

Introduction to multivariate polynomial approximation

Alvise Sommariva¹

¹Department of Mathematics Tullio Levi-Civita, University of Padua
Email: alvise@math.unipd.it

SSD: MATH/05-A

Credits: 4

Timetable: 24 hrs. Torre Archimede, Room 2BC30:

Nov 9, 2026	14:00-16:00	Nov 18, 2026	14:00-16:00
Nov 10, 2026	14:00-16:00	Nov 19, 2026	14:00-16:00
Nov 11, 2026	14:00-16:00	Nov 23, 2026	14:00-16:00
Nov 12, 2026	14:00-16:00	Nov 24, 2026	14:00-16:00
Nov 16, 2026	14:00-16:00	Nov 25, 2026	14:00-16:00
Nov 17, 2026	14:00-16:00	Nov 26, 2026	16:30-18:30

see also on Calendar of Activities on <https://dottorato.math.unipd.it/calendar>.

Examination and grading: seminar on a paper.

Introduction and background requirements: the course requires the knowledge of the basics of measure theory and multivariate calculus as well as some concepts of numerical analysis as interpolation, least-squares and one dimensional numerical integration; where necessary, the lecturer will introduce elementary notions, so to broad the course to a wider audience.

Aim: the goal is to provide some ideas on multivariate polynomial approximation on classical domains as simplex, sphere, cubes, but also on more complicated ones that arise in Finite or Virtual Elements Methods. In general these topics are not treated in university courses, though they are of fundamental interest in applied sciences. The outcomes are of practical and theoretical nature. The student will learn how to approach numerically multivariate polynomial approximation, but also deeper mathematical details concerning error analysis. Numerical experiments will be given to provide more insights on the topics.

Course contents:

- univariate and multivariate interpolation (on well-choosen sets) and Lebesgue constants,
- hyperinterpolation and variants for data denoising,
- numerical cubature on multivariate domains: application to hyperinterpolation, FEM and VEM,
- (weakly) admissible meshes and good interpolation/least squares pointsets (time permitting).

References:

1. P.J. Davis, P. Rabinowitz, *Methods of Numerical Integration*, Dover Publications, 2007.
2. N. Levenberg, J-P. Calvi, *Uniform approximation by discrete least squares polynomials*, Journal of Approximation Theory, 152, Issue 1 (2008), pp.82–100.
3. A. Sommariva and M. Vianello, *Computing approximate Fekete points by QR factorizations of Vandermonde matrices*, Computers Mathematics with Application, 57 (2009), pp.1324-1336.
4. I.H. Sloan, *Polynomial Interpolation and Hyperinterpolation over General Regions*, Journal of Approximation Theory, 83, Issue 2 (1995), pp.238–254.