The isoperimetric problem: techniques and applications

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Timetable: 16 hrs. First lecture on March 3rd, 2025, 09:00 (dates already fixed, see Calendar of Activities on `https://dottorato.math.unipd.it/calendar`), Room 2BC30, Torre Archimede. Zoom link: `https://unipd.zoom.us/j/81230802638`

Course requirements: basics in PDEs, calculus of variations.

Examination and grading: seminar about a research paper in the topic.

Aim: The course aims at providing the attendees a number of modern techniques that have been utilized in connection with the isoperimetric problem, with an eye to the analysis of metric spaces and to the open problems in contemporary research.

Course contents: In this course we deal with the isoperimetric problem and the isoperimetric inequality in the Euclidean space. We present a number of related techniques and results, such as the direct method of the calculus of variations, the Schwarz symmetrisation, the ABP method. We will describe the close relation between isoperimetric, Sobolev and Faber-Krahn inequalities. We are going to discuss these and other tools in connection with the isoperimetric problem beyond the Euclidean space, with a focus on Riemannian manifolds with curvature bounds and the Heisenberg group. Time permitting, we will describe mean curvature inequalities, the quantitative isoperimetric inequality, and the minimal partition problems.

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