Linear Parabolic Equations in Hilbert Spaces: analysis and numerical approximation

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SS: MATH-05/A

CFU/ECTS: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Xxx xx, 2027	xx:xx-xx:xx	Xxx xx, 2027	xx:xx-xx:xx
Xxx xx, 2027	xx:xx-xx:xx	Xxx xx, 2027	xx:xx-xx:xx
Xxx xx, 2027	xx:xx-xx:xx	Xxx xx, 2027	xx:xx-xx:xx
Xxx xx, 2027	xx:xx-xx:xx	Xxx xx, 2027	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Introduction and background requirements: real and functional analysis, basics of numerical analysis. All the essential notions will be briefly recalled during the lectures.

Aim: The course will offer, in the framework of linear parabolic PDEs, the opportunity of running into the whole scientific process of analyzing a mathematical problem, constructing its solution, and developing a robust numerical approximation method by exploiting the same properties of the problem that surfaced in the analysis step.

Examination and grading: either oral examination on the content of the course, or presentation of a related research paper. Whenener the background of the student includes some programming skills, the presentation of numerical experiments might be included in the exam.

Course contents: The course consists of two closely related and complementary parts: the first 10 hours will be devoted to the construction of the solutions of certain classes of linear parabolic equations formulated as evolution equations in Hilbert spaces. The second part of the course (6 hours) concerns the numerical approximation of such solutions. First we consider the semi-discrete approximation by the Faedo-Galerkin approach, then we exploit the properties of analytic semigroups to define a fully-discrete sequence of approximations by means of the Laplace transform and its numerical inversion.

Due to the time constraint, only the most important (and/or instructive) results will be proven, while many others will be only presented and discussed. Some classical equations (e.g., heat eq., convection-diffusion eq., Sobolev eq., and visco-elastic eq.) will be used as examples both to apply the presented theoretical results and verify their hypothesis, and to test the introduced approximation techniques.

Part 1 (10h - 5 lectures)

1) Quadratic forms and linear operators on Hilbert spaces;
2) Accretive operators, generation of contraction and analytic semigroups, relation with Laplace transform;
3) Solving first-order (in time) linear non-degenerte explicit parabolic equations;
4) More general linear parabolic equations than $u_t = \mathcal{L}u + f$: implicit and second-order equations;
5) Classical examples: heat, convection-diffusion, visco-elastic, and Sobolev equations.

Part 2 (6h - 3 lectures)

6) Galerkin method for elliptic problems, standard error estimates;
7) Semi-discretization by Faedo-Galerkin method, error analysis and convergence;
8) Fully discrete approximation by Laplace transform and quadrature, convergence analysis for sectorial operators.