Doctoral Program in Mathematical Sciences
Department of Mathematics "Tullio Levi-Civita"
University of Padova

Doctoral Program in Mathematical Sciences

Catalogue of the courses 2026-2027

Updated November 6th, 2026

INTRODUCTION

This Catalogue lists the activities offered to Graduate Students in Mathematical Sciences for the academic year 2026-2027.

The activities in this Catalogue are of three types.

1. Courses offered by the Graduate School (= Courses of the Doctoral Program)
2. Courses offered by one of its curricula.
3. Other activities:
    a) selected courses offered by the PhD school in Information Engineering;
    b) selected courses offered by other PhD schools or other Institutions;
    c) reading courses (if any).

Taking a course from the Catalogue automatically grants the corresponding credits., while crediting of courses not included in the Catalogue (such as courses offered by the Scuola Galileiana di Studi Superiori, Summer or Winter schools, Series of lectures devoted to young researchers, courses offered by other PhD Schools) is possible but it is subject to the approval of the Executive Board. **Moreover, at most one course of this type may be credited.**

We emphasize the importance of attending courses, both to **broaden your culture in Mathematics**, as well as to deepen your knowledge in your own area of interest.

HOW TO REGISTER AND UNREGISTER TO COURSES

The registration to a Course must be done online.
Students can access the **online registration form** in the dedicated page of the Doctoral Course website at https://dottorato.math.unipd.it/node/1682 clicking on "click to enroll" of the chosen courses.
To register, fill the registration form with all required data, and validate with the command "submit". The system will send a confirmation email message to the address indicated in the registration form; please save this message, as it will be needed in case of cancellation.

Registration to a course implies the commitment to follow the course.

Requests for **cancellation** to a course must be submitted in a timely manner, and **at least one month before the course** (except for courses that begin in November) using the link indicated in the confirmation email message.

REQUIREMENTS FOR PARTICIPANTS NOT ENROLLED IN THE GRADUATE SCHOOL OF MATHEMATICS

The courses in this Catalogue, although part of activities of the Graduate School in Mathematics, are open to all students, graduate students, researchers of this and other Universities.
For organizational reasons, external participants are required to communicate their intention to attend a course, by sending an email to phd.math@math.unipd.it, at least two months before the start date, if the course is scheduled from January 2027 onwards and as soon as possible for courses that take place until December 2026.
To **register,** follow the procedure described in the preceding section.
Possible **cancellations** to courses must also be notified.

List of Courses

Courses of the Doctoral Program

1. Prof. Marco Di Summa
   Convex polyhedra and their diameter **DP-1,2**

2. Dott. Sergej Monavari, Prof. Orsola Tommasi
   Introduction to moduli spaces **DP-3,4**

3. Prof.Franco Rampazzo
   Set separation, minimizing processes, and controllability **DP-5-6**

4. Prof.Alvise Sommariva
   Introduction to multivariate polynomial approximation **DP-7,8**

Courses of the "Mathematics" area

1. Dott. Anna Barbieri
   T-structures and stability conditions **M-1,2**

2. Prof.Martino Bardi
   Fully Nonlinear Partial Differential Equations: an introduction **M-3,4**

3. Prof.Francesco Fassò
   Nonholonomic Mechanical Systems **M-5,6**

4. Prof. Jack Kelly
   Foundations of derived analytic geometry **M-7,10**

5. Prof. Alessandra Pluda
   Geometric Flows and Applications **M-11,12**

6. Dott. Giorgio Stefani
   An invitation to Fractional Sobolev Spaces **M-13,14**

Courses of the "Computational Mathematics" area

1. Dott. Ofelia Bonesini
   Volterra Equations and their Markovian Lift(s) **MC-1,3**

2. Prof.Alberto Chiarini, Prof. Giambattista Giacomin
   Random Schrodinger operators **MC-4,5**

3. Dott. Vanessa Jacquier
   Large deviations and Metastability **MC-6,7**

4. Prof. Federico Piazzon
   Linear Parabolic Equations in Hilbert Spaces: analysis and numerical approximation **MC-8,9**

5. Prof. Benedetto Piccoli
   Mathematical Control of Multi-Agent Systems: From Traffic to Crowds **MC-10,13**

Courses in collaboration with the Doctoral School in "Information Engineering"

Please check regularly the website of the Doctoral Course in Information Engineering at the URL https://phd.dei.unipd.it/course-catalogues/

1. Prof. Alessandro Languasco
   Introduction to Modern Cryptography **DEI-1,3**

2. Prof. Gianluigi Pillonetto
   Applied Functional Analysis and Machine Learning **DEI-4,6**

Soft Skills

1. Lean for Working Mathematicians **SS-1,3**

2. Tools and Applications of Machine Learning for Social Challenges **SS-4,5**

Courses of the Doctoral Program

Convex polyhedra and their diameter

Marco Di Summa[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova*
Email: disumma@math.unipd.it

SS: MATH-03/B

Credits: 4

Timetable: 24 hrs.; Torre Archimede, Room 2BC30

Tue Nov 10, 2026	10:30-12:30	Tue Dic 1, 2026	10:30-12:30
Fri Nov 13, 2026	10:30-12:30	Fri Dic 4, 2026	10:30-12:30
Tue Nov 17, 2026	10:30-12:30	Wed Dic 9, 2026	10:30-12:30
Fri Nov 20, 2026	10:30-12:30	Fri Dic 11, 2026	10:30-12:30
Tue Nov 24, 2026	10:30-12:30	Tue Dic 15, 2026	10:30-12:30
Fri Nov 27, 2026	10:30-12:30	Fri Dic 18, 2026	10:30-12:30

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading: Seminar

Course requirements: Standard Mathematical knowledge will be sufficient.

Aim: A convex polyhedron is defined as the set of solutions to a given system of linear inequalities, and as such, it generalizes to n dimensions the commonly known notion of 3-dimensional polyhedron. Besides being natural and interesting mathematical objects, general n-dimensional polyhedra are fundamental in the field of optimization.

In this course we will introduce some basic properties of polyhedra and discuss their importance and usefulness. After reviewing some fundamental structural results, we will focus on the notion of diameter of a polyhedron. In order to understand what the diameter of a polyhedron is, one can visualize the 3-dimensional case, where the notions of vertex and edge should be clear: The diameter of a polyhedron is the maximum distance between any two vertices, where, for a fixed pair of vertices, the distance is measured by counting the minimum number of steps needed to move from one vertex to the other by traversing edges. (This can be properly extended to the n-dimensional case.) We will survey classical and famous results on the diameter of polyhedra, as well as some recent achievements. We will spend some time on the Hirsch conjecture, which was posed in 1957, proved for some special but very important cases in the subsequent decades, and finally disproved by Francisco Santos in 2010 in a paper that won the Fulkerson prize, one of the most important awards in the area of Discrete Mathematics. However, a weaker version, called the polynomial Hirsch conjecture, is still open, and its correctness would be extremely important to assess the efficiency of some algorithms for the solution of linear optimization problems.

The topic of this course can be interesting to students in various fields of Mathematics, as it connects (at least) combinatorial and discrete geometry, graph theory, and optimization. Graphs and their diameter are also studied in algebra and probability (e.g., random walks are sometimes the tool to prove the existence of a short path between two vertices).

Course contents:

- Polyhedra: basic facts, inequality description, vertex/ray description, fundamental examples
- The skeleton of a polyhedron and the notion of diameter
- Lower and upper bounds on the diameter of a polyhedron
- Hirsch conjecture and its link with the efficiency of linear programming algorithms
- Correctness of the conjecture and diameter bounds for some relevant cases (e.g., 0-1 polyhedra; polyhedra arising from network-flow, transportation, assignment and matching problems; polyhedra with bounded sub-determinants in the constraint matrix)
- Santos' counterexample to the Hirsch conjecture (sketch)
- The (weaker) polynomial Hirsch conjecture and some related results
- The recent notion of circuit diameter

Bibliography:

For the introductory part on polyhedra, students can refer to Chapter 3 of the following book, also accessible online when connected through the University network:

- M. Conforti, G. Cornuéjols, G. Zambelli, *Integer Programming*, Springer, 2014.

The notion of diameter and some classical related results are discussed, e.g., in Chapter 3 of the following book:

- G. M. Ziegler, *Lecture on Polytopes*, Springer, 2007.

More specific references (including those needed to be prepared for the exam, which is in the form of student seminar) will be provided during the course.

Introduction to moduli spaces

Sergej Monavari[1], Orsola Tommasi[2]

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[2] *Dipartimento di Matematica "Tullio Levi-Civita"*
Email: orsola.tommasi@unipd.it

SSD: MATH/02-B

Credits: 4

Timetable: 24 hrs. Torre Archimede, Room 2BC30:

Mar 1, 2027	10:30-12:30	Mar 22, 2027	10:30-12:30
Mar 3, 2027	10:30-12:30	Mar 24, 2027	10:30-12:30
Mar 8, 2027	10:30-12:30	Apr 5, 2027	10:30-12:30
Mar 10, 2027	10:30-12:30	Apr 7, 2027	10:30-12:30
Mar 15, 2027	10:30-12:30	Apr 12, 2027	10:30-12:30
Mar 17, 2027	10:30-12:30	Apr 14, 2027	10:30-12:30

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar)

Examination and grading: The final exam consists of giving a 45 minutes talk. The lecturers will provide a list of topics and references. Participants are also welcome to propose a different topic, in agreement with the lecturers.

Course requirements: Basic notions of geometry, as provided by a Mathematics degree. This course is open to any PhD student. Depending on the students' background, I will include a review of useful notions from geometry and topology.

Aim: In this course, I would like to present the basic ideas and formalism underlying the concept of moduli space, along with some main examples of interdisciplinary interest. Besides the moduli space of Riemann surfaces, interesting examples with applications in different fields include:

- mirror symmetry, a construction from theoretical physics that predicts that there exist pairs of topological spaces (X, X') such that the moduli space of complex structures on X is isomorphic to the moduli space of symplectic structures on X', and vice versa;
- modular curves in number theory, which are moduli spaces parametrizing elliptic curves with additional structures.

Course contents: A moduli space is a space parametrizing all possible objects of a certain fixed type. A classical example is the following: let us fix a compact oriented surface Σ. Then the space of all possible complex structures on Σ is the moduli space $\mathcal{M}_g$ of genus g Riemann surfaces, where g is the topological genus of Σ.
By construction, the points of $\mathcal{M}_g$ correspond to the isomorphism classes of Riemann surfaces

of genus g. This kind of construction generalizes to many other classification problems. In good cases, moduli spaces will turn out to be complex manifolds or varieties. However, in most cases the moduli space will not be truly a manifold, but rather a mild generalization of it, called an orbifold. Although the construction of moduli spaces originates in algebraic geometry, moduli spaces themselves are of interest also in other areas of mathematics, such as other areas of geometry, topology, group theory, analysis and mathematical physics.

References:

- Kock, Joachim; Vainsencher, Israel. An invitation to quantum cohomology. Kontsevich's formula for rational plane curves. Progress in Mathematics, 249. Birkh¨auser Boston, Inc., Boston, MA, 2007. xiv+159 pp.
- Newstead, P. E. Introduction to moduli problems and orbit spaces. Tata Institute of Fundamental Research Lectures on Mathematics and Physics, 51. Tata Institute of Fundamental Research, Bombay; by the Narosa Publishing House, New Delhi, 1978. vi+183 pp.
- Cox, David A.; Katz, Sheldon. Mirror symmetry and algebraic geometry. Mathematical Surveys and Monographs, 68. American Mathematical Society, Providence, RI, 1999. xxii+469 pp.
- Harris, Joe; Morrison, Ian. Moduli of curves. Graduate Texts in Mathematics, 187. Springer-Verlag, New York, 1998. xiv+366 pp.

Set separation, minimizing processes, and controllability

Franco Rampazzo

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova*
mail: rampazzo@math.unipd.it

SS: MATH-03/A

Credits: 4

Timetable: 24 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Feb xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Feb xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Feb xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Feb xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Feb xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Feb xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar)

Examination and grading: Knowledge of definitions and of the theorems' statements, together with a seminar on a research paper (or some book's section) provided by the instrructor.

Introduction and background requirements: Set-separation is a basic principle lying at the basis of several classical or more recent results on necessary conditions for solutions of a wide class of minimum problems. Actually, from Fermat's Theorem on extrema of real functions to (possibly higher order) Maximum Principles in constrained Control Theory and beyond, set-separation plays a foundational role. Dually, results on local controllability, namely the possibility of invading full neighborhoods of an initial point by given dynamical systems, can be formulated in terms of a lack of set-separation. In order to follow this course, no special competences are required besides a standard knowledge of real analysis.

Aim: The student will get a general understanding on how the application of set-separation, through suitable notions of (possibly non-smooth) tangential objects and the corresponding open mapping theorems, plays a crucial role in deducing necessary conditions for minimum problems of various kind. Questions of controllability will be also the occasion to get some insight into the use of non-commutativity of vector fields, e.g. in Sub-Riemannian geometry or in some questions concerning Sub-Elliptic PDEs.

Course contents:

- Set separation and open mapping theorems.
- Basic of ODEs with controls.
- Necessary conditions for various kinds of minimum problems.
- Non-commutativity of vector fields, Lie brackets and classical theorems on the subject.

- Controllability results and some connected applications in PDEs and Subriemannian Geometry.

References: printed notes provided by the teacher.

Introduction to multivariate polynomial approximation

Alvise Sommariva[1]

[1] *Department of Mathematics Tullio Levi-Civita, University of Padua*
Email: alvise@math.unipd.it

SSD: MATH/05-A

Credits: 4

Timetable: 24 hrs. Torre Archimede, Room 2BC30:

Nov 9, 2026	14:00-16:00	Nov 18, 2026	14:00-16:00
Nov 10, 2026	14:00-16:00	Nov 19, 2026	14:00-16:00
Nov 11, 2026	14:00-16:00	Nov 23, 2026	14:00-16:00
Nov 12, 2026	14:00-16:00	Nov 24, 2026	14:00-16:00
Nov 16, 2026	14:00-16:00	Nov 25, 2026	14:00-16:00
Nov 17, 2026	14:00-16:00	Nov 26, 2026	16:30-18:30

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading: seminar on a paper.

Introduction and background requirements: the course requires the knowledge of the basics of measure theory and multivariate calculus as well as some concepts of numerical analysis as interpolation, least-squares and one dimensional numerical integration; where necessary, the lecturer will introduce elementary notions, so to broad the course to a wider audience.

Aim: the goal is to provide some ideas on multivariate polynomial approximation on classical domains as simplex, sphere, cubes, but also on more complicated ones that arise in Finite or Virtual Elements Methods. In general these topics are not treated in university courses, though they are of fondamental interest in applied sciences. The outcomes are of practical and theoretical nature. The student will learn how to approach numerically multivariate polynomial approximation, but also deeper mathematical details concerning error analysis. Numerical experiments will be given to provide more insights on the topics.

Course contents:

- univariate and multivariate interpolation (on well-choosen sets) and Lebesgue constants,
- hyperinterpolation and variants for data denoising,
- numerical cubature on multivariate domains: application to hyperinterpolation, FEM and VEM,
- (weakly) admissible meshes and good interpolation/least squares pointsets (time permitting).

References:

1. P.J. Davis, P. Rabinowitz, *Methods of Numerical Integration*, Dover Publications, 2007.
2. N. Levenberg, J-P. Calvi, *Uniform approximation by discrete least squares polynomials*, Journal of Approximation Theory, 152, Issue 1 (2008), pp.82–100.
3. A. Sommariva and M. Vianello, *Computing approximate Fekete points by QR factorisations of Vandermonde matrices*, Computers Mathematics with Application, 57 (2009), pp.1324-1336.
4. I.H. Sloan, *Polynomial Interpolation and Hyperinterpolation over General Regions*, Journal of Approximation Theory, 83, Issue 2 (1995), pp.238–254.

Courses of the “Mathematics” area

T-structures and stability conditions

Anna Barbieri[1]

[1] *Dipartimento di Informatica, Universitá di Verona*
Email: anna.barbieri@univr.it

SS: MATH-02/A

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Dec xx, 2026	xx:xx-xx:xx	Jan xx, 2027	xx:xx-xx:xx
Dec xx, 2026	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Dec xx, 2026	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Jan xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading: (tentative) Seminar and discussion.

Introduction and background requirements: When studying a category, we are often interested in breaking it into simpler pieces. When we deal with abelian and triangulated categories, such as categories of modules over a finite dimensional algebra or their derived categories, respectively, we have various decomposition theorems, depending on the properties of the given category. A t-structure is a way of decomposing a category in such a way that any object can be viewed as the middle term of a short exact sequence (in the abelian case) or of a distringuished triangle (in the triangulated case) whose extremes belong to two orthogonal subcategories. This decomposition gives rise to the notion of torsion pairs in abelian categories, and of properly said t-structures in triangulated categories, first introduced in [1]. If a t-structure in a triangulated category $\mathcal{D}$ is bounded, moreover, it induces a cohomology functor to a smaller and abelian category $\mathcal{H}$, called the heart of the t-structure, that can be seen as "generating" $\mathcal{D}$. The notion of stability induces Jordan-Hölder type decompositions into semistable objectsforming well-behaved subcategories. A relevant notion of stability at the abelain level was introduced by A.King for module categories in representation theory. A King stability, on the other hand, defines a special t-structure called numerical torsion pair. In the triangulated case, we use Bridgeland stability conditions, providing a notion of slicing and a so-called Harder-Narasimhan filtration, as triangulated counterpart of a Jordan-Hölder filtration. Special t-structures are associated with a Bridgeland stability too. Stability conditions are used moreover to define and study moduli spaces of semistable objects, understand wall-crossing phenomena on the category, and have connections with other branches of mathematics, such as mathematical physics.

Aim: The goal of the course is to acquire familiarity with the notions of t-structures and of stability conditions for an abelian or a triangulated category, which are quite used tools for studying categories arising in different contextes

Course contents: The content of the course will be adapted to the background and interestes of

the students. Focus will be either put on t-structures and decomposition theorems for categories, with a more classical flavour, or on other applications of the notion of stability in triangulated categories. The notions introduced in the course will be studied in examples from representation theory and possibly geometry. The main topics that will be presented are:

- basic notions on abelian and triangulated categories and relevant examples from finite dimensional and dg algebras;
- decomposition theorems for abelian categories;
- t-structures in abelian and triangulated categories: torsion pairs, t-structures, bounded tstructures, cohomology;
- basics of tilting theory;
- numerical torsion pairs and King stability for module categories;
- slicings of triangulated categories and Bridgeland stability for triangulated categories.

References:

[1] A. A. Beeılinson, J. Bernstein, P. Deligne, Faisceaux pervers, Astérisque 100 (1982).

[2] Weibel, Introduction to Homological Algebra, Cambridge University Press (1994)

[3] Happel, Reiten, Smalø, Tilting in abelian categories and quasitilted algebras

[4] Tom Bridgeland, Stability conditions on triangulated categories

[5] selected research papers

Fully Nonlinear Partial Differential Equations: an introduction

Martino Bardi[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova*
Email: martino.bardi@unipd.it

SS: MATH-03/A

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30:

Nov 25, 2026	10:30-12:30	Dic 10, 2026	10:30-12:30
Nov 26, 2026	10:30-12:30	Dic 14, 2026	11:30-13:30 (Room 1AD100)
Nov 30, 2026	10:30-12:30	Dic 16, 2026	10:30-12:30
Dic 3, 2026	10:30-12:30	Dic 21, 2026	10:30-12:30

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading: seminar on a paper

Introduction and background requirements: I will start giving the main examples of fully nonlinear PDEs and their motivations. In the first part of the course I'll introduce the definitions and basic properties of viscosity solutions to degenerate elliptic equations. I'll state the main results on existence and uniqueness of solutions to Dirichlet and Cauchy boundary value problems, focusing in particular on the comparison principles, that are the cornerstone of the theory. The proofs will be given in some special cases, interesting for applications. The second part of the course will concern two important qualitative properties of solutions: the Strong Maximum Principle and the Liouville property. They are standard results for classical solutions of linear elliptic PDEs, and many extensions are known, especially for divergence form equations. My goal is explaining how the viscosity methods allow to turn around the difficulties of non-smooth solutions, fully nonlinear equations, and their possible degeneracies.

A solid **background** in advanced calculus is enough; some previous knowledge on PDEs helps to appreciate better the contents of the course.

Aim: Introducing the students to a theory of weak solutions for nonlinear problems with a very wide range of applications

Course contents:

- Second order fully nonlinear PDEs: notions of ellipticity. Main examples of equations: Hamilton-Jacobi, Monge-Ampere, geometric and Hessian equations, subelliptic PDEs,....
- Definition of viscosity solution, with motivations.
- Comparison principles: a review and some examples.

- Some results on existence and uniqueness of solutions to Dirichlet and Cauchy boundary value problems.
- Connections of some Hamilton-Jacobi equations with Calculus of Variations, deterministic and stochastic optimal control, and differential games.
- The Strong Maximum Principle (SMP): from classical to viscosity solutions; propagation of maxima for degenerate equations
- Examples of subelliptic PDEs: H¨ormander vector fields and the Heisenberg group. Subunit vector fields and a general SMP
- Liouville properties: from solutions of uniformly elliptic equations to subsolutions of degenerate equations.

References:

- D. Gilbarg and N.S. Trudinger: Elliptic PDEs of 2nd order, Springer 1983.
- M. Crandall, H. Ishii, P.L. Lions: Users guide to viscosity solutions, Bull. AMS 1992.
- L.A. Caffarelli and X. Cabre: Fully nonlinear elliptic equations, AMS Colloquium Publication, vol. 43, American Mathematical Society, 1995
- M. Bardi and I. Capuzzo-Dolcetta: Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equations, Birkh¨auser 1997.
- S. Koike: A beginner's guide to the theory of viscosity solutions, MSJ Memoirs 2004.
- M. Bardi, A. Goffi: New strong maximum and comparison principles for fully nonlinear degenerate elliptic PDEs, Calc. Var. PDE (2019)

Nonholonomic Mechanical Systems

Francesco Fassó[1]

[1]*Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova*
Email: francesco.fasso@unipd.it

SS: MATH-04/A

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Xxx xx, 202x	xx:xx-xx:xx	Xxx xx, 202x	xx:xx-xx:xx
Xxx xx, 202x	xx:xx-xx:xx	Xxx xx, 202x	xx:xx-xx:xx
Xxx xx, 202x	xx:xx-xx:xx	Xxx xx, 202x	xx:xx-xx:xx
Xxx xx, 202x	xx:xx-xx:xx	Xxx xx, 202x	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading: Verification of the comprehension and knowledge of the core topics covered in the course through a colloquium and/or the presentation of specific topics studied individually.

Introduction and background requirements: In mechanics, a nonholonomic constraint is a restriction on the possible velocities of a holonomic system without restricting its possible configurations. Examples are systems of rigid bodies whose contact points are constrained to have a preassigned velocity and the possibility to manoeuvre a car into parallel parking despite the fact that the wheels cannot move transversally. Nonholonomic mechanics is very different from Lagrangian mechanis because the equations of motion do not come from a variational principle. Many properties of the dynamics are still not completely understood and the subject is an active field of research, which is of interest in various areas of Mathematics and Engeneering. The aim of the course is to give an introduction to the subject, focussing mostly on the dynamical aspects and presenting the students to recent progress and open questions in the field. Prerequisites are basic notions of classical mechanics (Lagrangian mechanics), of the qualitative theory of ODEs and of analysis and differential geometry. More advanced topics (e.g., Frobenius theorem, Lie groups actions) will be reviewed.

Aim: The aim of the course is to provide a basic, though solid, knowledge of the geometric structure and of the dynamics of nonholonomic mechanical systems, leading the students to the level of understanding the current literature.

Course contents: Nonholonomic constraints. Distributions and Frobenius theorem. Ideal constraints and D'Alembert principle. The equations of motion for linear and nonlinear constraints. The role and structure of the reaction forces. Basic examples of nonholonomic mechanical systems. First integrals: energy, momenta, the nonholonomic Noether theorem. Based on the interest of the partecipants and on the time availability, one or more of the following topics might be covered: (non)existence of conserved measures; reduction under symmetry; integrability; control and trajectory planning of nonholonomic systems.

References:

1. Lecture notes will be prepared and made accessible to the students during the course.
2. L. García-Naranjo, Geometry and dynamics of nonholonomic systems. Lecture Notes for a course given in this Doctoral School in 2021.
3. Ju. I. Neĭmark and N.A. Fufaev, Dynamics of Nonholonomic Systems (AMS, 1972).
4. F. Bullo and A.D. Lewis, Geometric Control of Mechanical Systems (Springer, 2004).
5. R. Cushman, J.J. Duistermaat and J. Snyaticki, Geometry of Nonholonomically Constrained Systems (World Scientific, 2010).
6. A. Bloch, Nonholonomic Mechanics and Control (Springer, 2015).

Foundations of Derived Analytic Geometry

Jake Kelly[1]

[1] Lincoln College, Oxford
Email: jack.kelly@lincoln.ox.ac.uk

SS: MATH-02/B

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Sept 6, 2027	xx:xx-xx:xx	Sept xx, 2027	xx:xx-xx:xx
Sept xx, 2027	xx:xx-xx:xx	Sept xx, 2027	xx:xx-xx:xx
Sept xx, 2027	xx:xx-xx:xx	Sept xx, 2027	xx:xx-xx:xx
Sept xx, 2027	xx:xx-xx:xx	Sept 27, 2027	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading: Seminar on topics complementary to the course, or another form of assessment to be agreed upon with the teacher

Introduction and background requirements: A solid background in category theory is required. Basic knowledge of commutative algebra, algebraic geometry, and functional analysis is helpful but not essential

Aim: Derived analytic and C^∞-geometry are emerging fields of mathematics which have exciting implications for the study of moduli problems appearing in physics, complex geometry, and arithmetic geometry.
The goal of this course is to introduce a model of derived analytic geometry as algebraic geometry, in the sense of [16], relative to the category of complete bornological vector spaces over either an Archimedean or non-Archimedean field. We will define categories of derived analytic stacks, and explain how many important results - existence of cotangent complexes, six functor formalisms, descent for K-theory, representability theorems - translate to the analytic world. We will explain some applications, including the construction of analytic and C^∞-moduli spaces - moduli of Galois representations, of instantons, and of solutions to PDEs. The main reference will be the book draft [3].

Course contents:

Lecture 1: Motivations for derived analytic geometry, and homological algebra for functional analysis. In the first lecture we motivate the development of derived analytic geometry as relative algebraic geometry. For example, we will explain how one can define the category $\mathbf{QCoh}(X)$ of quasi-coherent sheaves on an analytic space in a way which resolves Gabber's counter-example to the naïve definition. The category $\mathbf{QCoh}(X)$ will necessarily be derived, and we will justify why by showing that natural maps in analytic geometry are not flat. We will also give examples of deep theorems which have been proven using this algebro-geometric model of derived analytic geometry, including HKR [9] and Koszul duality [4]. Having mo-

tivated the desire for an algebraic model of (derived) analytic geometry, we make a start on the technical details. The algebras of interest in analytic geometry - e.g., entire functions on complex analytic spaces - have natural topological, or rather bornological, structures. We explain how to marry functional analysis with homological algebra by introducing quasi-abelian categories.

Lecture 2: Homotopy theory in exact categories, and homological characterisations of the analytic topologies. Quasi-abelian categories are canonically Quillen exact categories. Following [7], here we will explain conditions on an exact category such that it is a convenient setting for homological/ homotopical algebra - precisely, such that they are homotopical algebra contexts in the sense of [16]. This will involve defining derived tensor products and derived mapping spaces. Next we will make a connection with geometry. Let $\{U_i \to U\}_{i\in I}$ be a collection of maps of Stein spaces over C or rigid affinoids over a non-Archimedean field K. We will recall the main results of [2], [1], which show how to characterise when such a collection is an open cover, purely in terms of the corresponding collection of maps of algebras of analytic functions $\{\mathcal{O}(U) \to \mathcal{O}(Ui)\}_{i\in I}$. This will necessitate the definition of homotopy epimorphisms of algebras.

Lecture 3: Higher algebra in derived algebraic contexts. In this lecture we will introduce the category-theoretic framework for generalised algebraic geometry furnished by Raksit's derived algebraic contexts [13]. This will include the definition of the category **DA**$\log$ **(C)** of derived algebras. We will also explain the relevant example for derived analytic geometry, namely, the ∞-category **Ch** (CBorn$_{\mathbb{Z}}$) of complexes of complete bornological abelian groups. Next we justify why derived algebraic contexts are convenient settings for higher algebra. In particular, we will prove the existence of the Tor-spectral sequence, and deduce important consequences for homotopy epimorphisms and étale maps.

Lecture 4: Derived algebraic contexts, Lawvere theories, and topologies. This is a technical lecture in which we show how to define rational localisations, étale maps, and smooth maps relative to a Lawvere theory. We will explain the relevance to analytic geometry using the Lawvere theories determined by Steins and affinoids - this will be further developed in later lectures. Finally, we will introduce the Grothendieck topologies of interest on $\mathbf{Aff}^{cn}$ **(C)** - namely, the G- and étale topologies associated to a Lawvere theory.

Lecture 5: Derived analytic geometry I - Affines and descent for modules. Here we discuss descent for modules. We introduce the notion of descendability, due to [10]. We explain why this is a reasonable condition to impose, and why it is satisfied by the topologies introduced in the previous lecture. We will then specialise to the usual topologies of open covers and of étale covers respectively, restricted to algebras of analytic functions on Steins/ affinoids. We will define categories of derived Stein algebras and derived affinoid algebras, and prove that, when restricted to these algebras, the G- and étale topologies can be characterised in the following way: { Spec$(_i)$ $\to$Spec$(A)\}_{i\in I}$ is a cover precisely if { Spec$(\pi_0(A_i))$ß Spec $(\pi_0(A))\}_{i\in I}$ is a cover, and for each $i \in \mathcal{I}$ and $n \in \mathbb{N}$, the natural map $\pi_0(A_i)\hat{\otimes}^{\mathbb{L}}_{\pi_0(A)}\pi_n(A) \to \pi_n(A_i)$ is an equivalence. In particular the topology really is a derived enhancement of the classical one.

Lecture 6: Derived analytic geometry II - analytic stacks and comparisons. We are finally ready to define schemes and Deligne-Mumford stacks. We explain why categories of classical analytic spaces embed fully faithfully in these new derived ones. We further show that a Deligne-Mumford stack admits an obstruction theory. In particular, such a stack has a cotangent complex. We recall related results for schemes such as the HKR theorem [9]. We will also sketch the existence of a moduli stack of Deligne-Mumford stacks, and a procedure for

constructing derived analytic blow-ups. Finally, we will compare our stacks with those of Porta and Yue Yu ([12]), and of Clausen and Scholze ([15]). In particular, in the complex analytic setting, we will show that Porta and Yu's geometric stacks embed fully faithfully in ours.

Lecture 7: Other geometries. In this lecture we explain how other geometries can be realised using the bornological approach. This will include C^∞-geometry, formal geometry, and a bornological enhancement of algebraic geometry. We will further explain how the formal geometry case allows us to compare with the non- Archimedean derived stacks of Porta and Yue Yu.

Lecture 8: Horizons - representability, K-theory, and six-functor formalisms. In the final lecture we explain the myriad possibilities presented by this theory of derived analytic and C^∞-geometry. We will survey recent results including K-theory [8], D-modules, and six functor formalisms (both [11]). We will also recall the representability theorem of [14] for derived stacks, and suggest how this can be used to construct moduli stacks in derived analytic and C^∞-geometry, including moduli of Galois representations and solutions to PDEs (following Pardon [6] and Steffens [5]).

References:

[1] Oren Ben-Bassat and Kobi Kremnizer, Non-Archimedean analytic geometry as relative algebraic geometry, Ann. Fac. Sci. Toulouse Math. (6) 26 (2017), no. 1, 49–126, doi: 10.5802/afst.1526 (English, with English and French summaries). MR 3626003

[2] Federico Bambozzi, Oren Ben-Bassat, and Kobi Kremnizer, Stein domains in Banach algebraic geometry, J. Func. Anal. 274 (2018), no. 7, 1865–1927.

[3] Oren Ben-Bassat, Jack Kelly, and Kobi Kremnizer, A perspective on the foundations of derived analytic geometry, arXiv preprint arXiv:2405.07936 (2024).

[4] Jack Kelly, Koszul duality in exact categories, arXiv preprint arXiv:1905.10102 (2019).

[5] Pelle Steffens, Representability of Elliptic Moduli Problems in Derived C^∞-Geometry, arXiv preprint arXiv:2404.07931 (2024).

[6] John Pardon, Derived moduli spaces of pseudo-holomorphic curves (2024).

[7] Jack Kelly, Homotopy in exact categories, Mem. Amer. Math. Soc. 298 (2024), no. 1490, v+160, doi: 10.1090/memo/1490. MR 4756824

[8] Jack and Mukherjee Kelly Devarshi, Localising invariants in derived bornological geometry, arXiv preprint arXiv:2505.15750 (2025).

[9] Jack Kelly, Kobi Kremnizer, and Devarshi Mukherjee, An analytic Hochschild-Kostant-Rosenberg theorem, Adv. Math. 410 (2022), 108694.

[10] Akhil Mathew, The Galois group of a stable homotopy theory, Advances in Mathematics 291 (2016), 403–541.

[11] Arun Soor, A six functor formalism for quasi-coherent sheaves and crystals on rigid analytic varieties, arXiv preprint arxiv:2409.07592 (2024).

[12] Mauro Porta, Derived complex analytic geometry I: GAGA theorems, arXiv preprint arXiv:1506.09042 (2015).

[13] Arpon Raksit, Hochschild homology and the derived de Rham complex revisited, arXiv preprint arXiv:2007.02576 (2020).

[14] Rhiannon Savage, A Representability Theorem for Stacks in Derived Geometry Contexts, arXiv preprint arXiv:2405.08361 (2024).

[15] Peter Scholze and Dustin Clausen, Condensed mathematics and complex geometry, Lecture notes.

[16] Bertrand Toën and Gabriele Vezzosi, Homotopical algebraic geometry I: Topos theory, Advances in mathematics 193 (2005), no. 2, 257–372.

Geometric Flows and Applications

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SS: MATH-03/A

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30

Mar 1, 2027	16:00-18:00	Mar 15, 2027	16:00-18:00
Mar 2, 2027	09:00-11:00	Mar 16, 2027	09:00-11:00
Mar 8, 2027	16:00-18:00	Mar 22, 2027	16:00-18:00
Mar 9, 2027	09:00-11:00	Mar 23, 2027	09:00-11:00

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading:

Introduction and background requirements:

Aim: The idea of deforming a geometric object via a parabolic PDE to $= E2 = 80 = 9C$ simplify $= E2 = 80 = 9D\ it = E2 = 80 = 94$ for instance, by reducing its topological complexity or increasing its symmetry $= E2 = 80 = 94$ dates back to Eells and Sampson, who introduced the harmonic map heat flow as the first geometric evolution equation. Since then, geometric flows have proven highly effective across a diverse range of areas, including topology, analysis, and physics, yielding significant and fruitful outcomes.
This course will examine a selection of these results, with a particular focus on geometric inequalities. Furthermore, it will explore the analytical tools developed in the literature to tackle these problems.

Course contents:

- Extrinsic and intrinsic geometric evolution equations; geometric gradient flows
- The role of flows in proving geometric and functional inequalities

The Mean Curvature Flow (MCF) and Network Flow

- Formal derivation of the MCF and the evolution of networks Comparison between different functionals and dissipation mechanisms.
- Basic properties: short-time existence, "geometric uniqueness" and invariance under reparametrization.

Flows Past Singularities

- Huisken's monotonicity formula and blow-up analysis.
- Restarting procedures for the network flow and surgery techniques for MCF.

The Curve Shortening Flow (CSF)

- A complete proof of Grayson=E2=80=99s Theorem.
- The rescaled flow and the isoperimetric inequality in the plane.

The Inverse Mean Curvature Flow (IMCF

- Definition of classical solutions and basic properties.
- Weak formulation: the level-set approach by Huisken and Ilmanen.

Applications to Geometric Inequalities

- Isoperimetric, Willmore, and Minkowski-type inequalities.
- The Riemannian Penrose Inequality.

Approximation of IMCF and MCF

- The p-harmonic approximation of the Inverse Mean Curvature Flow.
- Phase-field approximation of the Mean Curvature Flow.

References:

- G. Huisken and T. Ilmanen, The Inverse Mean Curvature Flow and the Riemannian Penrose Inequality, Journal of Differential Geometry, 59(3), 2001, pp. 353=E2=80=93437.
- C. Mantegazza, Lecture Notes on Mean Curvature Flow, Progress in Mathematics, Birkh=C3=A4user/Springer Basel AG, Basel, 2011.
- C. Mantegazza, M. Novaga, A. Pluda, and F. Schulze, Evolution of networks with multiple junctions, Ast=C3=A9risque, 452, 2024, pp. 1=E2=80=93266.

An invitation to Fractional Sobolev Spaces

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SS: MATH-03/A

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Mar xx, 2027	xx:xx-xx:xx	Mar xx, 2027	xx:xx-xx:xx
Mar xx, 2027	xx:xx-xx:xx	Mar xx, 2027	xx:xx-xx:xx
Mar xx, 2027	xx:xx-xx:xx	Mar xx, 2027	xx:xx-xx:xx
Mar xx, 2027	xx:xx-xx:xx	Mar xx, 2027	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading: Oral examination on the content of the course or presentation of a seminar based on a research paper related to the topics covered.

Introduction and background requirements: Basic knowledge ofmeasure theory and Lebesgue spaces, classical Sobolev spaces $W^{1,p}$, and elementary notions of partial differential equations.

Aim: The aim of the course is to introduce fractional Sobolev spaces $W^{s,p}$ and to present their main analytical properties, with a particular focus on applications to nonlocal partial differential equations. At the end of the course, students will be able to:

- understand and compare the definitions and characterizations of fractional Sobolev spaces;
- use the main functional-analytic tools associated with fractional Sobolev spaces, including embedding, approximation, compactness, and extension theorems;
- analyze variational problems and nonlocal partial differential equations, including problems involving the fractional Laplacian.

Course contents: The course provides an introduction to fractional Sobolev spaces and their applications to nonlocal partial differential equations. The following topics will be discussed:

(1) motivation, basic definitions, and comparison with integer-order Sobolev spaces;

(2) density of smooth functions and the Meyer–Serrin theorem;

(3) the Rellich–Kondrachov compactness theorem and the Poincaré inequality;

(4) extension theorems, embedding results, and interpolation inequalities;

(5) Bourgain–Brezis–Mironescu and Maz'ya–Shaposhnikova formulas;

(6) the fractional Laplacian, Riesz potentials, and the fractional Poisson equation;

(7) the Caffarelli–Silvestre extension.

References:

[1] J. Bourgain, H. Brezis, and P. Mironescu, Another look at Sobolev spaces, Optimal control and partial differential equations, IOS, Amsterdam, 2001, pp. 439–455. MR3586796

[2] L. Caffarelli and L. Silvestre, An extension problem related to the fractional Laplacian, Comm. Partial Differential Equations 32 (2007), no. 7-9, 1245–1260, DOI 10.1080/03605300600987306 MR2354493

[3] E. Di Nezza, G. Palatucci, and E. Valdinoci, Hitchhiker's guide to the fractional Sobolev spaces, Bull. Sci. Math. 136 (2012), no. 5, 521–573, DOI 10.1016/j.bulsci.2011.12.004. MR2944369

[4] G. Leoni, A first course in fractional Sobolev spaces, Graduate Studies in Mathematics, vol. 229, American Mathematical Society, Providence, RI, 2023. MR4567945

[5] V.Maz'ya and T. Shaposhnikova, On the Bourgain, Brezis, and Mironescu theorem concerning limiting embeddings of fractional Sobolev spaces, J. Funct. Anal. 195 (2002), no. 2, 230–238, DOI 10.1006/jfan.2002.3955. MR1940355

Courses of the “Computational Mathematics” area

Volterra Equations and their Markovian Lift(s)

Ofelia Bonesini[1]

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Credits: 3

SS: MATH-03/B

Timetable: 16 hrs. Torre Archimede, Room 2BC30:

Apr 12, 2027	14:30-16:30	Apr 19, 2027	14:30-16:30
Apr 13, 2027	11:00-13:00	Apr 20, 2027	11:00-13:00
Apr 14, 2027	14:30-16:30	Apr 21, 2027	14:30-16:30
Apr 15, 2027	11:00-13:00	Apr 22, 2027	11:00-13:00

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Course requirements: Real Analysis and Stochastic Calculus are considered foundational prerequisites, while a basic knowledge of stochastic modelling for finance is helpful yet not necessary

Examination and grading: A 30-minute seminar on one of the selected readings proposed in class

Aim:

1. Understand a generalisation of standard SDEs (SVEs) for which Markovianity and semimartingality do not hold.
2. Analyse Rough Volatility models to understand why fractional Brownian motion is particularly suitable for financial modeling.
3. Explore the infinite dimensional setting established by the use of Markovian lifts, in particular by comparing the different lifting techniques (measure-valued, curve-valued). Apply lifts to restore Markovianity and help in functional Itô calculus.
4. Indentify the connection between SVEs and Path-Dependent PDEs by linking the two to backward stochastic PDEs.
5. Identify the key challenges in the established methods to detect potential further developments and improvements in the established theory and discover new applications.

Course contents:

1. Preliminaries on Stochastic Volterra Equations (SVEs)
   - Definition and main features
   - Comparison with standard SDEs: loss of Markovianity and semimartingality
   - Existence and uniqueness results (in particular, for the singular kernel case)

2. Applications in finance: rough volatility models
    - Fractional Brownian motion and its role as the main player in volatility modelling
    - Rough Bergomi model: definition and its meaning
    - Connections between SVEs and (multifactor) fractional stochastic volatility models
3. Markovian lifts
    - Markovian lifts foundation: restoring Markovianity in SVEs
    - Measure-valued lift: genesis, how this is used in practice, and limitations
    - Curve-valued lift: link with forward variance curves in finance
    - Approximation techniques via Markovian representations
4. SVEs and Path-dependent PDEs
    - Connections between SVEs, PPDEs (path-dependent PDEs), and BSPDEs (backward stochastic PDEs)
    - Integral representations and Laplace transform methods
5. Advanced topics and open problems
    - Recent research on uniqueness and weak solutions for SVEs
    - Applications of Itô-type formulas in Volterra processes
    - Open questions in Markovian approximations and numerical methods

Bibliography:

1. E. Abi Jaber, Lifting the Heston model, Quantitative finance, 19 (2019), pp. 1995–2013.
2. E. Abi Jaber, C. Cuchiero, M. Larsson, and S. Pulido, A weak solution theory for stochastic Volterra equations of convolution type, The Annals of Applied Probability, 31 (2021), pp. 2924–2952.
3. E. Abi Jaber and O. El Euch, Markovian structure of the Volterra Heston model, Statistics and Probability Letters, 149 (2019), pp. 63–72.
4. E. Abi Jaber and O. El Euch, Multifactor approximation of rough volatility models, SIAM journal on financial mathematics, 10 (2019), pp. 309–349.
5. E. Abi Jaber, E. Miller, and H. Pham, Linear-quadratic control for a class of stochastic Volterra equations: solvability and approximation, The Annals of Applied Probability, 31 (2021), pp. 2244–2274.
6. F. E. Benth, N. Detering, and P. Kruehner, Stochastic Volterra integral equations and a class of first-order stochastic partial differential equations, Stochastics, 94 (2022), pp. 1054–1076.
7. M. A. Berger and V. J. Mizel, Volterra equations with Itô integrals -i and ii, The Journal of Integral Equations, (1980), pp. 187–245, 319–337.
8. O. Bonesini, A. Jacquier, and A. Pannier, Rough volatility, path-dependent PDEs and weak rates of convergence. ArXiv preprint, (2024).
9. P. Carmona, L. Coutin, and G. Montseny, Approximation of some gaussian processes, Statistical inference for stochastic processes, 3 (2000), pp. 161–171.
10. L. Coutin and P. Carmona, Fractional Brownian motion and the Markov property, Electronic Communications in Probability, 3 (1998), p. 12.

11. L. Coutin and L. Decreusefond, Stochastic Volterra equations with singular kernels, in Stochastic analysis and mathematical physics, Springer, 2001, pp. 39–50.
12. C. Cuchiero and J. Teichmann, Markovian lifts of positive semidefinite affine Volterra-type processes, Decisions in Economics and Finance, 42 (2019), pp. 407–448.
13. C. Cuchiero and J. Teichmann, Generalized Feller processes and Markovian lifts of stochastic Volterra processes: the affine case, Journal of Evolution Equations, 20 (2020), pp. 1301–1348.
14. J. Gatheral, T. Jaisson, and M. Rosenbaum, Volatility is rough, Quantitative finance, 18 (2018), pp. 933–949.
15. Y. Hamaguchi, Markovian lifting and asymptotic log-Harnack inequality for stochastic Volterra integral equations. ArXiv preprint, (2023).
16. Y. Hamaguchi, Weak well-posedness of stochastic Volterra equations with completely monotone kernels and non-degenerate noise. ArXiv preprint, (2023).
17. P. Harms and D. Stefanovits, Affine representations of fractional processes with applications in mathematical finance, Stochastic Processes and their Applications, 129 (2019), pp. 1185–1228.
18. F. Huber, Markovian lifts of stochastic Volterra equations in Sobolev spaces: Solution theory, an Itô formula and invariant measures, ArXiv preprint, (2024).
19. E. A. Jaber, C. Illand, et al., Joint SPX-VIX calibration with gaussian polynomial volatility models: deep pricing with quantization hints. ArXiv preprint, (2022).
20. L. Mytnik and T. S. Salisbury, Uniqueness for Volterra-type stochastic integral equations, ArXiv preprint, (2015).
21. D. J. Prömel and D. Scheffels, On the existence of weak solutions to stochastic volterra equations, Electronic Communications in Probability, 28 (2023), pp. 1–12.
22. D. J. Prömel and D. Scheffels, Stochastic Volterra equations with Hölder diffusion coefficients, Stochastic Processes and their Applications, 161 (2023), pp. 291–315.
23. D. J. Prömel and D. Scheffels, Pathwise uniqueness for singular stochastic Volterra equations with Hölder coefficients, Stochastics and Partial Differential Equations: Analysis and Computations, (2024), pp. 1–59.
24. F. Viens and J. Zhang, A martingale approach for fractional Brownian motions and related path dependent PDEs, The Annals of Applied Probability, 29 (2019), pp. 3489–3540.
25. H. Wang, J. Yong, and J. Zhang, Path dependent Feynman–Kac formula for forward backward stochastic Volterra integral equations, Annales de l'Institut Henri Poincaré: Probabilités et Statistiques, 58 (2022), pp. 603–638.
26. Z. Wang, Existence and uniqueness of solutions to stochastic Volterra equations with singular kernels and non-Lipschitz coefficients, Statistics and probability letters, 78 (2008), pp. 1062–1071.

Random Schrödinger operators

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SSD: MATH-03/B

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Mar xx, 2027	xx:xx-xx:xx	Mar xx, 2027	xx:xx-xx:xx
Mar xx, 2027	xx:xx-xx:xx	Mar xx, 2027	xx:xx-xx:xx
Mar xx, 2027	xx:xx-xx:xx	Mar xx, 2027	xx:xx-xx:xx
Mar xx, 2027	xx:xx-xx:xx	Mar xx, 2027	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar)

Examination and grading: Oral exam at a time to be agreed beforehand

Introduction and background requirements: Some basic knowledge of probability theory and functional analysis is desirable

Aim: The main goal of these lectures is to study random Schrödinger operators, mainly on $\mathbb{Z}^d$ and present some tools for the proof of the so-called Anderson localization. More precisely, we will analyze the spectrum of the random operator

$$H_\omega = -\Delta + \lambda V_\omega \tag{1}$$

acting on $\ell^2(\mathbb{Z}^d)$. Here, Δ is the discrete Laplacian and the potential V_ω is a multiplication operator, i.e. $(V_\omega f)(x) = V(x,\omega)f(x)$, with $(V(x,\omega))_{x\in\mathbb{Z}^d}$ being a family of i.i.d. random variables. The understanding of the spectrum and eigenfunctions of H_ω is crucial in determining the evolution of quantum systems with the help of the Schrödinger equation. In his seminal work P. W. Anderson posited in [2] that under random potential there would be a transition in the transport properties of the model which heavily depend on the dimension d of the underlying lattice, the strength λ of the disorder, and the energy. Anderson's work has been crucial in explaining how electrons get trapped in disordered materials and represents one of the greatest achievement in physics (for which P. W. Anderson was awarded the Nobel prize in 1977). At the same time, models and phenomena set forth in this line of research appeared from the start to be true challenges for mathematicians: this led, through the years, to the development of mathematical tools of importance also beyond the context of Anderson localization. The focus of the course will be on the *fractional moment method* that leads to showing that, in the high disorder regime, the spectrum is pure point and the corresponging eigenfunctions are localized, and further that localization happens at a weak disorder/high energy regime.

Course contents:

- Some basics: Self-adjoint operators, their spectrum and resolvent sets. The resolution of the identity and the spectral theorem. Weyl's criterion with application to random Schrödinger operators.
- A finer analysis of the spectrum: pure point, absolutely continuous, and singular continuous spectrum. An interpretation of the decomposition via the RAGE theorem.
- Resolvent identity and random walk representation of the resolvent.
- Herglotz-Nevalinna functions and Borel-Stieltjes transforms.
- The Simon-Wolff criterion. Some Localization results for random Schrödinger operators via the *fractional moments method*.

References

[1] Michael Aizenman and Simone Warzel. *Random operators*, volume 168. American Mathematical Soc., 2015.

[2] P. W. Anderson. Absence of diffusion in certain random lattices. *Phys. Rev.*, 109:1492–1505, Mar 1958.

[3] Werner Kirsch. An invitation to random Schrödinger operators. *arXiv preprint arXiv:0709.3707*, 2007.

[4] Gerald Teschl. *Mathematical methods in quantum mechanics*, volume 157. American Mathematical Soc., 2014.

Large deviations and Metastability

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SSD: MATH-03/B

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30:

Dec 15, 2026	14:30-16:30	Jan 26, 2027	14:30-16:30
Dec 17, 2026	10:30-12:30	Jan 28, 2027	10:30-12:30
Jan 12, 2027	14:30-16:30	Feb 9, 2027	14:30-16:30
Jan 14, 2027	10:30-12:30	Feb 11, 2027	10:30-12:30

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading: Presentation of an in-depth study of part of the course, or a seminar on a paper.

Introduction and background requirements: Basic knowledge of probability theory: discrete and continuous random variables, finite and countable probability spaces. Familiarity with the main modes of convergence of sequences of random variables, as well as with the law of large numbers and the central limit theorem. Knowledge of Markov chains is also required.

Aim: A solid understanding of large deviation theory and of its applications to statistical mechanics, with particular emphasis on phenomena of metastability. This includes familiarity with the mathematical framework underlying rare events and their probabilistic characterization.

Course contents: Large deviation theory:

- *Introduction to large deviation theory:* An overview of the motivations and core ideas behind large deviation theory, focusing on the study of rare events and their exponentially small probabilities in probabilistic systems.
- *Cramér's Theorem and its applications:* A detailed study of Cramér's Theorem, which characterizes the rate at which the probabilities of large deviations decay for sums of independent and identically distributed random variables. Applications illustrate how this theorem provides exponential estimates beyond classical limit results.
- *Large Deviation Principle:* Formalization of the Large Deviation Principle (LDP), including precise definitions and conditions under which it holds for sequences of probability measures.
- *Computation of main rate functions:* The rate functions quantify the cost or likelihood of deviations from typical behavior in probabilistic systems. In this part of the course, we focus on explicit computations of rate functions for sequences of Bernoulli, Poisson, and Gaussian random variables

- *Sanov's Theorem and applications:* Sanov's Theorem is presented as an extension of large deviation ideas to empirical measures, offering bounds on the probability of observing atypical distributions. This topic also discusses applications in information theory and statistical mechanics.
- *Varadhan's Lemma and the concept of free energy:* Varadhan's Lemma is explored as a key tool linking large deviation estimates to the asymptotics of exponential integrals, and its connection to the notion of free energy in physical systems is highlighted.
- *Metastability and selected applications:* An exploration of metastable phenomena, where systems remain in "quasi-stable" states for long times due to rare transitions, showing how large deviation techniques can describe such behavior. Selected applications illustrate the relevance of these ideas in statistical mechanics and dynamical systems.

References:

- *Large deviations*, F. den Hollander, 2000, American Mathematical Society.
- *Large deviations techniques and applications*, A. Dembo and O. Zeitouni, Springer.
- *Probability Theory: A Comprehensive Course* A. Klenke, Springer.
- *Large deviations and Metastability* E. Olivieri and M. E. Vares, Cambridge.
- *Metastability: a potential-theoretic approach*, A. Bovier and F. den Hollander, Springer.

Linear Parabolic Equations in Hilbert Spaces: analysis and numerical approximation

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SS: MATH-05/A

CFU/ECTS: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Xxx xx, 2027	xx:xx-xx:xx	Xxx xx, 2027	xx:xx-xx:xx
Xxx xx, 2027	xx:xx-xx:xx	Xxx xx, 2027	xx:xx-xx:xx
Xxx xx, 2027	xx:xx-xx:xx	Xxx xx, 2027	xx:xx-xx:xx
Xxx xx, 2027	xx:xx-xx:xx	Xxx xx, 2027	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Introduction and background requirements: real and functional analysis, basics of numerical analysis. All the essential notions will be briefly recalled during the lectures.

Aim: The course will offer, in the framework of linear parabolic PDEs, the opportunity of running into the whole scientific process of analyzing a mathematical problem, constructing its solution, and developing a robust numerical approximation method by exploiting the same properties of the problem that surfaced in the analysis step.

Examination and grading: either oral examination on the content of the course, or presentation of a related research paper. Whenener the background of the student includes some programming skills, the presentation of numerical experiments might be included in the exam.

Course contents: The course consists of two closely related and complementary parts: the first 10 hours will be devoted to the construction of the solutions of certain classes of linear parabolic equations formulated as evolution equations in Hilbert spaces. The second part of the course (6 hours) concerns the numerical approximation of such solutions. First we consider the semi-discrete approximation by the Faedo-Galerkin approach, then we exploit the properties of analytic semigroups to define a fully-discrete sequence of approximations by means of the Laplace transform and its numerical inversion.

Due to the time constraint, only the most important (and/or instructive) results will be proven, while many others will be only presented and discussed. Some classical equations (e.g., heat eq., convection-diffusion eq., Sobolev eq., and visco-elastic eq.) will be used as examples both to apply the presented theoretical results and verify their hypothesis, and to test the introduced approximation techniques.

Part 1 (10h - 5 lectures)

1) Quadratic forms and linear operators on Hilbert spaces;
2) Accretive operators, generation of contraction and analytic semigroups, relation with Laplace transform;
3) Solving first-order (in time) linear non-degenerte explicit parabolic equations;
4) More general linear parabolic equations than $u_t = \mathcal{L}u + f$: implicit and second-order equations;
5) Classical examples: heat, convection-diffusion, visco-elastic, and Sobolev equations.

Part 2 (6h - 3 lectures)

6) Galerkin method for elliptic problems, standard error estimates;
7) Semi-discretization by Faedo-Galerkin method, error analysis and convergence;
8) Fully discrete approximation by Laplace transform and quadrature, convergence analysis for sectorial operators.

Mathematical Control of Multi-Agent Systems: From Traffic to Crowds

Bnedetto Piccoli[1]

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Email: piccoli@camden.rutgers.edu

SSD: MATH-03/A

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Feb 23, 2027	14:00-16:00	Jun xx, 2027	xx:xx-xx:xx
Feb 24, 2027	14:00-16:00	Jun xx, 2027	xx:xx-xx:xx
Feb 25, 2027	10:30-12:30	Jun xx, 2027	xx:xx-xx:xx
Feb 26, 2027	10:30-12:30	Jun xx, 2027	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar)

Introduction and background requirements: Basics on Partial Differential Equations, Measure Theory.

Aim: Establishing the mathematical frameworks for vehicular traffic, network flows, and the transition between modeling scales

Examination and grading: Oral presentation of a research paper

Course contents:

Week 1: Fundamentals of Multi-Scale Modeling for Multi-Agents systems

Day 1: Introduction to Multi-Agent Systems & Scales

Topic: Overview of large-scale multi-agent systems and the three scales of modeling: Microscopic, Mesoscopic (Kinetic and mean-field), and Macroscopic (Fluid dynamics).

Key Concepts: Distinction between ”blind” gas particles (entropy) and ”intelligent” agents (self-organization), Introduction to Agent-Based Models (ABM) vs. Continuum Models.

Suggested Readings: Delle Monache, Maria Laura, et al. ”Modeling, monitoring, and controlling road traffic using vehicles to sense and act.” Annual Review of Control, Robotics, and Autonomous Systems 8.1 (2025): 211-233. B. Piccoli: Control of multi-agent systems: results, open problems, and applications, Open Mathematics, 21 (2023), math-2022-0585.

Day 2: Kinetic Models and Mean-Field Limits

Topic: Bridging the gap between micro and macro.

Key Concepts: Mean-Field Games (MFG) and mean-field control: Agents solving games with perfect knowledge and without. Vlasov-type Equations: How microscopic interactions with kernel Q lead to kinetic equations.

Suggested Readings: Paul, Thierry, and Emmanuel Trélat. "From microscopic to macroscopic scale equations: mean field, hydrodynamic and graph limits." arXiv preprint arXiv:2209.08832 (2022). Golse, François. "On the dynamics of large particle systems in the mean field limit." Macroscopic and large scale phenomena: coarse graining, mean field limits and ergodicity. Cham: Springer International Publishing, 2016. 1-144.

Day 3: Introduction to Measure Differential Equations

Topic: Overview of Wasserstein spaces and Measure Differential Equations.

Key Concepts: Wasserstein distance and Wasserstein spaces. Measure vector fields and measure differential equations.

Suggested Readings: Villani, Cédric. Topics in optimal transportation. Vol. 58. American Mathematical Soc., 2021. Piccoli, Benedetto. "Measure differential equations." Archive for rational mechanics and analysis 233.3 (2019): 1289-1317.

Day 4: Introduction to control theory

Topic: Overview of mathematical theory of controlled systems.

Key Concepts: Control system, open and closed-loop controls, existence of trajectories, controllability, optimal control.

Suggested Readings: Bressan, Alberto, and Benedetto Piccoli. Introduction to the mathematical theory of control. Vol. 1. Springfield: American institute of mathematical sciences, 2007.

Week 2: Crowd Dynamics, Social Interactions, and Control

Day 1: Phenomenology and modeling of Crowds

Topic: Empirical observations and experimental data.

Key Concepts: Self-Organization vs. Entropy: How pedestrians differ from gas particles (anisotropic, vision-based, decision-making). Experimental Settings: "Exiting the Metro" scenarios, path choice, and virtual maps. Microscopic: Helbing's Social Force Model, Maury-Venel. Macroscopic: Bellomo-Dogbé, Colombo-Rosini models. Time-Evolving Measures: Using flow maps and push-forward formulas.

Suggested Readings: Cristiani, Emiliano, Benedetto Piccoli, and Andrea Tosin. Multiscale modeling of pedestrian dynamics. Springer, 2014.

Day 2: Coupling controlled agents and measure differential equations

Topic: Rigorous analysis of evolving measures.

Key Concepts: Existence and uniqueness of solutions to MDEs. The continuity equation and superposition principles. Coupling microscopic controls with MDEs.

Suggested Readings: Piccoli, Benedetto. "A mathematical approach to crowd surveillance with drones." Mathematical Models and Methods in Applied Sciences 35.07 (2025): 1661-1679. Piccoli, Benedetto. "Surveillance Guidance of Crowds via Lagrangian Controls." Crowd

Dynamics, Volume 5: From Human Complexity to Scientific Machine Learning. Cham: Springer Nature Switzerland, 2025. 151-172.

Day 3 Control of Multi-Agent Systems

Topic: Influencing the system behavior.

Key Concepts: Sparse Control: Controlling a system by acting on a few agents (e.g., ”leaders” in Cucker-Smale). Curse of Dimensionality: Challenges in controlling large systems. Future Directions: Hybrid systems, Mixed ODE-PDE models, and human-in-the-loop control.

Suggested Readings: Fornasier, Massimo, Benedetto Piccoli, and Francesco Rossi. ”Mean-field sparse optimal control.” Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 372.2028 (2014): 20130400. B. Piccoli: Control of multi-agent systems: results, open problems, and applications, Open Mathematics, 21 (2023), math-2022-0585.

Day 4: Presentation of final projects

Courses in collaboration with the Doctoral School on “Information Engineering”

for complete Catalogue and class schedule see on

https://phd.dei.unipd.it/course-catalogues/

Please check regularly the website of the Doctoral Course

Calendar of activities on

`https://calendar.google.com/calendar/u/0/embed?src=fvsl9bgkbnhhkqp5mmqpiurn6c@group.calendar.google.com&ctz=Europe/Rome`

IE_CSC 4. Introduction to Modern Cryptography

Course Area: Information Engineering

Teacher in charge	Alessandro LANGUASCO
Teaching Hours	24
Number of ECTS credits allocated	5
Course period	Second semester
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☒ **Yes (75 % minimum of presence)** ☐ No
Course unit contents	First definition of a cryptosystem. Some historical examples. Fundamental crypto algorithms. Shannon's perfect cipher. A review about symmetric methods (historical ones, DES, AES). Asymmetric methods based on primality/factoring and discret log problems. Known attacks to some of the most used public key cryptosystems. How to use a public key system to build a digital signature algorithm. Digital Signatures with RSA and discrete log. Authentication protocols (Kerberos, Needham-Schroeder) and public key systems. Key exchange in three steps (Diffie-Hellman key exchange protocol), secret splitting, secret sharing, secret broadcasting, timestamping.
Learning goals	We present some of the main features about what a Modern Cryptosystem is. In particular we will focus on showing the internal characteristics of some of the now used public key cryptosystems.

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	We will overview the methods based on the primality/factorization and on the discrete logarithm problems. The focus will be on the actual implementation and its feasibility in terms of both time and space, while taking care of the needed mathematical concepts (congruences, finite fields) and explaining them along the course as needed. As a final topic, we will show how to use a public key system in an authentication/identification protocol. The goal of the course will be to evaluate pros and cons of the cryptographic choices performed in designing such protocols.
Teaching methods	In class; us
Course on transversal, interdisciplinary, transdisciplinary skills	☒ **Yes** ☐ No
Available for PhD students from other courses	☐ Yes ☐ No
Prerequisites **(not mandatory)**	none
Examination methods **(if applicable)**	a seminar on a related topic. For example (but on others of common interest we can agree upon): The Secure Hash Algorithm (SHA); other hash algorithms; primality algorithms; factoring algorithms; discrete log algorithms; homomorphic cryptography; elliptic curves cryptography; compression and hash functions; probabilistic cryptography; digital currencies, electronic voting.
Suggested readings	Books: 1) Languasco-Zaccagnini, “Manuale di Crittografia”, Hoepli, 2015. 2) Knospe, “A course in Cryptography”, AMS, 2019.

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	3) Schneier, “Applied Cryptography, Protocols, Algorithms, and Source Code in C”, Wiley, 1993.
Additional information	none

Schedule and room: see Class Schedule for details

Enrollment: add the course to the list of courses you plan to attend using the Course Enrollment Form (requires SSO authentication) and, if you are taking the course for credits, to the Study and Research Plan.

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IE_AUT 2. Applied Functional Analysis and Machine Learning

Course Area: Information Engineering

Teacher in charge	Gianluigi Pillonetto
Teaching Hours	24
Number of ECTS credits allocated	5
Course period	November-december 2026
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☒ Yes (80% minimum of presence) ☐ No
Course unit contents	Review of some notions on metric spaces and Lebesgue integration: Metric spaces. Open sets, closed sets, neighborhoods. Convergence, Cauchy sequences, completeness. Completion of metric spaces. Review of the Lebesgue integration theory. Lebesgue spaces. Banach and Hilbert spaces: Finite dimensional normed spaces and subspaces. Compactness and finite dimension. Bounded linear operators. Linear functionals. The finite dimensional case. Normed spaces of operators and the dual space. Weak topologies. Inner product spaces and Hilbert spaces. Orthogonal complements and direct sums. Orthonormal sets and sequences. Representation of functionals on Hilbert spaces. Reproducing kernel Hilbert spaces, inverse problems and regularization theory: Representer theorem. Reproducing Kernel Hilbert Spaces (RKHS): definition and basic properties. Examples of RKHS. Function estimation problems in RKHS. Tikhonov regularization. Support vector regression and classification. Extensions of the theory to deep kernel-

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	based networks: multi-valued RKHSs and the concatenated representer theorem.
Learning goals	The course is intended to give a survey of the basic aspects of functional analysis, machine learning, regularization theory and inverse problems. At the end of the course, the student will have the methodological tools to tackle various machine learning problems in both regression and classification (estimation of functions from scattered and noisy data) starting from very general hypothesis spaces.
Teaching methods	Blackboard lectures and various questions posed to students regarding previous lessons
Course on transversal, interdisciplinary, transdisciplinary skills	☒ Yes ☐ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	The classical theory of functions of real variable: limits and continuity, differentiation and Riemann integration, infinite series and uniform convergence. Some elementary set theory and linear algebra.
Examination methods (if applicable)	Two written exams, one in the middle of the course and the other at the end
Suggested readings	[1] G. Pillonetto, T. Chen, A. Chiuso, G. De Nicolao, L. Ljung. Regularized System Identification –learning dynamic models from data, Springer Nature 2022 [2] W. Rudin. Real and Complex Analysis, McGraw Hill, 2006 [3] C.E. Rasmussen and C.K.I. Williams. Gaussian Processes for Machine Learning. The MIT Press, 2006 [4] H. Brezis, Functional analysis, Sobolev spaces and partial differential equations, Springer 2010

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	[5] G. Pillonetto, A. Aravkin, D. Gedon, L. Ljung, A.H. Ribeiro and T.B. Schön, Deep networks for system identification: a Survey, eprint 2301.12832 arXiv, 2023 In addition, written notes will be made available to the students.
Additional information	None

Schedule and room: see Class Schedule for details

Enrollment: add the course to the list of courses you plan to attend using the Course Enrollment Form (requires SSO authentication) and, if you are taking the course for credits, to the Study and Research Plan.

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Soft Skills

1. Lean for theWorking Mathematician SS-1,3
2. Tools and Applications of Machine Learning for Social Challenges SS-4,5

Lean for the Working Mathematician

Dr. Stefano Rocca

Soft Skills activities

Timetable: 12 hrs, Torre Archimede, Room 2BC30:

Dec 2, 2026	14:30-16:30
Dec 3, 2026	14:30-16:30
Dec 9, 2026	14:30-16:30
Dec 11, 2026	14:30-16:30
Dec 16, 2026	14:30-16:30
Dec 18, 2026	14:30-16:30

See also the Calendar of Activities on https://dottorato.math.unipd.it/calendar

See next pages for Course requirements, Aim, Outline of the course

Lean for the Working Mathematician

Hands-on course in reading, navigating, and writing formal mathematics

Duration: 12 hours

Format: Live coding, guided exercises, code reading, and individual or paired practice, with a small mini-project connected with participants' mathematical interests if time permits. There will be no final assessment.

Course requirements

No previous experience with Lean, programming, or formal verification is required. Familiarity with Git/GitHub could be useful but is not mandatory. The course is intended primarily for mathematicians, although participants from other fields are also welcome: the vast majority of toy examples use undergraduate-level mathematics. Participants should bring a laptop on which they are permitted to install applications and which can run Visual Studio Code with the Lean extension. We will take our time going through the installation process together, and a browser-based environment can be used while installation is under way.

Aim

> The course is meant to help researchers get started with formalized mathematics and become comfortable using Lean4 in practice. Participants will learn how to read theorem statements and proof states, inspect definitions and hypotheses, find useful results in Mathlib, make informed use of automation, and take the first steps towards a formalization of their own. A recurring theme will be the difference between the theorem one has in mind, the statement written in Lean, and the proof object ultimately checked by the kernel. Formalization often brings hidden assumptions, ambiguous formulations, edge cases, and important choices of definition into view.

Outline of the course

The course will touch on the following topics, not necessarily in the order listed. The selection and emphasis may be adapted to the interests of the participants.

- **Working environment and first interaction.** Installation of Lean4 and a version-pinned Mathlib project; use of Visual Studio Code, the Infoview, and a browser editor for short experiments. First declarations with `#check`, `def`, `example`, `lemma`, and `theorem`, together with the interpretation of errors, warnings, and incomplete declarations.
- **Statements, specifications, and mathematical intent.** Comparison of informal claims with their formal counterparts, including examples in which Lean checks a correct proof of an unintended statement. Quantifiers, hidden hypotheses, exceptional cases, and the responsibility of the formalizer for choosing a faithful theorem statement.
- **Proof states and elementary proof construction.** The local context, target, goals, and subgoals. Core logical forms and tactics such as `intro`, `exact`, `apply`, `constructor`, `cases`, `rcases`, `use`, `refine`, and `have`, with continual comparison to the corresponding mathematical reasoning.
- **Mathematical language and abstraction.** Functions, predicates, relations, sets, images, preimages, and extensionality. Definitions, namespaces, overloaded notation, implicit arguments, structures, typeclasses, coercions, and type inference as they occur in ordinary mathematical code.
- **Equality, rewriting, and structured arguments.** Definitional and propositional equality, computation and `rfl`, unfolding, rewriting, and simplification through `unfold`, `rw`, and `simp`. Development of readable multi-step arguments using intermediate facts and `calc` blocks.
- **Reading and navigating Mathlib.** How to approach an unfamiliar source file through imports, sections, namespaces, variables, declarations, instances, and nearby lemmas. Use of hover information, completion, go to definition, source inspection, `#print`, `#synth`, and `#find` to reconstruct the mathematical interface of a development.

- **Searching for definitions and theorems.** Use of the Mathlib API documentation, LeanSearch, Moogle, and Loogle. Local search and suggestion commands such as `exact?`, `apply?`, `rw?`, and `simp?`, with emphasis on comparing candidate results and understanding their assumptions.
- **Automation and proof engineering.** Controlled simplification and domain-specific tactics including `norm_num`, `ring`, `linarith`, `nlinarith`, and `omega`, together with proof search using `aesop` and `grind`. Comparison of explicit, library-driven, and automated proofs in terms of readability, robustness, explanatory value, and maintainability.
- **Small-scale independent formalization.** Selection and scoping of a manageable mathematical example, reuse of existing definitions, decomposition into intermediate lemmas, use of scratch files and temporary `sorry`s. Refactoring, documentation, and reproducibility. The aim is to leave participants able to continue experimenting with a topic connected to their own interests.
- **AI-assisted and large-scale formalization.** A concluding discussion of how AI and increasingly automated methods are being used in larger formalization projects, including theorem proving, library search, proof repair, and the coordination of extended developments. We will discuss the possibilities opened by these methods, their current limitations, and the questions they raise for mathematical practice. Mathematicians who choose to formalize may need to strengthen complementary skills: specifying the intended theorem correctly; choosing suitable definitions and abstractions; decomposing a project into meaningful lemmas; checking that formal statements match mathematical intent; reviewing assumptions, axioms, imports, and dependencies; and deciding whether the resulting code is readable, reusable, and maintainable.

Teaching format

The course will be taught primarily through live coding, with a continuous back-and-forth between informal mathematical reasoning and its formal expression in Lean. Theoretical ideas will be introduced through concrete examples and immediately explored in practice. At least half of the course time will be devoted to hands-on work, including guided exercises, code reading, and direct experimentation in Lean.

References and resources

1. K. Buzzard, *Formalising Mathematics*. Course notes and exercises for mathematicians using Lean and Mathlib.
2. J. Avigad and P. Massot, *Mathematics in Lean*. A systematic introduction to mathematical formalization with Lean and Mathlib.
3. P. Massot, *A Glimpse of Lean*. A short, exercise-driven introduction designed for mathematical audiences.
4. J. Avigad, L. de Moura, S. Kong, S. Ullrich, and contributors, *Theorem Proving in Lean4*. Reference material on logic, proof terms, tactics, structures, and typeclasses.
5. F. van Doorn and M. Rothgang, *Bonn Lean Course, WiSe 2025/26*. Executable lecture files, assignments, installation guidance, and project material.
6. Mathlib API documentation, the tactic reference, and the search services LeanSearch, Moogle, and Loogle.

Course unit English denomination	**M6 - Tools and Applications of Machine Learning for Social Challenges**
Scientific Sector	INFO-01/A, PSIC-03/A
Teacher in charge (if defined)	Nicolò Navarin, Merylin Monaro
Teaching Hours	18
Number of ECTS credits allocated	3
Course period	See timetable on BMCS website
Course delivery method	in person
Language of instruction	English
Mandatory attendance	YES (70% minimum)
Course unit contents	• Introduction to machine learning tools and their correct use. • What machine learning tools can achieve: supervised vs unsupervised techniques. • Examples of applications and how to assess performances. • Introduction to cognitive services. • Case studies
Learning goals	The course is divided into two parts. The first part provides a practical understanding of basic machine learning tools and their use. The second part of the course is focused on understanding how machine learning techniques can be applied to answer research questions in psychology (case studies
Teaching methods	• Lessons • Groupwork
Course on transversal, interdisciplinary, transdisciplinary skills	No
Available for PhD students from other courses	YES Limited (see BMCS website for details)

Prerequisites (not mandatory)	Python for non computer scientists
Examination methods (if applicable)	Modality - The PhD students are asked to carry on a group interdisciplinary project involving the application of machine learning techniques to answer research questions in psychology. These projects are done collaboratively by cross-curricular groups. In the last lecture of the course, the students will present their projects. The projects will be evaluated based on the level of collaborative work required, and the quality of the experimental design and technical skills. Moreover, students are encouraged to interact with the presentations of their colleagues. Goal - To prepare students to conduct inter-disciplinary research, and practice their ability to apply theoretical knowledge to real-world problems. Skills acquired with the exam - Basic understanding of machine learning techniques; ability to recognise and properly formulate a machine learning problem; technical abilities in developing a machine learning pipeline; ability to work collaboratively with colleagues with a different background, Feedback - After each group's presentation, general feedback is provided in class. The exam feedback is provided to each student in the Moodle platform or via email.
Suggested readings	In the course Moodle
Additional information	-