Random Schrödinger operators

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SSD: MATH-03/B

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Mar xx, 2027	xx:xx-xx:xx	Mar xx, 2027	xx:xx-xx:xx
Mar xx, 2027	xx:xx-xx:xx	Mar xx, 2027	xx:xx-xx:xx
Mar xx, 2027	xx:xx-xx:xx	Mar xx, 2027	xx:xx-xx:xx
Mar xx, 2027	xx:xx-xx:xx	Mar xx, 2027	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar)

Examination and grading: Oral exam at a time to be agreed beforehand

Introduction and background requirements: Some basic knowledge of probability theory and functional analysis is desirable

Aim: The main goal of these lectures is to study random Schrödinger operators, mainly on $\mathbb{Z}^d$ and present some tools for the proof of the so-called Anderson localization. More precisely, we will analyze the spectrum of the random operator

$$H_\omega = -\Delta + \lambda V_\omega \tag{1}$$

acting on $\ell^2(\mathbb{Z}^d)$. Here, Δ is the discrete Laplacian and the potential V_ω is a multiplication operator, i.e. $(V_\omega f)(x) = V(x,\omega)f(x)$, with $(V(x,\omega))_{x\in\mathbb{Z}^d}$ being a family of i.i.d. random variables. The understanding of the spectrum and eigenfunctions of H_ω is crucial in determining the evolution of quantum systems with the help of the Schrödinger equation. In his seminal work P. W. Anderson posited in [2] that under random potential there would be a transition in the transport properties of the model which heavily depend on the dimension d of the underlying lattice, the strength λ of the disorder, and the energy. Anderson's work has been crucial in explaining how electrons get trapped in disordered materials and represents one of the greatest achievement in physics (for which P. W. Anderson was awarded the Nobel prize in 1977). At the same time, models and phenomena set forth in this line of research appeared from the start to be true challenges for mathematicians: this led, through the years, to the development of mathematical tools of importance also beyond the context of Anderson localization. The focus of the course will be on the *fractional moment method* that leads to showing that, in the high disorder regime, the spectrum is pure point and the corresponging eigenfunctions are localized, and further that localization happens at a weak disorder/high energy regime.

Course contents:

- Some basics: Self-adjoint operators, their spectrum and resolvent sets. The resolution of the identity and the spectral theorem. Weyl's criterion with application to random Schrödinger operators.
- A finer analysis of the spectrum: pure point, absolutely continuous, and singular continuous spectrum. An interpretation of the decomposition via the RAGE theorem.
- Resolvent identity and random walk representation of the resolvent.
- Herglotz-Nevalinna functions and Borel-Stieltjes transforms.
- The Simon-Wolff criterion. Some Localization results for random Schrödinger operators via the *fractional moments method*.

References

[1] Michael Aizenman and Simone Warzel. *Random operators*, volume 168. American Mathematical Soc., 2015.

[2] P. W. Anderson. Absence of diffusion in certain random lattices. *Phys. Rev.*, 109:1492–1505, Mar 1958.

[3] Werner Kirsch. An invitation to random Schrödinger operators. *arXiv preprint arXiv:0709.3707*, 2007.

[4] Gerald Teschl. *Mathematical methods in quantum mechanics*, volume 157. American Mathematical Soc., 2014.