.

Hyperbolic Geometry, Continued Fractions and Cluster Algebras

Prof. Daniel Labardini Fragoso[1]

[1]*Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova*
Email: daniel.labardinifragoso@unipd.it

Timetable: 24 hrs. First lecture on March 12th, 2025, 12:00 (dates already fixed, see Calendar of Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Credits: 4

Course requirements:
The prerequisites are reduced to the minimum:

- differential and integral calculus of one and several variables;
- elementary linear algebra;
- elementary algebra.

Knowledge of Differential or Riemannian Geometry would be helpful, but not necessary.

Examination and grading: Please contact the teacher of the course by e-mail.

SSD: MAT/02-MAT/03

Aim: To arrive at the notion of Cluster Algebra from two distinct, albeit related, elementary starting points: Hyperbolic Geometry and Continued Fractions.

Course contents:

1. Hyperbolic Geometry
    - Möbius transformations
    - Models and isometries of the hyperbolic plane
    - The Farey diagram and its symmetries
    - Ptolemy's identity in the hyperbolic plane
2. Continued Fractions
    - Recursive formalism
    - Finite continued fractions
    - Infinite continued fractions
    - Examples: the continued fractions of $1+\sqrt{2}$, $\frac{1+\sqrt{5}}{2}$ and e
    - Equivalent numbers
    - Continued fractions vs. perfect matchings of snake graphs
3. Cluster Algebras
    - Basic definitions and examples

- The Laurent phenomenon
- Cluster variables vs. perfect matchings of snake graphs

Bibliography:

1. James Anderson. Hyperbolic geometry. Springer-Verlag. Springer Undergraduate Mathematics Series. 2007.
2. Ilke Canakci, Ralf Schiffler. Cluster algebras and continued fractions. Compositio Mathematica 154 (2018), no. 3, 565–593. arXiv:1608.06568
3. Ilke Canakci, Ralf Schiffler. Snake graphs and continued fractions. European Journal of Combinatorics 86 (2020), 103081, 19 pp. arXiv:1711.02461
4. James W. Cannon, William J. Floyd, Richard Kenyon, Walter R. Parry. Hyperbolic Geometry. Flavors of Geometry (edited by Silvio Levy). Cambridge University Press, MSRI Publications, Volume 31. 1997.
5. Sergey Fomin, Andrei Zelevinsky. Cluster algebras IV: Coefficients. Compos. Math. 143 (2007), no. 1, 112–164. arXiv:math/0602259
6. Allen Hatcher. Topology of numbers. American Mathematical Society, 2022.
7. Serge Lang. Introduction to Diophantine approximations, 2nd ed. Springer-Verlag, 1995.
8. William J. LeVeque. Elementary Theory of Numbers. Dover Publications, 1990.
9. Toshihiro Nakanishi. An application of Penner's coordinates of Teichmüller space of punctured surfaces. RIMS Kokyuroku Bessatsu, Kyoto, 2010. B17: Infinite dimensional Teichmüller spaces and moduli spaces, 105–114.
10. Robert Penner. Lambda lengths. http://www.ctqm.au.dk/research/MCS/lambdalengths.pdf
11. Robert Penner. Decorated Teichmüller Theory. European Mathematical Society, the QGM Master Class Series. 2012. DOI 10.4171/075
12. Boris Springborn. The hyperbolic geometry of Markov's theorem on Diophantine approximation and quadratic forms. L'Enseignement Mathématique (2) 63 (2017), 333–373. arXiv:1702.05061
13. Lauren K. Williams. Cluster algebras: an introduction. Bull. Amer. Math. Soc. (N.S.) 51 (2014), no. 1, 1–26. arXiv:1212.6263
14. P.M.H. Wilson. Curved spaces, from Classical Geometries to Elementary Differential Geometry. Cambridge University Press. 2008.