Geometric Flows and Applications

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SS: MATH-03/A

Credits: 3

Timetable: 16 hrs. Torre Archimede, Room 2BC30

Mar 8, 2027	15:00-18:00	Mar 16, 2027	09:00-12:00
Mar 9, 2027	09:00-12:00	Mar 22, 2027	16:00-18:00
Mar 15, 2027	16:00-18:00	Mar 23, 2027	09:00-12:00

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar.

Examination and grading:

Introduction and background requirements:

Aim: The idea of deforming a geometric object via a parabolic PDE to $= E2 = 80 = 9C$ simplify $= E2 = 80 = 9D\ it = E2 = 80 = 94$ for instance, by reducing its topological complexity or increasing its symmetry $= E2 = 80 = 94$ dates back to Eells and Sampson, who introduced the harmonic map heat flow as the first geometric evolution equation. Since then, geometric flows have proven highly effective across a diverse range of areas, including topology, analysis, and physics, yielding significant and fruitful outcomes.
This course will examine a selection of these results, with a particular focus on geometric inequalities. Furthermore, it will explore the analytical tools developed in the literature to tackle these problems.

Course contents:

- Extrinsic and intrinsic geometric evolution equations; geometric gradient flows
- The role of flows in proving geometric and functional inequalities

The Mean Curvature Flow (MCF) and Network Flow

- Formal derivation of the MCF and the evolution of networks Comparison between different functionals and dissipation mechanisms.
- Basic properties: short-time existence, "geometric uniqueness" and invariance under reparametrization.

Flows Past Singularities

- Huisken's monotonicity formula and blow-up analysis.
- Restarting procedures for the network flow and surgery techniques for MCF.

The Curve Shortening Flow (CSF)

- A complete proof of Grayson=E2=80=99s Theorem.
- The rescaled flow and the isoperimetric inequality in the plane.

The Inverse Mean Curvature Flow (IMCF

- Definition of classical solutions and basic properties.
- Weak formulation: the level-set approach by Huisken and Ilmanen.

Applications to Geometric Inequalities

- Isoperimetric, Willmore, and Minkowski-type inequalities.
- The Riemannian Penrose Inequality.

Approximation of IMCF and MCF

- The p-harmonic approximation of the Inverse Mean Curvature Flow.
- Phase-field approximation of the Mean Curvature Flow.

References:

- G. Huisken and T. Ilmanen, The Inverse Mean Curvature Flow and the Riemannian Penrose Inequality, Journal of Differential Geometry, 59(3), 2001, pp. 353=E2=80=93437.
- C. Mantegazza, Lecture Notes on Mean Curvature Flow, Progress in Mathematics, Birkh=C3=A4user/Springer Basel AG, Basel, 2011.
- C. Mantegazza, M. Novaga, A. Pluda, and F. Schulze, Evolution of networks with multiple junctions, Ast=C3=A9risque, 452, 2024, pp. 1=E2=80=93266.