Bessel, Cox-Ingersoll-Ross, Ornstein-Uhlenbeck and Gaussian-Volterra processes with Wiener and fractional drivers

Prof. Yuliya Mishura[1]

[1] *Taras Shevchenko National University of Kyiv, Department of Probability, Statistics and Actuarial Mathematics*
Email: yuliyamishura@knu.ua

Timetable: 16 hours. First lecture on May 14, 2025, 14:30 (dates already fixed see the Calendar of Activities at https://dottorato.math.unipd.it/calendar) Room 2BC30, Torre Archimede.

Course requirements: A previous knowledge of the basic concepts of stochastic processes is required. Knowledge of stochastic calculus could help for the advanced parts of the course, but during the course the basic concepts will be introduced for the understanding.

Examination and grading: Seminar

Aim: To introduce PhD students to the most interesting and modern models of stochastic processes, which, firstly, have various and quite deep analytical, wise-trajectory and asymptotic properties, and secondly, serve as adequate models in financial mathematics, physics, cellular communications, biology, etc. We will consider stochastic differential equations with both the Wiener process and the fractional Brownian process and its generalizations.

Course contents: Standard Cox-Ingersoll-Ross and Bessel processes: local and global behaviour of the trajectories as the functionals of coefficients. Fractional Cox-Ingersoll-Ross and Bessel processes and their main properties in dependence of the value of Hurst index. Drift parameters estimation in the standard and fractional Cox-Ingersoll-Ross models. Exact and approximate option pricing under stochastic volatility modeled by fractional Ornstein-Uhlenbeck process. Functional limit theorems for financial markets driven by fractional long-range dependent processes. Fractional Gaussian noise: entropy and alternative entropy functionals, analytical and computational problems related to predictors. Gaussian-Volterra processes as the generalization of fractional Brownian motion. Tempered fractional processes.

References:

1. N. Ikeda, S. Watanabe. Stochastic Differential Equations and Diffusion Processes 2nd Edition - March 1, 1992
2. Cherny, A. S. and Engelbert, H.-J. (2005). Singular Stochastic Differential Equations. Springer Berlin Heidelberg, Berlin, Heidelberg.
3. Lecture Notes on the Yamada-Watanabe Condition for the Pathwise Uniqueness of Solutions of Certain Stochastic Differential Equations S. Altay and Uwe Schmock.
4. Cox, J. C., Ingersoll, J. E., and Ross, S. A. (1985). A theory of the term structure of interest rates. Econometrica: journal of the Econometric Society, 53(2):385
5. Mishura, Y., Pilipenko, A., and Yurchenko-Tytarenko, A. (2023). Low-dimensional Cox-Ingersoll-Ross process. Stochastics, 2024 arXiv:2303.12911 [math.PR].

6. .Mishura, Y., and Yurchenko-Tytarenko, A. (2023). Standard and fractional reflected Ornstein-Uhlenbeck processes as the limits of square roots of Cox-Ingersoll-Ross processes. Stochastics, 95(1):99-117.