Set separation, minimizing processes, and controllability

Franco Rampazzo

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova*
mail: rampazzo@math.unipd.it

SS: MATH-03/A

Credits: 4

Timetable: 24 hrs. Torre Archimede, Room 2BC30, calendar in progress:

Feb xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Feb xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Feb xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Feb xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Feb xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx
Feb xx, 2027	xx:xx-xx:xx	Feb xx, 2027	xx:xx-xx:xx

see also on Calendar of Activities on https://dottorato.math.unipd.it/calendar)

Examination and grading: Knowledge of definitions and of the theorems' statements, together with a seminar on a research paper (or some book's section) provided by the instrructor.

Introduction and background requirements: Set-separation is a basic principle lying at the basis of several classical or more recent results on necessary conditions for solutions of a wide class of minimum problems. Actually, from Fermat's Theorem on extrema of real functions to (possibly higher order) Maximum Principles in constrained Control Theory and beyond, set-separation plays a foundational role. Dually, results on local controllability, namely the possibility of invading full neighborhoods of an initial point by given dynamical systems, can be formulated in terms of a lack of set-separation. In order to follow this course, no special competences are required besides a standard knowledge of real analysis.

Aim: The student will get a general understanding on how the application of set-separation, through suitable notions of (possibly non-smooth) tangential objects and the corresponding open mapping theorems, plays a crucial role in deducing necessary conditions for minimum problems of various kind. Questions of controllability will be also the occasion to get some insight into the use of non-commutativity of vector fields, e.g. in Sub-Riemannian geometry or in some questions concerning Sub-Elliptic PDEs.

Course contents:

- Set separation and open mapping theorems.
- Basic of ODEs with controls.
- Necessary conditions for various kinds of minimum problems.
- Non-commutativity of vector fields, Lie brackets and classical theorems on the subject.

- Controllability results and some connected applications in PDEs and Subriemannian Geometry.

References: printed notes provided by the teacher.