T-structures and stability conditions

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SS: MATH-02/A

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Dec xx, 2026	xx:xx-xx:xx	Jan xx, 2027	xx:xx-xx:xx
Dec xx, 2026	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Dec xx, 2026	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Jan xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading: (tentative) Seminar and discussion.

Introduction and background requirements: When studying a category, we are often interested in breaking it into simpler pieces. When we deal with abelian and triangulated categories, such as categories of modules over a finite dimensional algebra or their derived categories, respectively, we have various decomposition theorems, depending on the properties of the given category. A t-structure is a way of decomposing a category in such a way that any object can be viewed as the middle term of a short exact sequence (in the abelian case) or of a distringuished triangle (in the triangulated case) whose extremes belong to two orthogonal subcategories. This decomposition gives rise to the notion of torsion pairs in abelian categories, and of properly said t-structures in triangulated categories, first introduced in [1]. If a t-structure in a triangulated category $\mathcal{D}$ is bounded, moreover, it induces a cohomology functor to a smaller and abelian category $\mathcal{H}$, called the heart of the t-structure, that can be seen as "generating" $\mathcal{D}$. The notion of stability induces Jordan-Hölder type decompositions into semistable objectsforming well-behaved subcategories. A relevant notion of stability at the abelain level was introduced by A.King for module categories in representation theory. A King stability, on the other hand, defines a special t-structure called numerical torsion pair. In the triangulated case, we use Bridgeland stability conditions, providing a notion of slicing and a so-called Harder-Narasimhan filtration, as triangulated counterpart of a Jordan-Hölder filtration. Special t-structures are associated with a Bridgeland stability too. Stability conditions are used moreover to define and study moduli spaces of semistable objects, understand wall-crossing phenomena on the category, and have connections with other branches of mathematics, such as mathematical physics.

Aim: The goal of the course is to acquire familiarity with the notions of t-structures and of stability conditions for an abelian or a triangulated category, which are quite used tools for studying categories arising in different contextes

Course contents: The content of the course will be adapted to the background and interestes of

the students. Focus will be either put on t-structures and decomposition theorems for categories, with a more classical flavour, or on other applications of the notion of stability in triangulated categories. The notions introduced in the course will be studied in examples from representation theory and possibly geometry. The main topics that will be presented are:

- basic notions on abelian and triangulated categories and relevant examples from finite dimensional and dg algebras;
- decomposition theorems for abelian categories;
- t-structures in abelian and triangulated categories: torsion pairs, t-structures, bounded tstructures, cohomology;
- basics of tilting theory;
- numerical torsion pairs and King stability for module categories;
- slicings of triangulated categories and Bridgeland stability for triangulated categories.

References:

[1] A. A. Beeılinson, J. Bernstein, P. Deligne, Faisceaux pervers, Astérisque 100 (1982).

[2] Weibel, Introduction to Homological Algebra, Cambridge University Press (1994)

[3] Happel, Reiten, Smalø, Tilting in abelian categories and quasitilted algebras

[4] Tom Bridgeland, Stability conditions on triangulated categories

[5] selected research papers