Nonholonomic Mechanical Systems

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SS: MATH-04/A

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Xxx xx, 202x	xx:xx-xx:xx	Xxx xx, 202x	xx:xx-xx:xx
Xxx xx, 202x	xx:xx-xx:xx	Xxx xx, 202x	xx:xx-xx:xx
Xxx xx, 202x	xx:xx-xx:xx	Xxx xx, 202x	xx:xx-xx:xx
Xxx xx, 202x	xx:xx-xx:xx	Xxx xx, 202x	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading: Verification of the comprehension and knowledge of the core topics covered in the course through a colloquium and/or the presentation of specific topics studied individually.

Introduction and background requirements: In mechanics, a nonholonomic constraint is a restriction on the possible velocities of a holonomic system without restricting its possible configurations. Examples are systems of rigid bodies whose contact points are constrained to have a preassigned velocity and the possibility to manoeuvre a car into parallel parking despite the fact that the wheels cannot move transversally. Nonholonomic mechanics is very different from Lagrangian mechanis because the equations of motion do not come from a variational principle. Many properties of the dynamics are still not completely understood and the subject is an active field of research, which is of interest in various areas of Mathematics and Engeneering. The aim of the course is to give an introduction to the subject, focussing mostly on the dynamical aspects and presenting the students to recent progress and open questions in the field. Prerequisites are basic notions of classical mechanics (Lagrangian mechanics), of the qualitative theory of ODEs and of analysis and differential geometry. More advanced topics (e.g., Frobenius theorem, Lie groups actions) will be reviewed.

Aim: The aim of the course is to provide a basic, though solid, knowledge of the geometric structure and of the dynamics of nonholonomic mechanical systems, leading the students to the level of understanding the current literature.

Course contents: Nonholonomic constraints. Distributions and Frobenius theorem. Ideal constraints and D'Alembert principle. The equations of motion for linear and nonlinear constraints. The role and structure of the reaction forces. Basic examples of nonholonomic mechanical systems. First integrals: energy, momenta, the nonholonomic Noether theorem. Based on the interest of the partecipants and on the time availability, one or more of the following topics might be covered: (non)existence of conserved measures; reduction under symmetry; integrability; control and trajectory planning of nonholonomic systems.

References:

1. Lecture notes will be prepared and made accessible to the students during the course.
2. L. García-Naranjo, Geometry and dynamics of nonholonomic systems. Lecture Notes for a course given in this Doctoral School in 2021.
3. Ju. I. Neĭmark and N.A. Fufaev, Dynamics of Nonholonomic Systems (AMS, 1972).
4. F. Bullo and A.D. Lewis, Geometric Control of Mechanical Systems (Springer, 2004).
5. R. Cushman, J.J. Duistermaat and J. Snyaticki, Geometry of Nonholonomically Constrained Systems (World Scientific, 2010).
6. A. Bloch, Nonholonomic Mechanics and Control (Springer, 2015).