The Maximal Subgroups of the Symmetric Group

Martino Garonzi[1]

[1] *Departamento de Matemática, Universidade de Brasília (DF, Brazil)*
Email: mgaronzi@gmail.com

Timetable: 16 hrs. First lecture on March 18, 2025, 08:15 (dates already fixed, see on Calendar of Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Credits: 3

Introduction and background requirements: This is a course on finite permutation groups, focusing on the classification of the maximal subgroups of the finite symmetric groups S_n and of the finite alternating groups A_n. The background required is a basic knowledge of group theory (including notions such as solvability and nilpotency of groups) and group actions.

Examination and grading: Seminar

SSD: MAT/02

Aim: It is expected that the students achieve the knowledge of the O'Nan-Scott theorem and its proof, so as to make them able to prove results about generation and the subgroup structure of the symmetric group and of permutation groups in general. For example, it is expected that the students understand that the primitive maximal subgroups of S_n (and A_n) represent a "local obstruction": as a corollary of the O'Nan-Scott theorem, the set of numbers n such that S_n (and A_n) presents no such local obstruction has density 1.

Course contents:

- Finite permutation groups,
- Wreath products,
- Transitive and primitive actions,
- Minimal normal subgroups and socle of a finite group,
- Description of the maximal subgroups of the symmetric groups, O'Nan-Scott theorem for finite primitive permutation groups: affine groups, almost-simple groups, product actions and diagonal actions.
- Applications and discussion of relevant recent results.

References:

1. Cameron, Peter J.; Permutation groups. London Mathematical Society Student Texts, 45. Cambridge University Press, Cambridge, 1999.
2. Dixon, John D.; Mortimer, Brian; Permutation groups. Graduate Texts in Mathematics, 163. Springer-Verlag, New York, 1996.
3. Liebeck, Martin W.; Praeger, Cheryl E.; Saxl, Jan; On the O'Nan-Scott theorem for finite primitive permutation groups. J. Austral. Math. Soc. Ser. A 44 (1988), no. 3, 389–396.