Volterra Equations and their Markovian Lift(s)

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Credits: 3

SS: MATH-03/B

Timetable: 16 hrs. Torre Archimede, Room 2BC30:

Apr 12, 2027	14:30-16:30	Apr 19, 2027	14:30-16:30
Apr 13, 2027	11:00-13:00	Apr 20, 2027	11:00-13:00
Apr 14, 2027	14:30-16:30	Apr 21, 2027	14:30-16:30
Apr 15, 2027	11:00-13:00	Apr 22, 2027	11:00-13:00

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Course requirements: Real Analysis and Stochastic Calculus are considered foundational prerequisites, while a basic knowledge of stochastic modelling for finance is helpful yet not necessary

Examination and grading: A 30-minute seminar on one of the selected readings proposed in class

Aim:

1. Understand a generalisation of standard SDEs (SVEs) for which Markovianity and semimartingality do not hold.
2. Analyse Rough Volatility models to understand why fractional Brownian motion is particularly suitable for financial modeling.
3. Explore the infinite dimensional setting established by the use of Markovian lifts, in particular by comparing the different lifting techniques (measure-valued, curve-valued). Apply lifts to restore Markovianity and help in functional Itô calculus.
4. Indentify the connection between SVEs and Path-Dependent PDEs by linking the two to backward stochastic PDEs.
5. Identify the key challenges in the established methods to detect potential further developments and improvements in the established theory and discover new applications.

Course contents:

1. Preliminaries on Stochastic Volterra Equations (SVEs)
   - Definition and main features
   - Comparison with standard SDEs: loss of Markovianity and semimartingality
   - Existence and uniqueness results (in particular, for the singular kernel case)

2. Applications in finance: rough volatility models
    - Fractional Brownian motion and its role as the main player in volatility modelling
    - Rough Bergomi model: definition and its meaning
    - Connections between SVEs and (multifactor) fractional stochastic volatility models
3. Markovian lifts
    - Markovian lifts foundation: restoring Markovianity in SVEs
    - Measure-valued lift: genesis, how this is used in practice, and limitations
    - Curve-valued lift: link with forward variance curves in finance
    - Approximation techniques via Markovian representations
4. SVEs and Path-dependent PDEs
    - Connections between SVEs, PPDEs (path-dependent PDEs), and BSPDEs (backward stochastic PDEs)
    - Integral representations and Laplace transform methods
5. Advanced topics and open problems
    - Recent research on uniqueness and weak solutions for SVEs
    - Applications of Itô-type formulas in Volterra processes
    - Open questions in Markovian approximations and numerical methods

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