Large deviations and Metastability

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SSD: MATH-03/B

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30:

Dec 15, 2026	14:30-16:30	Jan 26, 2027	14:30-16:30
Dec 17, 2026	10:30-12:30	Jan 28, 2027	10:30-12:30
Jan 12, 2027	14:30-16:30	Feb 9, 2027	14:30-16:30
Jan 14, 2027	10:30-12:30	Feb 11, 2027	10:30-12:30

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading: Presentation of an in-depth study of part of the course, or a seminar on a paper.

Introduction and background requirements: Basic knowledge of probability theory: discrete and continuous random variables, finite and countable probability spaces. Familiarity with the main modes of convergence of sequences of random variables, as well as with the law of large numbers and the central limit theorem. Knowledge of Markov chains is also required.

Aim: A solid understanding of large deviation theory and of its applications to statistical mechanics, with particular emphasis on phenomena of metastability. This includes familiarity with the mathematical framework underlying rare events and their probabilistic characterization.

Course contents: Large deviation theory:

- *Introduction to large deviation theory:* An overview of the motivations and core ideas behind large deviation theory, focusing on the study of rare events and their exponentially small probabilities in probabilistic systems.
- *Cramér's Theorem and its applications:* A detailed study of Cramér's Theorem, which characterizes the rate at which the probabilities of large deviations decay for sums of independent and identically distributed random variables. Applications illustrate how this theorem provides exponential estimates beyond classical limit results.
- *Large Deviation Principle:* Formalization of the Large Deviation Principle (LDP), including precise definitions and conditions under which it holds for sequences of probability measures.
- *Computation of main rate functions:* The rate functions quantify the cost or likelihood of deviations from typical behavior in probabilistic systems. In this part of the course, we focus on explicit computations of rate functions for sequences of Bernoulli, Poisson, and Gaussian random variables

- *Sanov's Theorem and applications:* Sanov's Theorem is presented as an extension of large deviation ideas to empirical measures, offering bounds on the probability of observing atypical distributions. This topic also discusses applications in information theory and statistical mechanics.
- *Varadhan's Lemma and the concept of free energy:* Varadhan's Lemma is explored as a key tool linking large deviation estimates to the asymptotics of exponential integrals, and its connection to the notion of free energy in physical systems is highlighted.
- *Metastability and selected applications:* An exploration of metastable phenomena, where systems remain in "quasi-stable" states for long times due to rare transitions, showing how large deviation techniques can describe such behavior. Selected applications illustrate the relevance of these ideas in statistical mechanics and dynamical systems.

References:

- *Large deviations*, F. den Hollander, 2000, American Mathematical Society.
- *Large deviations techniques and applications*, A. Dembo and O. Zeitouni, Springer.
- *Probability Theory: A Comprehensive Course* A. Klenke, Springer.
- *Large deviations and Metastability* E. Olivieri and M. E. Vares, Cambridge.
- *Metastability: a potential-theoretic approach*, A. Bovier and F. den Hollander, Springer.