Mathematical Control of Multi-Agent Systems: From Traffic to Crowds

Bnedetto Piccoli[1]

[1]*Department of Mathematical Sciences,Rutgers University–Camden, NJ, USA*
Email: piccoli@camden.rutgers.edu

SSD: MATH-03/A

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Feb 23, 2027	14:00-16:00	Jun xx, 2027	xx:xx-xx:xx
Feb 24, 2027	14:00-16:00	Jun xx, 2027	xx:xx-xx:xx
Feb 25, 2027	10:30-12:30	Jun xx, 2027	xx:xx-xx:xx
Feb 26, 2027	10:30-12:30	Jun xx, 2027	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar)

Introduction and background requirements: Basics on Partial Differential Equations, Measure Theory.

Aim: Establishing the mathematical frameworks for vehicular traffic, network flows, and the transition between modeling scales

Examination and grading: Oral presentation of a research paper

Course contents:

Week 1: Fundamentals of Multi-Scale Modeling for Multi-Agents systems

Day 1: Introduction to Multi-Agent Systems & Scales

Topic: Overview of large-scale multi-agent systems and the three scales of modeling: Microscopic, Mesoscopic (Kinetic and mean-field), and Macroscopic (Fluid dynamics).

Key Concepts: Distinction between ”blind” gas particles (entropy) and ”intelligent” agents (self-organization), Introduction to Agent-Based Models (ABM) vs. Continuum Models.

Suggested Readings: Delle Monache, Maria Laura, et al. ”Modeling, monitoring, and controlling road traffic using vehicles to sense and act.” Annual Review of Control, Robotics, and Autonomous Systems 8.1 (2025): 211-233. B. Piccoli: Control of multi-agent systems: results, open problems, and applications, Open Mathematics, 21 (2023), math-2022-0585.

Day 2: Kinetic Models and Mean-Field Limits

Topic: Bridging the gap between micro and macro.

Key Concepts: Mean-Field Games (MFG) and mean-field control: Agents solving games with perfect knowledge and without. Vlasov-type Equations: How microscopic interactions with kernel Q lead to kinetic equations.

Suggested Readings: Paul, Thierry, and Emmanuel Trélat. "From microscopic to macroscopic scale equations: mean field, hydrodynamic and graph limits." arXiv preprint arXiv:2209.08832 (2022). Golse, François. "On the dynamics of large particle systems in the mean field limit." Macroscopic and large scale phenomena: coarse graining, mean field limits and ergodicity. Cham: Springer International Publishing, 2016. 1-144.

Day 3: Introduction to Measure Differential Equations

Topic: Overview of Wasserstein spaces and Measure Differential Equations.

Key Concepts: Wasserstein distance and Wasserstein spaces. Measure vector fields and measure differential equations.

Suggested Readings: Villani, Cédric. Topics in optimal transportation. Vol. 58. American Mathematical Soc., 2021. Piccoli, Benedetto. "Measure differential equations." Archive for rational mechanics and analysis 233.3 (2019): 1289-1317.

Day 4: Introduction to control theory

Topic: Overview of mathematical theory of controlled systems.

Key Concepts: Control system, open and closed-loop controls, existence of trajectories, controllability, optimal control.

Suggested Readings: Bressan, Alberto, and Benedetto Piccoli. Introduction to the mathematical theory of control. Vol. 1. Springfield: American institute of mathematical sciences, 2007.

Week 2: Crowd Dynamics, Social Interactions, and Control

Day 1: Phenomenology and modeling of Crowds

Topic: Empirical observations and experimental data.

Key Concepts: Self-Organization vs. Entropy: How pedestrians differ from gas particles (anisotropic, vision-based, decision-making). Experimental Settings: "Exiting the Metro" scenarios, path choice, and virtual maps. Microscopic: Helbing's Social Force Model, Maury-Venel. Macroscopic: Bellomo-Dogbé, Colombo-Rosini models. Time-Evolving Measures: Using flow maps and push-forward formulas.

Suggested Readings: Cristiani, Emiliano, Benedetto Piccoli, and Andrea Tosin. Multiscale modeling of pedestrian dynamics. Springer, 2014.

Day 2: Coupling controlled agents and measure differential equations

Topic: Rigorous analysis of evolving measures.

Key Concepts: Existence and uniqueness of solutions to MDEs. The continuity equation and superposition principles. Coupling microscopic controls with MDEs.

Suggested Readings: Piccoli, Benedetto. "A mathematical approach to crowd surveillance with drones." Mathematical Models and Methods in Applied Sciences 35.07 (2025): 1661-1679. Piccoli, Benedetto. "Surveillance Guidance of Crowds via Lagrangian Controls." Crowd

Dynamics, Volume 5: From Human Complexity to Scientific Machine Learning. Cham: Springer Nature Switzerland, 2025. 151-172.

Day 3 Control of Multi-Agent Systems

Topic: Influencing the system behavior.

Key Concepts: Sparse Control: Controlling a system by acting on a few agents (e.g., "leaders" in Cucker-Smale). Curse of Dimensionality: Challenges in controlling large systems. Future Directions: Hybrid systems, Mixed ODE-PDE models, and human-in-the-loop control.

Suggested Readings: Fornasier, Massimo, Benedetto Piccoli, and Francesco Rossi. "Mean-field sparse optimal control." Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 372.2028 (2014): 20130400. B. Piccoli: Control of multi-agent systems: results, open problems, and applications, Open Mathematics, 21 (2023), math-2022-0585.

Day 4: Presentation of final projects