rapetti, High order edge elements on simplicial meshes, ESAIM: M2AN, 41 (2007)
4. R. A. Nicolaides, On a class of
   nite elements generated by Lagrange interpolation, SIAM J. Numer. Anal., 9 (1972)
5. A. Alonso Rodríguez and F. Rapetti, On a generalization of the Lebesgues constant, Journal of Computational Physics, 428 (2021)
6. D. N. Arnold, R. S. Falk, and R. Winther, Finite element exterior calculus, homological techniques, and applications, Acta Numer., 15 (2006)

The isoperimetric problem: techniques and applications

Mattia Fogagnolo[1], Valentina Franceschi[2]

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Email: mattia.fogagnolo@unipd.it
[2] Università di Padova
Email: valentina.franceschi@unipd.it

Timetable: 16 hrs. First lecture on March 3rd, 2025, 09:00 (dates already fixed, see Calendar of Activities on https://dottorato.math.unipd.it/calendar), Room 2BC30, Torre Archimede.

Course requirements: basics in PDEs, calculus of variations.

Examination and grading: seminar about a research paper in the topic.

Aim: The course aims at providing the attendees a number of modern techniques that have been utilized in connection with the isoperimetric problem, with an eye to the analysis of metric spaces and to the open problem in contemporary research.

Course contents: In this course we deal with the isoperimetric problem and the isoperimetric inequality in the Euclidean space. We present a number of related techniques and results, such as the direct method of the calculus of variations, the Schwarz symmetrisation, the ABP method. We will describe the close relation between isoperimetric, Sobolev and Faber-Krahn inequalities. We are going to discuss these and other tools in connection with the isoperimetric problem beyond the Euclidean space, with a focus on Riemannian manifolds with curvature bounds and the Heisenberg group. Time permitting, we will describe mean curvature inequalities, the quantitative isoperimetric inequality, and the minimal partition problems.

Bibliography

L. AMBROSIO, E. BRUÈ AND D. SEMOLA, Lectures on Optimal Transport, Springer, 2021.

G. ANTONELLI, E. PASQUALETTO, M. POZZETTA AND D. SEMOLA, *Asymptotic isoperimetry on noncollapsed spaces with lower Ricci bounds*

X. CABRÉ, *Elliptic PDE's in probability and geometry: Symmetry and regularity of solutions*, Discrete and Continuous Dynamical Systems, 2008

I. CHAVEL, Eigenvalues in Riemannian geometry, Elsevier, 1984.

S. GALLOT, D. HULIN AND J. LAFONTAINE, Riemannian Geometry, Springer, 2002.

M. FOGAGNOLO AND L. MAZZIERI, *Minimising hulls, p-capacity and isoperimetric inequality* J. Functional Anal. 2022.

V. FRANCESCHI AND R. MONTI, *Isoperimetric Problem in H-type groups and Grushin spaces*, Rev. Mat. Iberoam., 2014.

BRUCE KLEINER, *An isoperimetric comparison theorem*, Inventiones Math., 1992

F. MAGGI, Sets of Finite Perimeter and Geometric Variational Problems, Cambridge University Press, 2012.

R. MONTI, *Heisenberg isoperimetric problem. The axial case*, Adv. Calc. Var., 2008

S. NARDULLI, *Generalized existence of isoperimetric regions in non-compact Riemannian manifolds and applications to the isoperimetric profile*, Asian J. of Math. 2014.

R. MAGNANINI AND G. POGGESI, *Serrin's problem and Alexandrov's Soap Bubble Theorem: enhanced stability via integral identities*, Indiana Univ. Math. Journal, 2020.

M. RITORÉ, Isoperimetric Inequalities in Riemannian Manifolds, Birkhäuser, 2023.

M. RITORÉ AND C. ROSALES, *Existence and characterization of regions minimizing perimeter under a volume constraint inside Euclidean cones*, TAMS, 2004.

M. RITORÉ AND C. ROSALES, *Area-stationary surfaces in the Heisenberg group* $\mathbb{H}^1$, Adv. in Math., 2008.

The Maximal Subgroups of the Symmetric Group

Martino Garonzi[1]

[1] *Departamento de Matemática, Universidade de Brasília (DF, Brazil)*
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Timetable: 16 hrs. First lecture on March 18, 2025, 08:15 (dates already fixed, see on Calendar of Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Credits: 3

Introduction and background requirements: This is a course on finite permutation groups, focusing on the classification of the maximal subgroups of the finite symmetric groups S_n and of the finite alternating groups A_n. The background required is a basic knowledge of group theory (including notions such as solvability and nilpotency of groups) and group actions.

Examination and grading: Seminar

SSD: MAT/02

Aim: It is expected that the students achieve the knowledge of the O'Nan-Scott theorem and its proof, so as to make them able to prove results about generation and the subgroup structure of the symmetric group and of permutation groups in general. For example, it is expected that the students understand that the primitive maximal subgroups of S_n (and A_n) represent a "local obstruction": as a corollary of the O'Nan-Scott theorem, the set of numbers n such that S_n (and A_n) presents no such local obstruction has density 1.

Course contents:

- Finite permutation groups,
- Wreath products,
- Transitive and primitive actions,
- Minimal normal subgroups and socle of a finite group,
- Description of the maximal subgroups of the symmetric groups, O'Nan-Scott theorem for finite primitive permutation groups: affine groups, almost-simple groups, product actions and diagonal actions.
- Applications and discussion of relevant recent results.

References:

1. Cameron, Peter J.; Permutation groups. London Mathematical Society Student Texts, 45. Cambridge University Press, Cambridge, 1999.
2. Dixon, John D.; Mortimer, Brian; Permutation groups. Graduate Texts in Mathematics, 163. Springer-Verlag, New York, 1996.
3. Liebeck, Martin W.; Praeger, Cheryl E.; Saxl, Jan; On the O'Nan-Scott theorem for finite primitive permutation groups. J. Austral. Math. Soc. Ser. A 44 (1988), no. 3, 389–396.

Topics of Control Theory from a Differential Geometric point of view

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[2] *Dipartimento di Scienze Matematiche "G. L. Lagrange" (DISMA) Politecnico di Torino*
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Timetable: 16 hrs. First lecture on March 20, 2025, 16:30 (dates already fixed, see on Calendar of Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Credits: 4

Course requirements: Even if the course concerns topics in Control Theory and Differential Geometry, the lectures will be designed to be accessible by a general audience, with no special training in either of those two areas.

Examination and grading: seminar about a research paper in the topic.

SSD: MAT/05-07-03

Aim:
We would like to offer an introduction to a few fundamental results of Control Theory and Differential Geometry, with the purpose of illustrating how certain classical Differential Geometric tools can be used to find solutions to problems in Control Theory. We also have the intent of discussing some very recent results by Cardin, Giannotti, Spiro and Zoppello on the Pontryagin Maximum Principle and the local controllability problem of systems with non-linear controls.

Course contents:

- *Preliminaries (I)*. A quick overview of some basic notions and facts of Differential Geometry, as e.g. manifolds, submanifolds, vector fields, push-forwards under diffeomorphisms, tensor fields, p-forms and their integrations on submanifolds, flows, Lie derivatives and Lie brackets, etc.
- *Preliminaries (II).* A brief introduction to a few topics of Control Theory: first order control systems in normal forms, controllability and accessibility, the three variants (Mayer, Lagrange and Bolza) of cost minimising problems under first order differential constraints, Kalman Theorem, needle variations, Pontryagin Maximum Principle (PMP).
- Comparison between the classical and the differential geometric proofs of the PMP for Mayer problems.
- Frobenius Theorem and its applications to the integrability problems of systems of partial differential equations.
- Non-integrable distributions, Rashevskij's Theorem and its generalisations by Chow and Sussmann.

- Chow's criterion for the controllability of systems which are linear in controls; Controllability of systems on Lie groups; Discussions of several examples of control systems and comparisons of criterions for local controllabliity. The Chaplygin sleigh and its fluidodynamic variant: results and open problems.
- Non-linear control systems and representation of solutions as oriented curves in the extended state-control space. Graphic completions. New criterions for local accessibility and small time local controllability. Examples.

Bibliography:

1. A. Bressan and B. Piccoli, Introduction to the mathematical theory of control, American Institute of Mathematical Sciences (AIMS), Springfield, MO, 2007.
2. F. Cardin and A. Spiro, Pontryagin maximum principle and Stokes theorem, J. Geom. Phys. 142, (2019), 274–286.
3. F. Cardin, C. Giannotti and Spiro, Andrea, On the Pontryagin maximum principle under differential constraints of higher order, Ann. Polon. Math. 130 (2023), 97–147.
4. W.-L. Chow, Über Systeme von linearen partiellen Differentialgleichungen erster Ordnung, Math. Ann. 117 (1939), 98–105.
5. J. M. Coron, Control and nonlinearity, American Mathematical Society, Providence, RI, 2007.
6. F. Rampazzo, Lecture notes on Control, Set Separation, and Minima (Optimal control and controllability), (unpublished lectures notes), 2018.
7. P.K. Rashevskij, About connecting two points of complete non-holonomic space by admissible curve (in Russian), Uch. Zapiski Ped. Inst. K. 2 (1938), 83–94.
8. N. Sansonetto and M. Zoppello, On the trajectory generation of the hydrodynamic Chaplygin sleigh, IEEE Control Syst. Lett. 4 (2020), 922–927.
9. S. Sternberg, Lectures on differential geometry, Chelsea Publishing Co., New York, 1983.
10. H. J. Sussmann, Orbits of families of vector fields and integrability of distributions, Trans. Amer. Math. Soc. 180 (1973), 171–188.
11. F. W. Warner, Foundations of Differentiable Manifolds and Lie groups, Springer-Verlag, New York, 1983.

Polyhedral structures in algebraic geometry

Stefano Urbinati[1]

[1] *Dipartimento di Scienze Matematiche, Informatiche e Fisiche, Universitá di Udine*
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Timetable: 16 hrs. First lecture on March 2025, Torre Archimede, Room 2BC30

Credits: 3

Course requirements: Good knowledge of commutative algebra, projective geometry and algebraic geometry

Examination and grading: Presentation of a part of a research paper

SSD: MAT/03

Aim: Understand how some deep geometric problem can be translated by a simple polyhedral object.

Course contents:

Algebraic geometry studies the zero locus of polynomial equations connecting the related algebraic and geometrical structures. In several cases, nevertheless the theory is extremely precise and elegant, it is hard to read in a simple way the information behind such structures. A possible way of avoiding this problem is that of associating to polynomials some polyhedral structures that immediately give some of the information connected to the zero locus of the polynomial. In relation to this strategy I will introduce Newton-Okounkov bodies and Tropical Geometry, underlying the connection between the two theories

Bibliography:

KL Küronya, A., Lozovanu, V., Local positivity of linear series, arXiv:1411.6205v1 (2014)

PAGI Lazarsfeld, Robert, Positivity in algebraic geometry. I, Classical setting: line bundles and linear series, Springer-Verlag, Berlin, 2004.

PAGII Lazarsfeld, Robert, Positivity in algebraic geometry. II, Positivity for vector bundles, and multiplier ideals, Springer-Verlag, Berlin, 2004.

LM Lazarsfeld, R., Mustaţă, M., Convex bodies associated to linear series, Ann. Sci. Ec.Norm. Supér. (4) 42 (2009), no. 5, 783-835.

Courses of the “Computational Mathematics” area

Stability of Queueing Networks

Bernardo D'Auria[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova*
Email: bernardo.dauria@unipd.it

Timetable: 16 hrs. First lecture on 2nd half of April 2025, Torre Archimede, Room 2BC30

Credits: 3

Course requirements: Basic knowledge of continuous-time Markov Chains

Examination and grading: Solving the exercises assigned during the course

SSD: MAT/06

Aim: Queueing networks pervade modern systems, playing a crucial role in diverse fields such as telecommunications, computer networks, manufacturing processes, and service systems. This course aims to foster a profound comprehension of these stochastic models, with a specific emphasis on exploring issues related to stability. Following a formal introduction to queueing networks, the course will delve into the intricacies of stability properties. It will shed light on this concept by presenting examples of unstable networks while also introducing techniques for establishing the stability of stochastic networks through the application of fluid models and Lyapunov functions.

Course contents:

- Introduction to queueing networks
- The classical networks
- Instability of Subcritical Queueing Networks
- Stability of Queueing Networks

Bibliography:

M. Bramson (2008). Stability of queueing networks. Probab. Surv., 5:169-345. DOI:10.1214/08-PS137
M. Bramson, B. D'Auria and N. Walton (2021). Stability and Instability of the MaxWeight Policy. Math. Oper. Res., 46(4): 1611-1638. DOI:10.1287/moor.2020.1106

Mathematical Climate Finance

Andrea Macrina

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Website: amacrina.wixsite.com

Timetable: 16 hrs. First lecture on February 25, 2025, 15:00 (dates already fixed, see Calendar of Activities on https://dottorato.math.unipd.it/calendar/), Torre Archimede, Room 2BC30.

Course requirements: Probability and stochastic process theory in continuous time, and MSc-level financial mathematics knowledge/skills.

Examination and grading: None

Aim: This is an introduction to mathematical climate finance by in-depth treatment of some of the recent advances in this burgeoning field of research in financial mathematics. The focus will be on climate transition risk that, together with physical risk, is a major source of climate change risk impacting economies and financial markets. The mathematics, especially the formulation and modelling aspects, on which climate finance is based, is at the core of this course. The aim is thus the study and development of climate finance anchored in financial mathematics. Mathematical climate finance is an active area of research, and this course aims at keeping up-to-date with new insights and ongoing research progress in academia and the industry. A taste of the topic treated in this course can be sampled from the cover story in the *Fields Notes*, Vol. 12:4, Spring/Summer 2024.

Course contents:

1. *The carbon equivalence principle and the design of a probability space for climate policy-making*

   The carbon equivalence principle (CEP) is introduced that requires carbon-equivalent flows enabled, or caused, by a financial product to have equal status with cash flows. This reveals that existing financial products already have environmental impact and so are ESG-linked, without the need for any add-on terms. We show how treating enabled, or caused, carbon-equivalent flows enables position-keeping, and management to carbon net-zero. We apply the CEP to minimize the cost to achieving carbon net zero on financing portfolios that are of interest to Net Zero Banking Association members. We present a mathematical programming implementation for loan financing and associated data generation.

2. *Project finance under climate transition risk*

   The Carbon Equivalence Principle (CEP) states: all financial products shall contain a termsheet of the equivalent carbon flows from greenhouse gases that the financial products cause or enable, as well as their existing termsheet in terms of currency or other flows. This reveals that all existing financial products may already be environment-related. We apply the CEP to project finance, taking the example of power generation. The financial requirements to offset the costs of the future carbon flows radically change project costs, and risk that the assets become stranded, thus further increasing costs. For financial viability we introduce project re-designs that are financially net-zero, where a carbon net-zero constraint can also be included. The numerical project finance examples give a blueprint for wider application and insights into project finance impacted by carbon price risk.

3. *Internalisation of Scope 3 emissions in the trading book*

   Methods are presented for steering derivative portfolios based on the mitigation cost (benefit) of CO2-equivalent emissions (sequestration) these derivatives enable with counterparties. Thus, we price the cost-benefit of transitioning carbon externalities into changes affecting deals won and lost to steer the portfolios to net-zero based on a CO2-equivalent valuation adjustment (CO2eVA). This CO2eVA can be used in return-on-capital calculations, or as a shadow cost-benefit. We provide numerical examples considering interest rate swaps with a stylized regional airline, and a stylized shipping company that demonstrate highly significant effects in both cases.

4. *Climate-contingent convertible bonds*

   Adjustments to governmental climate transition policies and uncertainties linked to the ability for industry sectors and consumers to manufacture and purchase low carbon technologies, respectively, have led to an increasing need to embed a richer framework for uncertainty in climate transition outcomes. Such uncertainties create a number of financial risks for firms, their investors, the banking sector, and potentially sovereign risks. The need thus arises for the development of a bespoke pricing setup and financial instruments, which offer a mechanism to share climate transition risk. As an example, the so-called climate-contingent convertible (CLoCo) bond is proposed. This instrument enables firms to reduce the risk of default due to adverse climate transition policies over the product's lifetime. The proposed financial innovation implies reduced risk of default for firms, thereby increasing the expected firm value, but it also reduces the dependency of firm failures on the banking sector and potential bail-out costs incurred by sovereign nations.

5. *Stochastic integrated assessment models*

   Integrated assessment models (IAM) and climate-economic systems are developed in a deterministic setting. The most known example is the DICE model, by which optimal consumption and greenhouse emissions/reduction rates are calculated. We shall consider stochastic extensions to IAMs, and derive the optimal control levels. Applications to asset pricing are envisaged. Moreover, an alternative method to inject randomness into DICE shall be treated, too.

Bibliography (selected):

1. D. Brigo & F. Mercurio (2006) Interest Rate Models – Theory and Practice. Springer Berlin, Heidelberg
2. C. Cormack & A. Macrina (2024) Climate Transition Mitigation: Introducing the CLoCo Bond. SSRN:4851975.
3. C. Cormack & A. Macrina (2024) Sovereign climate-contingent convertible bonds (S-CloCo). SSRN:4959445.

4. C. Kenyon, M. Berrahoui & A. Macrina (2024) The Carbon Equivalence Principle: Minimising the Cost to Carbon Net Zero. Risk, Cutting Edge: Climate Finance, Risk.net Feb. 2024. Full version at SSRN:3979608.
5. C. Kenyon, A. Macrina & M. Berrahoui (2023) The Carbon Equivalence Principle: Methods for Project Finance. Risk, Cutting Edge: Climate Finance, Risk.net May 2023. Full version at SSRN:4035833.
6. C. Kenyon, A. Macrina & M. Berrahoui (2023) CO2eVA: Pricing the Transition of Scope 3 Emissions. Risk, Cutting Edge: Climate Finance, Risk.net Oct. 2023. Full version at SSRN:4136710.
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8. A. Macrina (2024) Mathematical climate finance: an investment in our future. Fields Notes, Vol. 12:4, Spring/Summer 2024.
9. G. Kassis & A. Macrina (2024) Arcade processes for informed martingale interpolation. ArXiv:2301.05936
10. S. E. Shreve (2004) Stochastic Calculus for Finance II: Continuous-Time Models. Springer Science & Business Media, LLC.

Bessel, Cox-Ingersoll-Ross, Ornstein-Uhlenbeck and Gaussian-Volterra processes with Wiener and fractional drivers

Prof. Yuliya Mishura[1]

[1] Taras Shevchenko National University of Kyiv, Department of Probability, Statistics and Actuarial Mathematics
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Timetable: 16 hours. First lecture on March 2025, Room 2BC30, Torre Archimede.

Course requirements: A previous knowledge of the basic concepts of stochastic processes is required. Knowledge of stochastic calculus could help for the advanced parts of the course, but during the course the basic concepts will be introduced for the understanding.

Examination and grading: Seminar

Aim: To introduce PhD students to the most interesting and modern models of stochastic processes, which, firstly, have various and quite deep analytical, wise-trajectory and asymptotic properties, and secondly, serve as adequate models in financial mathematics, physics, cellular communications, biology, etc. We will consider stochastic differential equations with both the Wiener process and the fractional Brownian process and its generalizations.

Course contents: Standard Cox-Ingersoll-Ross and Bessel processes: local and global behaviour of the trajectories as the functionals of coefficients. Fractional Cox-Ingersoll-Ross and Bessel processes and their main properties in dependence of the value of Hurst index. Drift parameters estimation in the standard and fractional Cox-Ingersoll-Ross models. Exact and approximate option pricing under stochastic volatility modeled by fractional Ornstein-Uhlenbeck process. Functional limit theorems for financial markets driven by fractional long-range dependent processes. Fractional Gaussian noise: entropy and alternative entropy functionals, analytical and computational problems related to predictors. Gaussian-Volterra processes as the generalization of fractional Brownian motion. Tempered fractional processes.

References:

1. N. Ikeda, S. Watanabe. Stochastic Differential Equations and Diffusion Processes 2nd Edition - March 1, 1992
2. Cherny, A. S. and Engelbert, H.-J. (2005). Singular Stochastic Differential Equations. Springer Berlin Heidelberg, Berlin, Heidelberg.
3. Lecture Notes on the Yamada-Watanabe Condition for the Pathwise Uniqueness of Solutions of Certain Stochastic Differential Equations S. Altay and Uwe Schmock.
4. Cox, J. C., Ingersoll, J. E., and Ross, S. A. (1985). A theory of the term structure of interest rates. Econometrica: journal of the Econometric Society, 53(2):385
5. Mishura, Y., Pilipenko, A., and Yurchenko-Tytarenko, A. (2023). Low-dimensional Cox-Ingersoll-Ross process. Stochastics, 2024 arXiv:2303.12911 [math.PR].

6. .Mishura, Y., and Yurchenko-Tytarenko, A. (2023). Standard and fractional reflected Ornstein-Uhlenbeck processes as the limits of square roots of Cox-Ingersoll-Ross processes. Stochastics, 95(1):99-117.

Numerical cubature and its applications

Alvise Sommariva[1]

[1] Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova
Email: alvise@math.unipd.it

Timetable: 16 hrs. First lecture on November 11, 2024, 14:30 (dates already fixed, see on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30

Credits: 3

Course requirements: Basic knowledge of analysis, linear algebra and numerical analysis.

Examination and grading: Oral examination.

SSD: MAT/08

Aim: The aim of this course is to provide an overview of numerical cubature over multivariate domains, even with complex geometries. Next we show its application to the approximation of continuous functions and to the solution of integral equations.

Course contents:

- Quadrature rules: from Newton-Cotes formula to Gaussian rules
- Cubature rules on multivariate domains
- Quasi-Montecarlo methods
- Cubature rules compression by Tchakaloff theorem (time permitting)
- Hyperinterpolation on general domain (time permitting)
- Numerical solution of Fredholm integral equations of second kind on multivariate domains (time permitting)

Bibliography:

1. K. Atkinson, Numerical integration on the sphere, J. Austral. Math. Soc. Ser. B 23 (1981/82), pp. 332–347.
2. K. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press; 1997.
3. J. Dick, F. Kuo, I.H. Sloan, High-dimensional integration: The quasi-Monte Carlo way, Acta Numerica, Cambridge University Press, 2013.
4. C.L. Lawson, R.J. Hanson, Solving Least Squares Problems, Classics in Applied Mathematics 15, SIAM, Philadelphia, 1995.
5. I.H. Sloan, Interpolation and hyperinterpolation over general regions, J. Approx. Theory 83 (1995), pp. 238–254.
6. A. Sommariva, Some cubature rules in Matlab, https://www.math.unipd.it/~alvise/sets.html.
7. A. Sommariva and M. Vianello, Compression of multivariate discrete measures and applications, Numer. Funct. Anal. Optim. 20 (2015), pp. 1198–1223.

Courses in collaboration with the Doctoral School on “Information Engineering”

for complete Catalogue and class schedule see on

https://phd.dei.unipd.it/course-catalogues/

Please check regularly the website of the Doctoral Course

Calendar of activities on

`https://calendar.google.com/calendar/u/0/embed?src=`
`fvsl9bgkbnhhkqp5mmqpiurn6c@group.calendar.google.com&ctz=Europe/Rome`

Applied Functional Analysis and Machine Learning

Prof. Gianluigi Pillonetto[1]

[1] Department of Information Engineering, Univ. Padova
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Timetable: 28 hrs (see Class Schedule on https://phd.dei.unipd.it/course-catalogues/)

Course requirements: The classical theory of functions of real variable: limits and continuity, differentiation and Riemann integration, infinite series and uniform convergence. Some elementary set theory and linear algebra.

Examination and grading: Two written exams, one in the middle of the course and the other at the end

Credits: 6

Aim: The course is intended to give a survey of the basic aspects of functional analysis, machine learning, regularization theory and inverse problems. At the end of the course, the student will have the methodological tools to tackle various machine learning problems in both regression and classification (estimation of functions from scattered and noisy data) starting from very general hypothesis spaces.

Course contents:

Review of some notions on metric spaces and Lebesgue integration: Metric spaces. Open sets, closed sets, neighborhoods. Convergence, Cauchy sequences, completeness. Completion of metric spaces. Review of the Lebesgue integration theory. Lebesgue spaces. Banach and Hilbert spaces: Finite dimensional normed spaces and subspaces. Compactness and finite dimension. Bounded linear operators. Linear functionals. The finite dimensional case. Normed spaces of operators and the dual space. Weak topologies. Inner product spaces and Hilbert spaces. Orthogonal complements and direct sums. Orthonormal sets and sequences. Representation of functionals on Hilbert spaces. Reproducing kernel Hilbert spaces, inverse problems and regularization theory: Representer theorem. Reproducing Kernel Hilbert Spaces (RKHS): definition and basic properties. Examples of RKHS. Function estimation problems in RKHS. Tikhonov regularization. Support vector regression and classification. Extensions of the theory to deep kernel-based networks: multi-valued RKHSs and the concatenated representer theorem.

References:

1. G. Pillonetto, T. Chen, A. Chiuso, G. De Nicolao, L. Ljung. Regularized System Identification – learning dynamic models from data, Springer Nature 2022
2. W. Rudin. Real and Complex Analysis, McGraw Hill, 2006
3. C.E. Rasmussen and C.K.I. Williams. Gaussian Processes for Machine Learning. The MIT Press, 2006
4. H. Brezis, Functional analysis, Sobolev spaces and partial differential equations, Springer 2010

5. G. Pillonetto, A. Aravkin, D. Gedon, L. Ljung, A.H. Ribeiro and T.B. Schön, Deep networks for system identification: a Survey, eprint 2301.12832 arXiv, 2023

In addition, written notes will be made available to the students.

Elements of Deep Learning

Prof. Gian Antonio Susto[1]

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Timetable: 24 hrs (see Class Schedule on https://phd.dei.unipd.it/course-catalogues/)

Course requirements: Basics of Machine Learning and basic Programming.

Examination and grading: Project based on the topics covered during the course or summary of scientific papers on advanced topics not covered directly during the course. Both the project and the summary will then be presented and discussed with the lecturer and the other students.

Credits: 5

Aim: The course will serve as an introduction to Deep Learning (DL) for students who already have a basic knowledge of Machine Learning. The course will move from the fundamental architectures (e.g. CNN and RNN) to hot topics in Deep Learning research.

Course contents:

- Introduction to Deep Learning: context, historical perspective, differences with respect to classic Machine Learning.
- Feedforward Neural Networks (stochastic gradient descent and optimization).
- Convolutional Neural Networks.
- Neural Networks for Sequence Learning.
- Elements of Deep Natural Language Processing.
- Elements of Deep Reinforcement Learning.
- Unsupervised Learning: Generative Adversarial Neural Networks and Autoencoders.
- Laboratory sessions in Colab.
- Hot topics in current research.

References:

- “Dive into Deep0“ Learning’ https://d2l.ai/
- “Deep Learning“ https://www.deeplearningbook.org/
- Notebook colab and slides from the lecturer