

2nd Dolomites Workshop on Constructive Approximation and Applications

Alba di Canazei (1517 m), Val di Fassa (Trento), Italy
September 4–9, 2009

Dedicated to Borislav Bojanov. In memoriam.

Book of abstracts

<http://www.math.unipd.it/~dwcaa09/>

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I=Invited talk, **A**=Session A, **B**=Session B, **C**=Session C, **D**=Session D

Thursday, September 3

21:00–22:30 Registration

Friday, September 4

10:30–12:30	Registration	
12:30–15:00	Lunch	
15:00–15:45	Registration	
15:45–16:00	Opening	
16:00–17:00	C. de Boor, I	The error in multivariate polynomial interpolation, 16
17:00–18:00	N. Dyn, I	Nonlinear subdivision schemes—case study, 17
18:00–19:00	Welcome Reception	

Saturday, September 5

8:00–9:00	Registration	
9:00–10:00	R. Schaback, I	Nonstandard kernels and their applications, 19
10:00–10:30	I. Cação, A	Complete orthonormal sets of polynomial solutions of the Riesz and Moisil-Teodorescu systems in \mathbb{R}^3 , 28
	W. Gautschi, C	Gauss quadratures for two classes of logarithmic weight functions, 68
	A. Chesnokov, D	A numerical solution of the constrained weighted energy problem, 85
10:30–11:00	M. A. Piñar, A	Orthogonal polynomials relative to measures with mass points in several variables, 31
	A. Corbo Esposito, C	Quadrature with respect to balanced measures, 66
	P. Novati, D	Implementation of Lawson-Adams multistep methods for large stiff problems, 94
11:00–11:30	Coffee break	

Saturday, September 5

11:30–12:00	L. Demaret, A	Wedgelet segmentations and approximation classes, 33
	G. Nikolov, C	Adapting the Peano kernel theory to estimate the error of product cubature formulae, 72
	P. Mathé, D	Approximation theoretic aspects of variable Hilbert scales, 93
12:00–12:30	G. Tamberg, A	On approximation properties of the generalized Shannon sampling operators generated by band-limited kernels, 42
	R. Cools, C	Construction of lattice rules: from K -optimal lattices to extremal lattices, 65
	F. Sgallari, D	Edge-preserving multilevel methods for deblurring, denoising, and segmentation, 97
12:30–15:00	Lunch	
15:00–16:00	I. H. Sloan, I	Multiscale approximation on the sphere, 20
16:00–16:30	M. Berzins, A	Nonlinear Data-Bounded Polynomial Approximations and their Applications, 27
	K. Hesse, B	Smoothing Approximation on the Sphere from Noisy Scattered Data, 55
	C. Guardasoni, C	Efficient numerical integration schemes for the discretization of hypersingular BIEs related to wave propagation problems, 64
16:30–17:00	F. Rapetti, A	Spectral element methods on unstructured meshes, 39
	F. Filbir, B	Scattered data approximation on manifolds, 51
	A. Salvadori, C	Recent developments on the 3D BEM approximation of hyperbolic problems, 79
17:00–17:30	Coffee break	
17:30–18:00	H. R. Malonek, A	Constructing complete sets of hypercomplex Appell sequences, 37
	S. Morigi, B	Composed Segmentation and Reconstruction by an Anisotropic PDE Model, 60
	F. D. d’Almeida, C	Error Bounds and Discretization Grids in the Solution of Weakly Singular Integral Equations, 67
18:00–18:30	A. Mazzia, B	Tensor Product Based Meshless Solution of Potential Problems, 59
	K. Orav-Puurand, C	Central part interpolation and product integration method for weakly singular Fredholm integral equations, 74

Sunday, September 6

	Excursion
21:00	Wine tasting

Monday, September 7

9:00–10:00	N. Trefethen, I	Approximation theory and approximation practice, 21
10:00–10:30	M. Floater, A	Barycentric rational interpolation with no poles and high rates of approximation, 34
	R. Beatson, B	The two stage method with an adaptive spline second stage, 49
	T. Okayama, C	Improvement of a Sinc-collocation method for Fredholm integral equations of the second kind, 73
10:30–11:00	J. Delgado, A	Two problems where the choice of the weights plays a key role, 32
	W. zu Castell, B	Kernel based methods for vector data with correlated components, 62
	B. Quatember, C	Assessment of Coronary Haemodynamics: Treatment of Tracer Kinetic Problems by Solving Integral Equations, 76
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	N. Petrovskaya, B	Discontinuous Weighted Least-Squares Approximation with Anisotropic Support, 61
	U. Hämarik, D	On solution of linear problems by the extrapolated Tikhonov method, 92
12:00–12:30	Q. Wang, A	A high order multivariate approximation scheme on arbitrary grids, 46
	A. Iske, B	Analysis of High-Dimensional Signal Data by Manifold Learning and Convolution Transforms, 53
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17:00–18:30	N. Trefethen	The Chebfun software system (tutorial)

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	G. Vainikko, C	A class of Volterra equations with noncompact weakly singular integral operators (part I), 81
	A. Frommer, D	Verified Computation of Square Roots of a Matrix, 90
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16:00–16:30	T. Hangelbroek, B	Stability of Boundary-Free Surface Spline Interpolation, 54
	M. Rebelo, C	Computational methods for a weakly singular integral equation based on extrapolation procedures, 77
	P. G. Constantine, D	Spectral Methods for Parameterized Matrix Equations, 86
16:30–17:00	F. Bernal, B	An RBF Method for the Numerical Solution of Delayed Differential Equations, 50
	W. Sproessig, C	Quaternionic Analysis applied to Initial-Boundary-Value, 80
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	L. Morgado, C	Numerical solution of a singular boundary value problem for the p -Laplace operator using a finite difference method, 71
	M. Popolizio, D	On approximating matrix functions in the solution of fractional differential equations, 95
10:30–11:00	K. Deckers, A	A rational variant of Fejér's quadrature rule with arbitrary complex poles, 30
	K. Ruotsalainen, C	Boundary element methods for time-fractional diffusion equations, 78

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	B. Baxter, B	On spherical averages of radial basis functions, 48
	P. M. Lima, C	Numerical Solution of Backward-Forward Equations: Comparative analysis of methods, 69
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	E. Larsson, B	A scaling perspective on accuracy and convergence in RBF approximations, 58
	G. Mantica, C	Dynamical Systems and Numerical Analysis: Measures and Jacobi Matrices generated by uncountable I.F.S., 70
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15:00–15:30	J. M. Carnicer, A	Weighted interpolation for equidistant nodes, 29
	E. H. Georgoulis, B	On the suboptimality of the p -version interior penalty discontinuous Galerkin method, 52
15:30–16:00	V. Kim, A	The Lebesgue constants from \mathbb{C} to \mathbb{C} for interpolation \mathcal{L} -splines of third order, 36
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16:00	Coffee break and good-byes	

Invited talks

The error in multivariate polynomial interpolation

C. de Boor*

University of Wisconsin, Madison, USA

A survey of known error formulas and a discussion of some desirable features of a general error formula, finished with a puzzling example.

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Nonlinear subdivision schemes—case study

N. Dyn*

Tel-Aviv University, Israel

Subdivision schemes are processes for the generation of curves/surfaces from discrete data by repeated refinements. The implementation of these schemes is simple, but their mathematical analysis is rather involved. This talk will present briefly the "classical" case of linear univariate schemes refining points and their analysis. In particular two basic schemes will be considered, and then extended to nonlinear schemes, refining points, curves and sets. Motivation for the need for such schemes will be given, together with hints on their analysis and few examples of their performance.

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Matrices, moments and quadrature

G. Meurant*

The aim of this lecture is to describe and explain the beautiful mathematical relationships between matrices, moments, orthogonal polynomials, quadrature rules and the Lanczos and conjugate gradient algorithms. The main topic is to obtain numerical methods to estimate or in some cases to bound quantities like $I[f] = u^T f(A)v$ where u and v are given vectors, A is a symmetric nonsingular matrix and f is a smooth function. There are many instances in which one would like to compute bilinear forms like $u^T f(A)v$. A first application is the computation of some elements of the matrix $f(A)$ when it is not desired or feasible to compute all of $f(A)$. Computation of quadratic forms $r^T A^{-i} r$ for $i = 1, 2$ is interesting to obtain estimates of error norms when one has an approximate solution of a linear system $Ax = b$ and r is the residual vector $b - Ax$. Bilinear or quadratic forms arise naturally for the computation of parameters in problems like least squares, total least squares and regularization methods for solving ill-posed problems. We will describe the algorithms and give some examples of applications.

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Nonstandard kernels and their applications

R. Schaback*

Georg-August-Universität Göttingen, Germany

Besides using standard radial basis functions, there are a few good reasons to look for kernels with special properties. This talk will provide several examples, starting from an introduction into kernel construction techniques. We shall review the use of kernels as particular or fundamental solutions of PDEs, look at harmonic kernels and their application to Poisson problems, ask for reproducing kernels connected to Taylor's formula and finally look for the missing Wendland functions, if time permits.

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Multiscale approximation on the sphere*

I. H. Sloan[†]

University of New South Wales, Sydney, Australia

This talk describes recent joint work with Q. Thong Le Gia and Holger Wendland, in which we describe, analyse and illustrate a multiscale method for the unit sphere $S^n \subseteq R^{n+1}$. The multiscale approximation is constructed using scaled versions of a single compactly supported radial basis function (RBF) $\Psi(\mathbf{x}, \mathbf{y}) = \psi(|\mathbf{x} - \mathbf{y}|)$ for $\mathbf{x}, \mathbf{y} \in R^{n+1}$. It uses a sequence of decreasing scales $\delta_1, \delta_2, \dots$, and a sequence of point sets X_1, X_2, \dots , with the mesh norm of X_j proportional to δ_j . The approximation is a linear combination of scaled RBFs $\Psi_\delta(\mathbf{x}, \mathbf{y}) := \Psi(\frac{|\mathbf{x} - \mathbf{y}|}{\delta})$ with different scales δ , restricted to the sphere S^n . The j th term is the correction at stage j obtained by interpolating the error at stage $j-1$ using RBFs for the finer scale δ_j and the larger point set X_j . While the idea of a multiscale scheme has appeared previously, for example in papers of Schaback, Narcowich/Schaback/Ward, Floater/Iske and Hales/Levesley, there seems to be no existing analysis of a multiscale approximation based on scaled versions of a single compactly supported RBF, for either spherical or Euclidean regions. In this talk I shall outline the ideas behind our error analysis for the sphere, and illustrate the method with a problem from geophysics, that of approximating height above sea level on Earth.

*Research supported by the Australian Research Council

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Approximation theory and approximation practice

N. Trefethen*
Oxford University, UK

Many of us know and love the classical theory of polynomial approximation. This talk will be a fast tour of this subject from an unusual point of view: everything will be computational, illustrated in the chebfun system by polynomials of degrees up to the millions. Topics touched upon will include barycentric interpolation, convergence rates for analytic functions, Lebesgue functions and constants, Caratheodory-Fejer approximation, the Remez algorithm, best vs. near-best, Gauss quadrature, and splines. Some surprises will turn up along the way.

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A Meshfree Approximation Method for Semilinear Parabolic PDEs

H. Wendland*

University of Sussex, UK

We introduce an approximation scheme for (systems of) semilinear parabolic problems, which works both on bounded domains and surfaces. The solution of the PDE is approximated by a method-of-line type approach. The space discretization of the scheme is based upon a (local) radial basis function approximation of the involved partial differential operators. We analyze the stability, the error and the complexity of the scheme and present a series of different test cases to demonstrate the relevance and flexibility of the scheme. These test cases comprise the Allen-Cahn equation for phase separation and the Gray-Scott model for pattern formation.

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Discrete Fourier analysis on Lattice, Cubature and Interpolation

Y. Xu*

University of Oregon, USA

We consider a discrete Fourier analysis on the fundamental domain of a lattice that tiles the Euclidean space by translation, such as regular hexagon in \mathbb{R}^2 and rhombic dodecahedron in \mathbb{R}^3 . Our results include orthogonality, cubature formulas and Lagrange interpolation by trigonometric functions. Here are two highlights: (1) a compact formula for the Lagrange interpolation by trigonometric functions on the simplex in \mathbb{R}^d with the Lebesgue constants in the order of $(\log n)^d$ as n goes to infinity; (2) our cubature formula for trigonometric functions on the simplex is shown to equivalent to a Gaussian cubature formula for algebraic polynomials on a domain in \mathbb{R}^d .

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Contributed talks: Session A

Polynomial and rational approximation

Organizers: J. M. Carnicer and A. Cuyt

Fighting Gibbs' phenomenon through quotienting*

J.-P. Berrut[†]

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Gibbs' phenomenon, the overshooting at jumps, is a very annoying drawback of infinitely smooth approximants. Many methods for its alleviation have been suggested, in the past as well as in recent years, see, e.g., the book by Jerri and the articles by Gottlieb, Gelb, Brezinski, Beckermann and their coauthors. Many of these methods do not act in physical space, but rather in a transformed space.

A very simple method working in physical space seems to have been overlooked so far. It is based on the following observation: for a given approximation operator, the quotient of the approximant and the approximated function f is very similar for various f . In this talk I shall present some conjectures precizing this observation and demonstrate how it may be used to alleviate, and in many cases even eliminate, the phenomenon.

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Nonlinear Data-Bounded Polynomial Approximations and their Applications

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The derivation and theoretical properties of a class of high-order data-bounded polynomials on general meshes will be given. Such polynomials make it possible to circumvent the problem of Runge-type oscillations on evenly spaced meshes by adaptively varying the stencil and order used, but at the cost of only enforcing C0 solution continuity at data points. It will be shown that the use of these high-order provably data-bounded polynomials, based on extensions of the work of [1], provides a way to develop positivity preserving polynomial approximations as well as methods of potentially high orders for hyperbolic equations. The central idea [1] is to use ENO (Essentially Non Oscillatory) type approximations but to enforce additional restrictions on how the polynomial order is increased by using ideas similar to the limiters employed in numerical methods in hyperbolic equations. The question of how high a polynomial order should be used will be considered, by reference to typical numerical examples.

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Complete orthonormal sets of polynomial solutions of the Riesz and Moisil-Teodorescu systems in \mathbb{R}^{3*}

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Consider the pair $F = (F_0, \vec{F})$, where F_0 is a real-valued continuously differentiable function defined in an open domain $\Omega \subset \mathbb{R}^3$ and $\vec{F} = (F_1, F_2, F_3)$ is a continuously differentiable vector-field in Ω .

The so-called Riesz system can be written as

$$\begin{cases} \operatorname{div} \vec{F} &= 0 \\ \operatorname{curl} \vec{F} &= 0 \end{cases}$$

and describes the velocity field of a stationary flow of a non-compressible fluid without sources nor sinks. The Moisil-Teodorescu system is represented by

$$\begin{cases} \operatorname{div} \vec{F} &= 0 \\ \operatorname{grad} F_0 + \operatorname{curl} \vec{F} &= 0 \end{cases}$$

and it is closely related with the Lamé equations of the theory of elasticity.

Both systems can be viewed as natural generalizations to \mathbb{R}^3 of the classical Cauchy-Riemann system in \mathbb{R}^2 .

Rewriting the Riesz and the Moisil-Teodorescu systems in quaternionic language, we construct complete orthonormal sets (CONS) of polynomial solutions of both systems on the sphere. Those CONS can be viewed as analogues to the complex case of the Fourier exponential functions $\{e^{in\theta}\}_{n \geq 0}$ on the unit circle and constitute a refinement of the well-known spherical harmonics.

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Weighted interpolation for equidistant nodes*

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Weighted Lagrange interpolation is proposed for solving Lagrange interpolation problems of degree on equidistant or almost equidistant data. Good condition numbers are found in the case of rational interpolants

$$r(x) = \frac{\sum_{i=0}^n y_i w(x_i) \prod_{j \neq i} \frac{x - x_j}{x_i - x_j}}{w(x)},$$

whose denominator

$$w(x) = \left(1 + \alpha \left(\frac{2x - a - b}{b - a}\right)^4\right)^k,$$

has degree $4k \approx 2n$, $n \leq 100$. Since the degree of the denominator is higher than the numerator, simple functions like constants and linear polynomials will not be exactly reproduced. Furthermore, some kinds of barycentric formulae cannot be used to express the interpolant. As a counterpart, the interpolation algorithm is simple and leads to low Lebesgue constants.

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A rational variant of Fejér's quadrature rule with arbitrary complex poles*

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In [2], we constructed efficient rational generalizations of the well-known classical Gauss-Chebyshev quadrature formula. These quadrature rules integrate rational functions with arbitrary but prefixed complex poles outside the interval $[-1, 1]$, with respect to the different Chebyshev weight functions $(1-x)^\alpha(1+x)^\beta$, where α and β belong to $\{\pm\frac{1}{2}\}$. These rational quadrature formulas were derived within the framework of orthogonal rational functions as described in [1, Chapter 11]. If there are only m distinct poles (possibly repeated), then these formulas can be constructed in order $\mathcal{O}(mn)$ operations, for arbitrarily high degree n . The main reason for the efficient computation of these rules is that we have explicit representations for the so-called Chebyshev (para-)orthogonal rational functions, which were also derived in [2]. Further, in [4] an algorithm was presented to compute the nodes and weights in these rational quadrature formulas.

In the special case of all real poles, in [3] was illustrated how the nodes and weights from the Gauss-Chebyshev quadrature formula can be used in the construction of a rational quadrature rule to approximate integrals without weight function; i.e.; integrals of the form $\int_a^b f(x)dx$, where $-\infty < a < b < \infty$ and $f(x)$ has real singularities near the interval of integration. In this contribution we show how the results from [3] can be generalized to the case in which $-\infty \leq a < b \leq \infty$ and $f(x)$ has arbitrary complex singularities near the interval of integration.

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Orthogonal polynomials relative to measures with mass points in several variables

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In one variable, the study of orthogonal polynomials relative to measures with mass points appears in a work of A. M. Krall ([1]) when he studied the orthogonal polynomials that are eigenfunctions of a fourth order differential operator. The purpose of the present contribution is to study this problem in several variables.

Let Π^d denote the space of polynomials in d -variables, and let $G \subset \mathbb{R}^d$ be a simply connected domain (having a nonempty interior), and let $d\mu(x)$ be a measure defined on the domain G . Let $\langle \cdot, \cdot \rangle$ denote the inner product defined on Π^d by means of

$$\langle p, q \rangle = \int_G p(x)q(x)d\mu(x).$$

Let $N \geq 1$ be a positive integer number, Λ be a symmetric and positive definite matrix of order N , and $\xi_1, \xi_2, \dots, \xi_N$, be N fixed points on \mathbb{R}^d . We define a new inner product $\langle \cdot, \cdot \rangle_K$ by means of

$$\langle p, q \rangle_K = \langle p, q \rangle + (p(\xi_1), p(\xi_2), \dots, p(\xi_N))\Lambda(q(\xi_1), q(\xi_2), \dots, q(\xi_N))^T.$$

Our main result expresses orthogonal polynomials with respect to $\langle \cdot, \cdot \rangle_K$ in terms of the orthogonal polynomials with respect to $\langle \cdot, \cdot \rangle$. Furthermore, we also obtain the relation between the corresponding reproducing kernels. In the particular case where $d\mu(x)$ denotes the classical measure on the simplex, explicit representations and asymptotic properties are obtained.

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Two problems where the choice of the weights plays a key role*

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Let us consider a sequence of parameters $\{t_i\}_{i=0}^n$ and a sequence of points $\{P_i\}_{i=0}^n$ in \mathbf{R}^2 or \mathbf{R}^3 such that the point P_i is assigned to the parameter t_i for all $i = 0, 1, \dots, n$. Then, given a blending basis (u_0, \dots, u_n) , we generate a starting curve as $\gamma^0(t) = \sum_{i=0}^n P_i^0 u_i(t)$ where $P_i^0 = P_i$ for $i = 0, 1, \dots, n$. Then, by calculating the adjusting vector for each control point $\Delta_i^0 = P_i^0 - \gamma^0(t_i)$, and taking $P_i^1 = P_i^0 + \Delta_i^0$ for $i = 0, 1, \dots, n$ we get the curve $\gamma^1(t) = \sum_{i=0}^n P_i^1 u_i(t)$. Iterating this process we can get a sequence of curves $\{\gamma^k\}_{k=0}^\infty$. Then, when the curve sequence converges to a curve interpolating the given initial sequence of points the initial curve is said to have the *progressive iteration approximation* (PIA) property.

Here we will make a survey of the well known results on the PIA (see [4] and [2]). In addition, we will present some rational representations satisfying the PIA (see [3]). Finally, we will make a suggestion for the choice of the weights for some of the representations and we will show the high rate of convergence of the PIA for this choice.

The choice of the weights is also very important when we want a rational curve to be near its control polygon. Respect to this problem, we will recall some results bounding the mean distance between a curve and its control polygon. Then we will apply these results to the particular case of rational curves and we will provide a suggestion for the choice of the weights (see [1]).

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Wedgelet segmentations and approximation classes

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The search for a *good* paradigm for detecting and representing curvilinear singularities of bivariate functions remains a fundamental issue in image analysis. Efficient representations should use atomic decompositions which are local in space (like wavelets), but also possess appropriate directional properties (unlike wavelets). Wedgelet partitions, as introduced by Donoho in 1999 [1], belong to the class of shape-preserving image segmentation methods. The decompositions are based on local polynomial approximation on some adaptively selected leaves of a quadtree structure. Adaptivity is obtained by the minimisation of a global variational functional, that can be reduced to several elementary regression problems. This, together with a suitable data structure, enables the development of fast algorithm for wedgelet decomposition (see our previous work [2]).

In this talk, we are concerned with the characterisation of functional classes defined by the decay of their n -term best-approximations via a continuous wedgelet transform. In particular we discuss how the approximation classes for the wedgelet transform are different from those classes provided by wavelets or by adaptive piecewise polynomial approximations (in both cases isotropic Besov or Sobolev spaces). The case of piecewise polynomial quadtree approximation was already treated in 1967 in [3] with the help of a well-chosen sub-additive functional. The use of modified sub-additive functionals for wedgelets lead to a straightforward characterisation of wedgelet approximation classes. The resulting functional classes contain but are much larger than the prototypical horizon functions considered by Donoho in his seminal work [1].

Explicit constructions of well approximating functions belonging to the wedgelet decompositions have applications, for instance, in image compression. In this context the connection with entropic considerations of the functional classes and discretisation problems are discussed.

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Barycentric rational interpolation with no poles and high rates of approximation

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It is well known that rational interpolation sometimes gives better approximations than polynomial interpolation, especially for large sequences of points, but it is difficult to control the occurrence of poles. In this talk we propose and study a family of barycentric rational interpolants that have no real poles and arbitrarily high approximation orders on any real interval, regardless of the distribution of the points. These interpolants depend linearly on the data and include a construction of Berrut as a special case.

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High order parametric polynomial approximation of conic sections

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Conic sections, particularly circular arcs, are fundamental objects in computer aided geometric design. Although they can be exactly represented as rational parametric curves, they have no exact parametric polynomial representation (except for parabola, of course). Thus it is an interesting and important problem to look for good parametric polynomial approximants.

In this talk, a new class of parametric polynomials of degree $\leq n$ will be presented, which approximate the whole circle with the radial error of order $2n$. The approximants have some surprising properties and can be constructed by knowing the sines of particular fractions of π only. A similar result can be derived for ellipse and hyperbola.

Using those results a high order polynomial approximation of a sphere can be obtained. The results are a generalization of [1] and [2].

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The Lebesgue constants from \mathbb{C} to \mathbb{C} for interpolation \mathcal{L} -splines of third order

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Cardinal \mathcal{L} -splines of third order belong to the kernel of $\mathcal{L}(D) = D(D - \alpha)(D - \beta)$, where $\alpha < 0$, $\beta > 0$, D — operator of differentiation, with interpolation nodes $\{2hk, k \in \mathbb{Z}\}$ and possible discontinuity of second derivative in $\{2hk - \omega, k \in \mathbb{Z}\}$, where $\omega = \frac{1}{\beta - \alpha} \cdot \ln((1 + e^{2h\beta})/(1 + e^{2h\alpha}))$. This choice of ω is caused by [1]. It also follows from [1] that for each continuous function, bounded on \mathbb{R} , exists the only cardinal \mathcal{L} -spline of third order.

Theorem. The Lebesgue constants for cardinal \mathcal{L} -splines of third order equals $\|L_\infty^\mathcal{L}\|_\infty =$

$$-\frac{1}{\alpha\beta} \cdot [(e^{2h\alpha}I_{-1} - (1 + e^{2h\alpha})I_0 + I_1)^{\frac{\alpha}{\alpha-\beta}} \cdot (e^{2h\beta}I_{-1} - (1 + e^{2h\beta})I_0 + I_1)^{\frac{\beta}{\beta-\alpha}} - (e^{2h(\alpha+\beta)}I_{-1} - (e^{2h\alpha} + e^{2h\beta})I_0 + I_1)], \text{ when } a \geq c,$$

$$\|L_\infty^\mathcal{L}\|_\infty = -\frac{1}{\alpha\beta} \cdot [(e^{2h\alpha}J_{-1} - (1 + e^{2h\alpha})J_0 + J_1)^{\frac{\alpha}{\alpha-\beta}} \cdot (e^{2h\beta}J_{-1} - (1 + e^{2h\beta})J_0 + J_1)^{\frac{\beta}{\beta-\alpha}} - (e^{2h(\alpha+\beta)}J_{-1} - (e^{2h\alpha} + e^{2h\beta})J_0 + J_1)], \text{ when } a \leq c.$$

where $I_{-1} = [1/(1 + x_1) + 1/(1 + x_2)]/\sqrt{D}$, $I_0 = [x_1/(1 + x_1) + x_2/(1 + x_2)]/\sqrt{D}$,
 $I_1 = [(2x_1^2 + x_1)/(1 + x_1) - x_2/(1 + x_2)]/\sqrt{D}$, $J_1 = [x_1/(1 + x_1) + x_2/(1 + x_2)]/\sqrt{D}$,
 $J_0 = [1/(1 + x_1) + 1/(1 + x_2)]/\sqrt{D}$, $J_{-1} = [-1/(1 + x_1) + (2 + x_2)/(x_2(1 + x_2))]/\sqrt{D}$,
 $x_2 < -1$, $-1 < x_1 < 0$ — roots and D — discriminant of $ax^2 + bx + c = 0$,

$$a = \frac{1}{\alpha\beta(\beta-\alpha)}[\beta - \alpha - \beta e^{\omega\alpha} + \alpha e^{\omega\beta}], \quad c = \frac{e^{2h(\alpha+\beta)}}{\alpha\beta(\beta-\alpha)}[\beta - \alpha - \beta e^{-2h\alpha+\omega\alpha} + \alpha e^{-2h\beta+\omega\beta}],$$

$$b = \frac{1}{\alpha\beta(\beta-\alpha)}[-(\beta - \alpha)(e^{2h\alpha} + e^{2h\beta}) + \beta(1 + e^{2h\beta})e^{\omega\alpha} - \alpha(1 + e^{2h\alpha})e^{\omega\beta}].$$

2hN-periodical \mathcal{L} -splines of third order belong to the kernel of $\mathcal{L}(D) = D(D^2 - \beta^2)$, where $\beta > 0$, D — operator of differentiation, with interpolation nodes $\{2hk, k \in \mathbb{Z}\}$ and possible discontinuity of second derivative in $\{2hk - h, k \in \mathbb{Z}\}$. Existence and uniqueness of interpolation periodical \mathcal{L} -splines of third order follows from [2].

Theorem. The Lebesgue constants for 2hN-periodical \mathcal{L} -splines of third order equals

$$\|L_{2n}^\mathcal{L}\|_\infty = \Delta \cdot (1 - |x_1|^n)/(1 + |x_1|^n), \text{ when } N = 2n \text{ (even), and}$$

$$\|L_{2n}^\mathcal{L}\|_\infty = \Delta \cdot (1 - |x_1|^{2n+1})(1 + |x_1|^{2n+1}), \text{ when } N = 2n + 1 \text{ (odd),}$$

where $\Delta = (1 + e^{-h\beta})(1 + e^{h\beta})/\sqrt{(1 + e^{-h\beta})^2 + (1 + e^{h\beta})^2}$
and $x_1 = -(1 + e^{-h\beta} + e^{h\beta}) + \sqrt{(1 + e^{-h\beta})^2 + (1 + e^{h\beta})^2}$.

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Constructing complete sets of hypercomplex Appell sequences*

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Hypercomplex function theory (HFT) generalizes the theory of holomorphic functions of one complex variable by using Clifford algebras and provides the fundamentals of Clifford Analysis as a refinement of Harmonic Analysis. For this purpose one considers, e.g., the standard orthonormal basis $\{e_1, e_2, \dots, e_n\}$ of \mathbf{R}^n as imbedded in a associated 2^n -dimensional real Clifford algebra $Cl_{0,n}$ with unit $e_0 \cong 1$ and subject to the multiplication rules $e_i e_j + e_j e_i = -2e_0 \delta_{ij}$, $(i, j = 1, \dots, n)$, where δ_{ij} is the Kronecker symbol). The main object of HFT are then $Cl_{0,n}$ -valued functions of the hypercomplex variable $z = x_0 e_0 + x_1 e_1 + \dots + x_n e_n$. Until the 80-ties the opinion persisted that the only way to introduce a class of generalized holomorphic functions in the context of HFT was as the solution set of a generalized Cauchy-Riemann system (see [3]). Later, the appropriate concept of hypercomplex differentiability, introduced in [2], led to hypercomplex monogenic polynomials in terms of the monogenic variables $z_l = x_l - x_0 e_l$, $l=1, \dots, n$. They are given with the help of generalized powers defined by $z_1^{\nu_1} \times \dots \times z_n^{\nu_n} := \frac{1}{k!} \sum_{\pi(i_1, \dots, i_n)} z_{i_1} \dots z_{i_n}$, where the sum is taken over *all* $k! = |\nu|!$ permutations of (i_1, \dots, i_n) , $i_j \in \{1, \dots, n\}$ ($j = 1, \dots, n$) (see [3]).

However, for numerical reasons, e.g. the reduction of numerical costs, the use of hypercomplex monogenic Appell polynomials (power-like monomials) and their corresponding generating function as an alternative to generalized powers is of special interest. The construction of a complete set of such polynomials is our main goal.

Note that Appell polynomials play an important rôle, for instance, in the development of constructive methods for approximation problems in Geometry or PDE (c.f. [1]), as well as for the application of Clifford Analysis techniques in discrete mathematics.

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Rational Approximation and Pole Location in the Complex Plane

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A central feature of the chebfun system is the representation of analytic functions on a real interval by polynomials of degree high enough that they are accurate to machine precision. This is a starting point to develop algorithms for numerical analytic continuation which we take further by rational approximation methods such as Padé-type and Carathéodory-Fejér approximation. The chebfun system is quite suitable for implementing these analytic continuation algorithms. For example, we use it to extrapolate the chebfun of the trajectory of the Lorentz equations to locate its singularities in complex time.

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Spectral element methods on unstructured meshes

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The main advantage of standard spectral methods relies on the exponential convergence property as soon as smooth solutions are involved. The main drawback is their inability to handle complex geometries. Different strategies are however possible to overcome this difficulty. The most famous one is certainly the quadrangle based spectral element method, developed in the 80's and then largely adopted. However, in order to handle highly complex geometries the use of triangular (tetrahedral in 3D) elements is generally preferred. Therefore, *hp*-finite element methods have been deeply investigated during this last decade.

In the field of spectral methods, it was suggested to use a change of coordinates to transform the quadrangle (and its quadrature points) into a triangle [1]. Here we are interested in an enhanced version of the triangle based SEM introduced in [2], relying on the use of Koornwinder-Dubiner polynomials and Fekete points, and in its efficient implementation [3]. The question of how to distribute interpolation nodes in a triangle which are suitable for high order polynomial interpolation is still a somewhat open problem. There have been several attempts to produce nodal sets utilizing direct and indirect methods to minimize the corresponding Lebesgue constant. We will summarize new existing nodal sets, comparing them with the Fekete set from the point of view of the Lebesgue constant, of the Vandermonde matrix conditioning and of the accuracy of the approximation spectral element method based on these sets.

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Representation of functions on unbounded domains in the Chebfun system

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Chebfun is a system for computing with functions and solving differential equation in Matlab. For bounded intervals, the mathematical basis is Chebyshev interpolants and series. We are in the process of extending the system to handle unbounded domains, such as $(-\infty, \infty)$, via rational mappings. Two of the algorithmic challenges that arise in the process are efficient scaling and accurate quadrature. The mapping technology we introduce is general enough to allow users to employ their own maps for applications like improved Gauss quadrature and adaptive spectral methods (Tee, Trefethen, and Hale).

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Rational Interpolation of Vertical Segments

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In many practical applications, exact interpolation is often not necessary or desirable. On the other hand, finding a rational least squares approximation is typically a difficult problem, since it reduces to a nonlinear optimization problem where the objective function may have many local minima. We present a somewhat different approach by allowing a bounded tolerance on the function values. Hence it is assumed that the uncertainty in the independent variables is negligible and that for each observation an uncertainty interval can be given which contains the (unknown) exact value. To approximate such data we look for rational functions which intersect all uncertainty intervals. We show how to reduce the problem to a quadratic programming problem with a strictly convex objective function, yielding a unique rational function which intersects all uncertainty intervals. The applicability of the technique is illustrated with multivariate engineering examples where the problem arises naturally .

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On approximation properties of the generalized Shannon sampling operators generated by band-limited kernels*

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In this talk we consider some generalized Shannon sampling operators, which are defined by band-limited kernels. For the uniformly continuous and bounded functions $f \in C(\mathbb{R})$ the generalized sampling series (see [2], [1] and references cited there) with a kernel function $s \in L^1(\mathbb{R})$ are given by ($t \in \mathbb{R}; W > 0$)

$$(S_W f)(t) := \sum_{k=-\infty}^{\infty} f\left(\frac{k}{W}\right) s(Wt - k), \text{ where } \sum_{k=-\infty}^{\infty} s(u - k) = 1. \quad (1)$$

In this talk we study an even band-limited kernel s , defined as the Fourier transform of an even window function $\lambda \in C_{[-1,1]}$, $\lambda(0) = 1$, $\lambda(u) = 0$ ($|u| \geq 1$) by the equality $s(t) := s(\lambda; t) := \sqrt{\frac{\pi}{2}} \lambda^\wedge(\pi t)$. Here the kernel function s belongs to a Bernstein class B_π^1 of functions of exponential type provided $s \in L^1(\mathbb{R})$. Many kernels are represented in this form, see [3], [4], [5].

We studied in [5] an dilated kernel $s_\alpha(t) = \alpha s(\alpha t)$. In the case of $s \in B_\pi^1$ for $0 < \alpha \leq 2$ we get an sampling operator $S_{W,\alpha} : C(\mathbb{R}) \rightarrow B_{\alpha\pi W}^\infty \subset C(\mathbb{R})$.

We can define an averaged kernel in form $\bar{s}_m(t) := \frac{1}{m} \int_{-m/2}^{m/2} s(t+v) dv$. If $s \in B_\pi^1$ then also $\bar{s}_m \in B_\pi^1$ (see [4] Th. 3).

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Multivariable orthogonal polynomials and structured matrix computations

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The recurrence coefficients of polynomials in one variable with respect to a discrete inner product can be computed by solving a structured inverse eigenvalue problem. In this talk we will investigate how this inverse eigenvalue problem is modified in case of multivariable orthogonal polynomials. We will also indicate how this inverse eigenvalue problem can be solved only using orthogonal similarity transformations. Some numerical experiments will show the validity of this approach.

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Strange convergence phenomena in rational interpolation

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Interpolation points which are near best for rational interpolation with fixed poles can be studied in terms of certain conformal maps involving inverse finite Blaschke products. Using these maps in the context of barycentric rational interpolation in transformed Chebyshev points (where the poles are no longer fixed) leads to rather strange convergence results. The expected geometric convergence seems to be broken for fixed multiples of the number of interpolation points.

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Lattices on Simplicial Partitions with Holes

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It is well known that the existence and the uniqueness of the Lagrange interpolant in Π_n^d , ($d > 1$), the space of polynomials in d variables of total degree $\leq n$, heavily depends on the geometry of the interpolation points. This fact makes interpolation in several variables much more complicated than the univariate one. Although a simple algebraic characterization states that a set of interpolation points is correct in Π_n^d if and only if they do not lie on an algebraic hypersurface of degree $\leq n$, it is useless in practical computations, since in general it can not be verified in the floating point arithmetic. Thus a considerable research has been focused on finding configurations of points which guarantee the existence and the uniqueness of the Lagrange interpolant. Perhaps the most frequently used configurations of this type are lattices which satisfy the well-known GC (geometric characterization) condition. Among them, principal lattices and their generalization, $(d+1)$ -pencil lattices, are the most important.

In this talk, $(d+1)$ -pencil lattices on simplicial partitions in R^d will be considered. The barycentric representation of the lattice enables us to extend the lattice from a simplex to a simplicial partition in such a way, that at least continuity over common faces of simplices is ensured. The number of free shape parameters will be considered for simplicial partitions with (or without) holes. It will be shown, how the lattice can be extended in order to fill the holes. Some important applications of lattices will be presented, too.

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A high order multivariate approximation scheme on arbitrary grids*

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We construct a high order multi-variate interpolation scheme for arbitrary scattered data sets. The estimated approximation error is minimized by solving a equality constrained least squares. The approximation function is an interpolation when the data points are exact, or a regression function when there are measurement errors. Using this formulation, the gradient information on each datapoint can be used to significantly reduce the interpolation error. The approximation converges exponentially on smooth functions for a variety of grids, including randomly scattered nodes. The output of the approximation scheme includes a prediction interval, which estimates the error of the approximation.

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Contributed talks: Session B

Meshfree methods

Organizers: A. Iske and J. Levesley

On spherical averages of radial basis functions

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A radial basis function (RBF) has the general form

$$s(x) = \sum_{k=1}^n a_k \phi(x - b_k), \quad x \in \mathbf{R}^d,$$

where the coefficients a_1, \dots, a_n are real numbers, the points, or centres, b_1, \dots, b_n lie in \mathbf{R}^d , and $\phi : \mathbf{R}^d \rightarrow \mathbf{R}$ is a radially symmetric function. Such approximants are highly useful and enjoy rich theoretical properties. The important special case of *polyharmonic splines* results when ϕ is the fundamental solution of the iterated Laplacian operator, and this class includes the Euclidean norm $\phi(x) = \|x\|$ when d is an odd positive integer, the thin plate spline $\phi(x) = \|x\|^2 \log \|x\|$ when d is an even positive integer, and univariate splines. Now B-splines generate a compactly supported basis for univariate spline spaces, but an analyticity argument implies that a nontrivial polyharmonic spline generated by cannot be compactly supported when $d > 1$. However, a pioneering paper of Jackson [1] established that the *spherical average* of a radial basis function generated by the Euclidean norm can be compactly supported when the centres and coefficients satisfy certain moment conditions; Jackson then used this compactly supported spherical average to construct approximate identities, with which he was then able to derive some of the earliest uniform convergence results for a class of radial basis functions. Our work extends this earlier analysis, but our technique is entirely novel, and applies to all polyharmonic splines. Furthermore, we observe that the technique provides yet another way to generate compactly supported, radially symmetric, positive definite functions. Specifically, we find that the spherical averaging operator commutes with the Fourier transform operator, and we are then able to identify Fourier transforms of compactly supported functions using the Paley–Wiener theorem. We are then able to generalize the technique to constructing rotation-invariant Borel measures whose convolution with polyharmonic radial basis functions is compactly supported.

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The two stage method with an adaptive spline second stage

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This talk considers a variation on the two stage method in which the first stage is made up from overlapping local radial basis function fits and the second by an adaptive spline fit. For the second stage we consider adaptive fits where there is some constraint on the geometry of local refinement. For example in 2D using binary triangle trees. This limiting of the geometry avoids long thin triangles and corresponding tetrahedra, and opens up the possibility of gaining speed via the use of some precomputation and table lookup. Algorithmic aspects and computational experience with the method will be discussed.

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An RBF Method for the Numerical Solution of Delayed Differential Equations

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A general and easy-to-code numerical method based on the collocation of radial basis functions (RBFs) is proposed for the solution of delay- and neutral differential equations (DDEs and NDEs respectively). It relies on the well-known interpolation properties of infinitely smooth RBFs, which allow for a large accuracy over a scattered and relatively small discretization support. In order to further the advantages of RBF collocation, reported heuristic observations concerning the tunable shape parameter [1] have been exploited and combined with the Residual Subsampling Algorithm of Driscoll and Heryudono [2] for support adaptivity. This allows for an accuracy close to machine precision while keeping a relatively short computer code. The implemented RBF has been Hardy's multiquadric and the nonlinear collocation systems that arise have been addressed with Powell's method, without resorting to analytical Jacobians. Therefore, the proposed RBF numerical scheme is very general. The performance of this code is demonstrated against a cross-section of benchmark DDEs which include topical features such as stiffness, vanishing-, and state delay, and which are known to pose numerical difficulties [3]. As shown by comparison with existing general-purpose (MATLAB's dde23 and ddesd), as well as more specialized numerical schemes for DDEs, the proposed RBF method is competitive -as long as the solution of the DDE is smooth, or can be partitioned in advance into smooth segments.

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Scattered data approximation on manifolds*

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Let $\{\phi_j\}$ be an orthonormal system on a quasi-metric measure space \mathbb{X} , $\{\ell_j\}$ be a non-decreasing sequence of numbers with $\lim_{j \rightarrow \infty} \ell_j = \infty$. A diffusion polynomial of degree L is an element of the span of $\{\phi_k : \ell_k \leq L\}$. We study approximation processes of the form

$$\sigma_L f(x) = \sum_{j=0}^{\infty} H\left(\frac{\ell_j}{L}\right) \langle f, \phi_j \rangle \phi_j(x),$$

where H is a suitable function. First, we address the problem under which conditions we can expect convergence, i.e. $\|\sigma_L f - f\|_p \rightarrow 0$, $L \rightarrow \infty$. Secondly, we will consider the relation between the localization of the kernel

$$\Phi_L(x, y) = \sum_{j=0}^{\infty} H\left(\frac{\ell_j}{L}\right) \phi_j(x) \phi_j(y),$$

the wave kernel $W(t, f_1, f_2) = \sum_{j=0}^{\infty} \cos(\ell_j t) \langle f_1, \phi_j \rangle \langle f_2, \phi_j \rangle$, and a generalized heat kernel $K_t(x, y) = \sum_{j=0}^{\infty} \exp(-\ell_j^2 t) \phi_j(x) \phi_j(y)$ respectively.

Finally we will address the problem of deriving Marcinkiewicz-Zygmund type inequalities for scattered data on a Riemannian manifold.

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On the suboptimality of the p -version interior penalty discontinuous Galerkin method

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We address the question of the rates of convergence of the p -version interior penalty discontinuous Galerkin method (p -IPDG) for second order elliptic problems with non-homogeneous Dirichlet boundary conditions. It is known that the p -IPDG method admits slightly sub-optimal a-priori bounds with respect to the polynomial degree (in the Hilbertian Sobolev space setting). An example for which the suboptimal rate of convergence with respect to the polynomial degree is both proven theoretically and validated in practice through numerical experiments is presented. Moreover, the performance of p -IPDG on the related problem of p -approximation of corner singularities is assessed both theoretically and numerically, witnessing an almost doubling of the convergence rate of the p -IPDG method. The results presented are based on joint work with Edward Hall (University of Nottingham, UK), Jens Markus Melenk (TU Vienna, Austria) and Endre Süli (University of Oxford, UK).

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Analysis of High-Dimensional Signal Data by Manifold Learning and Convolution Transforms*

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Recent advances in nonlinear dimensionality reduction and manifold learning have provided novel methods in the analysis of high-dimensional signals. In this problem, a very large data set $U \subset \mathbf{R}^n$ of scattered points is given, where the data points are assumed to lie on a compact submanifold \mathcal{M} of \mathbf{R}^n , i.e. $U \subset \mathcal{M} \subset \mathbf{R}^n$. Moreover, the dimension $k = \dim(\mathcal{M})$ of \mathcal{M} is assumed to be much smaller than the dimension of the ambient space \mathbf{R}^n , $k \ll n$. Now, the primary goal in the data analysis through dimensionality reduction is to construct a low-dimensional representation of U . To this end, we combine suitable techniques from manifold learning with signal transformations to construct a projection map $P : \mathbf{R}^n \rightarrow \mathbf{R}^k$ which then outputs the desired low-dimensional representation $P(U) \subset \mathbf{R}^k$. But the projection map P is required to preserve intrinsic geometrical and topological properties of the manifold in order to obtain a sufficiently accurate (low-dimensional) approximation to U . Therefore, the construction of P needs particular care. In our construction of P , customized convolution filters and suitable wavelet transformations are utilized to analyze the geometric distortion of the manifold. The good performance of the resulting nonlinear dimensionality reduction method is illustrated by numerical examples concerning low-dimensional parameterizations of scale modulated signals and solutions to the wave equation at varying initial conditions.

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Stability of Boundary-Free Surface Spline Interpolation*

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New results about the stability of boundary-free surface spline interpolation are presented. Specifically, we treat functions defined in a bounded region, Ω , that vanish in a neighborhood of the boundary. After a careful analysis of the decay of Lagrange functions, boundary-free interpolation can be shown (under mild assumptions on the centers, $\Xi \subset \Omega$) to have a bounded Lebesgue constant:

$$\sup_{x \in \Omega} |I_{\Xi} f(x)| \leq C \sup_{x \in \Omega} |f(x)|$$

However, a much stronger result holds. When the spacing of the centers is permitted to vary spatially, interpolation is stable in a more refined sense, one that takes into account a local measure of the density. Namely,

$$\sup_{x \in \Omega} |w(x) I_{\Xi} f(x)| \leq C \sup_{x \in \Omega} |w(x) f(x)|$$

holds for certain weight functions $w : \Omega \rightarrow (0, \infty)$ that reflect the local density of centers. Thus, the local Lebesgue constant C is bounded, independent of Ξ and f . This leads to a Lebesgue lemma showing that the (theoretical) approximation results obtained recently by Devore and Ron [1] hold for interpolation as well.

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Smoothing Approximation on the Sphere from Noisy Scattered Data*

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In geophysical applications, measured scattered data usually contains noise, and any approximation method should take this into account. In this talk we discuss the properties of a ‘smoothing approximation’ of noisy scattered data on the sphere by a ‘hybrid approximant’ that is the sum of a low to medium degree polynomial and a radial basis function approximant. This hybrid approximant is computed via solving a large linear system and has the property that it minimizes a certain functional which depends on a smoothing parameter $\lambda > 0$ that balances between fitting the data and getting a smooth solution. For $\lambda \rightarrow 0$, we obtain the interpolation scenario. A crucial question is how this smoothing parameter λ should be chosen depending on the level of the noise, and in this talk we discuss one a-posteriori strategy for choosing λ , namely Morozov’s discrepancy principle. For λ chosen with Morozov’s discrepancy principle, we give error estimates for the uniform error in terms of powers of the mesh norm and in terms of the noise level. We show that Morozov’s discrepancy principle is a valid parameter choice strategy, in that the uniform error tends to zero if the noise level and the mesh norm tend to zero. Numerical tests are presented that illustrate the theoretical work. – This talk is about joint work in progress with Ian Sloan and Rob Womersley.

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A Simple Fast Evaluation Method for Thin-Plate Splines

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Radial basis function interpolation is a paradigm for numerically solving many problems ranging from scattered data interpolation to partial differential equations. The theoretical foundations are remarkable in their simplicity and applicability, and although this theoretical simplicity does carry over to simple algorithms (e.g. interpolation is a few simple lines of Matlab code), the execution time grows rather dramatically with the size of the problem. A major culprit in this is the high cost of evaluating a radial basis function. For example, each evaluation of the rbf $s(x) = \sum_{j=1}^n \lambda_j \phi(x - \xi_j)$ requires n evaluations of the radial function ϕ . Beatson and his collaborators have been pioneers in developing fast evaluation methods for radial basis functions, however the methods developed so far are so complicated that one quickly loses sight of the initial “simplicity and elegance” of rbf interpolation.

In this talk, I’ll present a fast evaluation method for thin-plate splines which aims at being simple, conceptually, and practical to implement. It is also *safe* in the sense that there is a user defined parameter ε , which bounds the evaluation error, and this allows the user to control the trade-off between accuracy and speed with the assurance that the evaluation error does not exceed ε . The preprocessing phase is similar to that employed by Beatson et al in that the centers $\Xi = \{\xi_j\}$ are decomposed into a multilevel tree of subsets $\{\Xi_k\}$ (called panels) and for each panel Ξ_k a *summary* of $s_k(x) = \sum_{\xi \in \Xi_k} \lambda_\xi \phi(x - \xi)$ is constructed. It differs, however, in the actual nature of the summaries and the determination, for each summary, of its usable region. Evaluations commence immediately after the summaries have been constructed; there is no need to combine the summaries into near and far-field expansions for each panel. After describing the method, I’ll present some results concerning the asymptotic cost estimates as ε tends to 0, along with some experimental runs which shed further light on the actual performance of the method.

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Meshless simulations of two-dimensional flows with discontinuous fronts

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Two-dimensional hyperbolic partial differential equations (PDEs) are simulated with meshless multiquadric radial basis functions (MQ-RBFs); the solution space is enriched to contain both continuous and discontinuous solutions appropriate for nonlinear hyperbolic PDEs. It is hypothesized that local rotational and translational transformations can be applied to each discretized data center in the subdomain, and the discontinuous fronts, that transform nonlinear PDEs into linearized PDEs moving at an appropriate “characteristic” velocity, see [1]. Then the method of lines solution for the time dependent solutions is exactly integrated. MQ-RBFs are prewavelets that are translationally, dilationally, and rotationally invariant, see [2].

A new Greedy Algorithm developed in [3] permits the wavelet selection of shape parameters, $[c_j^2]$, that resolves features at their proper length scales. The formation of new discontinuities is readily tracked by using the set of “characteristic” velocities, and projecting forward in time the location of new discontinuities. Front tracking is computationally more efficient than the standard front tracking if a meshless approach is used; no unphysical artificial viscosity is required. A front is a piecewise continuous curve that separates two continuous subdomains. It is straightforward to construct unit normal and tangential vectors at the front, and use the Rankine-Hugoniot jump conditions to propagate the front forward in time. To allow the insertion or deletion of discretization points, any redistribution of points must preserve the inherent physics of the PDEs. The strict conservation is enforced by integrating the MQ-RBF basis functions over each subdomain physical and frontal boundaries. The integrated basis functions form a constraint condition that guarantees strict conservation.

Numerical results will be presented for the numerical simulation of turbulent combustion under vortical flow, and wave breaking.

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A scaling perspective on accuracy and convergence in RBF approximations*

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Radial basis function (RBF) approximation is a meshless technique using scattered node points and scattered data. Infinitely smooth RBFs can be highly effective for approximating or interpolating smooth functions. However, the shape parameter ε associated with the RBFs must be chosen with some care in order to obtain accurate results. In particular, convergence as the separation distance $h \rightarrow 0$ can typically only be achieved if the product εh does not increase.

For smooth functions, the most accurate result for a given set of node points is typically achieved for small values of ε corresponding to nearly flat RBFs. A standard collocation approach (RBF-Direct) in these cases results in badly conditioned interpolation matrices, and numerical instability. However, with the recently developed RBF-QR method, stable computations for any small value of ε can be performed for problem sizes up to several thousands of node points in two dimensions. The RBF-QR method is applied here for numerical exploration of convergence and accuracy in regions that were previously unreachable by direct computation. The implications for how to choose the shape parameter appropriately are discussed.

RBFs can also be used for generating scattered node finite difference stencils. Also in this case, scaling has important effects on accuracy, convergence, and stability. Individual stencils should be scaled according to the local node density, but as nodes are refined globally, the same rules as for the global interpolation problem apply.

The flat RBF limit ($\varepsilon = 0$) is a special case, closely related to polynomial approximation. A discussion of the limit effects will be interwoven with the other topics of the talk.

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Tensor Product Based Meshless Solution of Potential Problems*

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Meshless methods have been explored in a huge number of problems, and they have been shown to be as accurate as Finite Element Methods (FEM) [1, 2, 3, 4].

Usually, radial functions are exploited for providing basis and test functions. Integration domains in the ensuing variational equations are complex intersections of circles, hence providing accurate evaluations of integrals is challenging. We show that accurate meshless solutions of potential problems can be attained, by appropriately operating on tensor product functions in order to identify appropriate shape and test functions. Using tensor product functions, rectangular integration domains are involved, hence optimal quadrature rules can be enrolled, which improves accuracy and performance of our meshless method.

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Composed Segmentation and Reconstruction by an Anisotropic PDE Model*

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We consider the problem of shape reconstruction from organized and unorganized data set which has many important applications in medical imaging, scientific computing, reverse engineering and geometric modelling. Most of these applications rely on the efficient computation of partial differential equations (PDE) on curves or surfaces implicitly represented by a level set function. The advantages of these techniques are the topological flexibility, the possibility to easily capture geometry property of the surface, and to realize in a simple way the classical Boolean operations, while it is a challenge to deal with open or incomplete surfaces. In this work the reconstruction is obtained by continuously deforming an initial distance function following the PDE-based diffusion model derived from a minimal volume-like variational formulation. The gradient flow for this functional leads to a nonlinear curvature motion model. An anisotropic variant is provided which includes a diffusion tensor aimed to follow the object geometry and to deal with non-uniform and incomplete data set in order to construct an arbitrary topology surface with a controlled hole filling strategy.

Moreover, we introduce the concept of composed segmentation (C-segmentation), that is a priori composition of sources to obtain a single one segmentation result according to specific logic combinations. Numerical examples demonstrate the ability of the proposed method to produce high quality 2D/3D reconstructions of complex and eventually incomplete synthetic and real data.

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Discontinuous Weighted Least-Squares Approximation with Anisotropic Support

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Discontinuous weighted least-squares (DWLS) approximation is a modification of a weighted least-squares (LS) method that computes coefficients of local weighted LS approximation at each point belonging to a set of points selected over a computational grid. A DWLS reconstruction is very similar to a moving least-squares (MLS) method, as the coefficients of the approximation depend on the location of a point where the reconstruction is made. However, the difference between a DWLS and an MLS procedure is that in the latter case a local support for the approximation is prescribed by the definition of a weight function in the problem, while for the DWLS approximation a local support (also called a reconstruction stencil) is entirely determined by the edge data structure on a computational grid. The weight function in a DWLS method is only used to improve the accuracy of the approximation on a given stencil by reducing the uncertainty of the measurement. Weight coefficients are allocated to DWLS data in order to suppress data points where the observation error can be large. In many applications the uncertainty of the measurement is associated with geometrically distant points in the data set, so that the weight of each observation is often chosen to be a function of the inverse distance between two given points.

In our talk we discuss the issues related to DWLS approximation on an anisotropic stencil, as it recently turned out that weighting of stencil points is not efficient on anisotropic meshes used in practical computations [1]. It will be demonstrated in the talk that a decision about elimination of distant points from a reconstruction stencil should not rely upon geometric shape of the stencil only, as the definition of such points should also be based on data used for the approximation (numerically distant points). We explain the difference between geometrically distant points and numerically distant points and conclude that the implementation of a weight function in DWLS approximation requires recognition of numerically distant points in a reconstruction stencil. These results are also important for MSL approximation as they allow one to better understand which support set would be optimal for a given function.

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Kernel based methods for vector data with correlated components

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This talk discusses kernel based methods for vector data with correlated components. We give conditions for a matrix kernel to be conditionally positive definite in an appropriate sense. The conditions allow construction of matrix kernels from non-symmetric mixtures and scalings of scalar kernels. In particular the kernel used to model the influence of component i on component j can be different from that used to model the influence of component j on component i . Such non-symmetric models are appropriate in many circumstance, think for example of the influence of acid rain on a forest and vice versa. The technique is particularly appropriate when there are relatively few measurements of one quantity and relatively many of another “correlated” quantity. The talk concludes with some numerical tests on model problems.

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Contributed talks: Session C
Quadrature and cubature
formulas, numerical methods for
integral equations

Organizer: E. Venturino

Efficient numerical integration schemes for the discretization of hypersingular BIEs related to wave propagation problems

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Time-dependent problems that are frequently modelled by hyperbolic partial differential equations can be dealt with the boundary integral equations (BIEs) method. Boundary element methods (BEMs) have been successfully applied in the discretization phase. The consideration of the time-domain (transient) problem yields directly the unknown time-dependent quantities. In this case, the representation formula in terms of single layer and double layer potentials uses the fundamental solution of the hyperbolic partial differential equation and jump relations, giving rise to retarded boundary integral equations. Recently in [2], we have considered 2D Dirichlet or Neumann problems for a temporally homogeneous (normalized) scalar wave equation outside an obstacle, reformulated as a BIE with retarded potential. Special attention has been devoted to a natural energy identity related to the differential problem, that leads to a space-time weak formulation for the BIE, having, under suitable constraints, precise continuity and coerciveness properties. Galerkin BEM coming from energetic weak formulation produces a linear system that has a block lower triangular Toeplitz structure. Matrix elements, after a double analytic integration with piecewise constant or linear test and shape functions in the time variables, contain double integrals in space variables having both weakly singular and hypersingular kernels. These double integrals have been efficiently calculated modifying numerical quadrature schemes widely used for BIEs related to elliptic problems [1]: in fact, Gauss-Legendre formula, product quadrature rules and Hadamard Finite Part quadrature formula have been coupled with a suitable regularization technique [3], after a careful subdivision of the integration domain due to the presence of the Heaviside function in the integral kernels. Several numerical results for 2D wave propagation problems will be presented.

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Construction of lattice rules: from K -optimal lattices to extremal lattices

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The concept of K -optimal lattices was introduced by Cools and Lyness [2] to set up searches for lattice rules for multivariate integration with a low number of points for a given trigonometric degree of precision. The original searches were restricted to 3 and 4 dimensions. Refinements were later made to make the searches possible in 5 and 6 dimensions [1, 3].

The idea behind K -optimal lattices was extended in Russia using extremal lattices, see, e.g., [4, 5].

In this talk these approaches to lattice rule construction will be described and a survey of the obtained results will be presented.

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Quadrature with respect to balanced measures

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In this talk we will consider the calculation of $\int_K f(x) d\mu$ where μ is a balanced measure, i.d. the unique measure such that:

$$\int_K f d\mu = \sum_{i=0}^M p_i \int_K f(\phi_i(x)) d\mu \quad (1)$$

with $\sum p_i = 1$, $\phi_i(x) = \delta_i x + \beta_i$; $0 < \delta_i < 1$, $\beta_i \in K$.

The automatic calculation of the n -points Gauss rules (already available in [5]) and a proper extension of Clenshaw Curtis formulae is presented together with the convergence behaviour with respect to the number of quadrature points. Moreover the case of composite rules is considered, as it naturally comes from the balancing equation (1).

Then local error estimates are proven and the computation of the same is made in the framework of null rules [1, 4]. With this ingredients, a reliable and efficient globally adaptive quadrature algorithm is presented for quadrature with respect to a general balanced measure. All calculations are made in a stable manner with an overall cost of $o(n^2)$ operations.

Part of the results are included in the works [2, 3].

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Error Bounds and Discretization Grids in the Solution of Weakly Singular Integral Equations*

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In the solution of weakly singular second kind Fredholm integral equations defined on the space of Lebesgue integrable complex valued functions by projection-type methods such as Petrov-Galerkin or Kantorovitch methods [1], the choice of the discretization grids is crucial. We will present the proof of an error bound in terms of the mesh size of the underlying discretization grid on which no regularity assumptions are made and compare it with other recently proposed error bounds [2]. This allows us to use non regular grids which is convenient when there are boundary layers or discontinuities in the right hand side function of the equation. We present some results using a simplified model of the radiative transfer in stellar atmospheres which illustrates the actual behaviour of the error in terms of the distribution of the points in the grid.

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Gauss quadratures for two classes of logarithmic weight functions

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By means of the Matlab symbolic/variable-precision facilities, routines are developed that generate an arbitrary number of recurrence coefficients to any given precision for polynomials orthogonal with respect to weight functions of Laguerre and Jacobi type containing logarithmic terms. The vehicle used is a symbolic modified Chebyshev algorithm based on ordinary as well as modified moments, executed with sufficiently high precision. The results are applied to Gaussian quadrature of integrals involving weight functions of the type mentioned.

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Numerical Solution of Backward-Forward Equations: Comparative analysis of methods*

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In the present work we are concerned with the approximate solution of functional differential equations with both delayed and advanced arguments. Such equations are often referred to in the literature as mixed type functional differential or forward-backward equations. The analysis of this type of equation is recent and some important questions remain open.

Consider the equation

$$x'(t) = \alpha(t)x(t) + \beta(t)x(t-1) + \gamma(t)x(t+1). \quad (1)$$

One looks for a solution x , defined for $t \in [-1, k], (k \in \mathbb{N})$, which takes given values on the intervals $[-1, 0]$ and $(k-1, k], (k \in \mathbb{N})$.

In [1], the autonomous case of equation (1) was studied, when α, β, γ are constants and $\alpha = 0$. As remarked in that paper, one of the main difficulties with the numerical approximation of this problem results from its ill-conditioning. Even for not very high values of k , numerical methods become unstable. The same problem has been reported in [2] and [3], where the collocation method was applied to equation (1). Here we give an overview of the computational approaches developed so far and discuss the advantages and drawbacks of each method.

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Dynamical Systems and Numerical Analysis: Measures and Jacobi Matrices generated by uncountable I.F.S.

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Theorems linking the asymptotic behavior (convergence, periodicity) of the components of the Jacobi matrix [1] to the continuity properties of the associated measure and to its Fourier transform are classical. When the measure is singular continuous, these relations become more complex and intriguing [2, 3].

In this communication I want to describe an example of the fruitful interplay between the theory of dynamical systems and numerical analysis, by showing how singularity (or continuity) with respect to Lebesgue of the orthogonality measure of a Jacobi matrix associated with I.F.S. (see below) can be determined numerically, and viceversa how the dynamics affects numerical analysis.

I shall study the iteration of maps $\phi_\lambda : X \rightarrow X$ from a compact metric space X to itself, labelled by the variable λ which belongs to a measure space Λ , on which the probability measure σ is given [4]. This process gives rise to what is called an Iterated Function System, or I.F.S., with invariant measure μ , whose existence and uniqueness is ensured by suitable hypotheses.

I will describe two techniques to study such an invariant measure. On the one hand, I will define the transfer operator T on the space $C(X)$ of continuous functions on X , via $(Th)(s) = \int d\sigma(\lambda)(h \circ \phi_\lambda)(s)$. Then, if we let T^* be the adjoint operator in the space of regular Borel measures on X , the invariant measure of the I.F.S. is the fixed point of T^* , $T^*(\mu) = \mu$. I will show how to treat numerically this equation in well specified circumstances.

In a second approach, I will describe the Fourier transform of the measure μ , its Mellin transform and its Sobolev dimension, a set of quantities that can be used to discuss the continuity of μ . I will then put in relation these results with the asymptotic behavior of the Jacobi matrix associated to μ , for which I will present a stable numerical algorithm.

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Numerical solution of a singular boundary value problem for the p -Laplace operator using a finite difference method

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Let us consider the following nonlinear ordinary differential equation:

$$r^{1-N}(r^{N-1}|g'(r)|^{p-2}g'(r))' = ar^\sigma g^n(r), \quad 0 < r < r_0, \quad (1)$$

where $n < 0$, $N \geq 1$, $a < 0$, $p > 1$, $r_0 > 0$ and $\sigma \in \mathbb{R}$. We search for a positive solution of (1) that satisfies the boundary conditions

$$\begin{aligned} \lim_{r \rightarrow 0^+} g'(r) &= 0 \\ g(r_0) &= \lim_{r \rightarrow r_0^-} [(r_0 - r)g'(r)] = 0. \end{aligned} \quad (2)$$

The differential operator on the left-hand side of (1) is the radial part of the N -dimensional p -laplacian, $\Delta_p g$, which reduces to the classical laplacian when $p = 2$ and, for $p \neq 2$, is used in nonlinear models of physical phenomena, as for example, the deformation of a nonlinear elastic membrane and problems arising in non-newtonian fluid mechanics. In [1] and we have studied the behavior of solutions of (1) in the neighborhood of the singularities and obtained asymptotic expansions of the families of solutions that satisfy each of the boundary conditions. These expansions were then used to construct stable shooting algorithms for the solution of the considered problems. This approach has enabled us to obtain accurate solutions for a number of different cases, in spite of the singularities. Here, we improve further the accuracy of the discretization method by using a variable substitution that takes into account the behavior of the solution in the neighborhood of the singular points. After introducing the variable substitution, the boundary value problem is discretized by a finite-difference scheme and reduced to a system of nonlinear equations. This system is then solved by the Newton method, using upper and lower solutions (studied in [1]) as initial approximations.

The accuracy of the method is tested in numerical examples and the results are compared with the ones obtained in previous works.

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Adapting the Peano kernel theory to estimate the error of product cubature formulae^{*}

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In the univariate case, there is a well-developed theory of the error estimation of the quadrature formulae for integrands from the Sobolev classes of functions. It is based on the Peano kernel representation of linear functionals, which yields sharp error bounds for the quadrature remainder. The product cubature formulae are the usual tool for the approximation of double integrals over a rectangular domain. We suggest a modification of the product cubature formulae, based on blending interpolation of bivariate functions. Besides the usual point evaluations, the modified cubature formulae involve few line integrals. Our approach allows application of the Peano kernel theory for derivation of error bounds for both standard cubature formulae and their modifications. We also provide some sufficient conditions for the definiteness of the modified product cubature formulae, and describe classes of integrands for which a product cubature formula is inferior to its modified version.

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Improvement of a Sinc-collocation method for Fredholm integral equations of the second kind*

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We consider Fredholm integral equations of the second kind of the form:

$$\lambda u(t) - \int_a^b k(t, s)u(s) ds = g(t), \quad a \leq t \leq b, \quad (1)$$

where λ is a known non-zero constant, $k(t, s)$ and $g(t)$ are known functions, and $u(t)$ is the solution to be determined. A collocation method by means of the Sinc approximation [5] for (1) has been developed by Rashidinia–Zarebnia [4]. Their method enjoys *exponential* convergence, $O(\exp(-c\sqrt{N}))$, not only in the case where known functions $k(t, s)$ and $g(t)$ are analytic, but also even in the case where the functions have endpoint singularities (see, for example, [1, Example 4.2.5b]).

In this talk we refine their method in the following two points. First we modify the basis functions. Rashidinia–Zarebnia have proposed to switch the basis functions depending on whether the solution $u(t)$ is zero or non-zero at the endpoints. From a practical point of view, however, the solution $u(t)$ is *unknown*, and it is not straightforward to implement their method. Therefore we change the basis functions with the ones which have been utilized in the present authors [3], so that they do not depend on $u(t)$. The second point is modification of the variable transformation. In the Rashidinia–Zarebnia method, the standard “tanh transformation” is employed, but we replace it with a stronger transformation, the so-called “double exponential transformation.” This is motivated by the fact that its replacement improves the convergence rate from $O(\exp(-c\sqrt{N}))$ to $O(\exp(-cN/\log N))$ in many cases [2]. Numerical examples that confirm the efficiency will be presented as well in the talk.

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Central part interpolation and product integration method for weakly singular Fredholm integral equations

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In the first part of the talk the regularity of a solution to a linear Fredholm integral equation of the second kind is discussed. The kernel of the integral equation may have weak diagonal singularities. Assuming certain differentiability properties of the kernel and free term, we discuss the growth rates of the derivatives of the exact solution near the boundary of the domain of integration. In the second part we perform a change of variables which improves the boundary behaviour of the exact solution. After that, using a central part interpolation by polynomials on the uniform grid, we solve the transformed equation by the product integration method. Global convergence estimates are presented and a collection of numerical results is given.

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Error Estimation for Approximate Solution of the Volterra First Kind Integral Equation

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In [2] the author uses for solving the first kind Volterra integral equation whose kernel has a weak singularity, a method based on the articles [4] and [1]. This study presents the error estimate of the approximate solution for such type of equation when noise in the right hand side for the discrete problem, is present. The ill-posedness of the continuous problem is transferred to the discrete one, so we use a Tikhonov regularization, like the one proposed in [3].

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Assessment of Coronary Haemodynamics: Treatment of Tracer Kinetic Problems by Solving Integral Equations

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We will give an overview of our patient-specific simulation studies of the flow of blood in the entire coronary circulation. In the epicardial arteries, especially around sections with a stenosis (narrowing), the blood flow is simulated three-dimensionally. In the other parts of the coronary circulation, however, we preferably use lumped parameter simulation models. The patient-specific simulation studies are based on information contained in medical images (angiograms and perfusion images). The geometry of the epicardial arteries is derived from biplane angiograms. Based on the knowledge of this geometry, this part of the coronary circulation can be simulated in a straightforward manner.

However, it is more difficult to build a computational flow model of the intramyocardial coronary circulation. The knowledge of the intramyocardial flow must be deduced from perfusion images, preferably PET perfusion images [1]. The measurement of the regional intramyocardial blood flow requires a thorough analysis of these images and the solution of a difficult tracer kinetic problem. This tracer-kinetic problem is an inverse problem by its nature which can be formulated as a non-linear integral equation [2][3]. We will present a pragmatic approach for its solution [4].

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Computational methods for a weakly singular integral equation based on extrapolation procedures*

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This talk is concerned with the numerical analysis of a weakly singular integral equation [1], which arises in connection with some problems of heat-conduction, boundary-layer heat transfer, chemical kinetics and superfluidity. The singular behaviour of the solution causes the loss of the optimal (global) convergence orders of product integration and piecewise polynomial collocation methods. The optimal orders can be recovered if we use collocation methods based on graded meshes. However, one drawback of these methods is that intervals of small length will be used near the singular point thus leading to possible round-off error problems when using high order polynomials. In [3] a hybrid collocation method was introduced in order to overcome these difficulties. In the present work we shall be concerned with the use of low order product integration methods in conjunction with extrapolation procedures.

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Boundary element methods for time-fractional diffusion equations

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In this paper we discuss the numerical solution of the space-time boundary integral equation

$$S_\Gamma u_\Gamma(x, t) = \int_0^t \int_\Gamma u_\Gamma(y, \tau) E(x - y, t - \tau) ds_y d\tau = f(x, t), \quad x \in \Gamma, 0 < t < T,$$

where Γ is a smooth plane curve. The kernel of the integral operator,

$$E(x, t) = \frac{1}{\pi} t^{\alpha-1} |x|^{-2} H_{12}^{20} \left(\frac{1}{4} |x|^2 t^{-\alpha} \right)_{(1,1),(1,1)}^{(\alpha,\alpha)}, \quad 0 < \alpha \leq 1,$$

is the fundamental solution of the time-fractional diffusion equation

$$\begin{aligned} \partial_t^\alpha \Phi - \Delta \Phi &= 0, \text{ in } Q_T = \Omega \times (0, T), \\ B(\Phi) &= g, \text{ on } \Sigma_T = \Gamma \times (0, T) \\ \Phi(x, 0) &= 0, \quad x \in \Omega, \end{aligned} \tag{1}$$

where the boundary operator $B(\Phi) = \Phi|_{\Sigma_T}$, and ∂_t^α is the Caputo time derivative of the fractional order $0 < \alpha \leq 1$.

We shall consider the spline collocation method for the numerical approximation of the solution on quasi-uniform meshes with tensor product splines as the approximation space. We will show that the spline collocation method is stable in a suitable anisotropic Sobolev space, and it furnishes quasi-optimal error estimates.

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Recent developments on the 3D BEM approximation of hyperbolic problems

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The present note deals with 3D hyperbolic problems and their numerical approximation via Boundary Element Methods [1]. Space-time collocation schemes as well as variational approaches are pursued. Space discretization is made of (flat) triangulation of the boundary, adopting polynomial test and shape functions of arbitrary degree. Time marching schemes are allowed to make use of polynomial test and shape functions of arbitrary degree in time.

Analytical integrations are performed for both the singular and the regular part. The final closed form shows recurrence outcomes that link the hyperbolic to the quasi-static integration schemes and seem to be promising towards the formulation of fast integral operators in hyperbolic problems.

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Quaternionic Analysis applied to Initial-Boundary-Value

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The talk is devoted to applications of quaternionic analysis to problems in initial boundary value problems. This is done with examples from fluid dynamics. We introduce a necessary quaternionic equipment and formulate important fluid flow equations in quaternionic form. After modification of a quaternionic operator calculus we turn to a time-discretisation method. On the basis of new splitting iteration methods corresponding initial-boundary value problems will be discussed. We also intent to give representations of using the so called harmonic continuation.

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A class of Volterra equations with noncompact weakly singular integral operators*

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Consider the class of Volterra operators $(Vu)(t) = \int_0^t K(t, s)u(s)ds$, $0 \leq t \leq T$, having the following property (A): V is a bounded operator in $C[0, T]$ and $u_r(t) = t^r$, $0 \leq r < \infty$, are eigenfunctions of V . Such an operator is noncompact in $C[0, T]$. It occurs that V has property (A) if and only if its kernel has the form $K(t, s) = t^{-1}\varphi(s/t)$, $0 \leq s \leq t \leq T$, where $\varphi \in L^1(0, 1)$. Thus we actually study the class of operators

$$(V_\varphi u)(t) = \int_0^t t^{-1}\varphi(s/t)u(s)ds = \int_0^1 \varphi(x)u(tx)dx, \quad 0 \leq t \leq T.$$

We call $\varphi \in L^1(0, 1)$ the *core* of V_φ and V_φ itself an operator with a core, or simply *cordial operator*. We introduce in $L^1(0, 1)$ a multiplication operation $\varphi \star \psi$ so that $V_\varphi V_\psi = V_{\varphi \star \psi}$. So $L^1(0, 1)$ and the class of cordial operators become commutative Banach algebras which are isometrically isomorphic. In this way we establish formulae for the spectrum $\sigma_m(V_\varphi)$ of V_φ as an operator in $C^m[0, T]$:

$$\sigma_0(V_\varphi) = \{0\} \cup \{\hat{\varphi}(\lambda) : \lambda \in \mathbf{C}, \operatorname{Re} \lambda \geq 0\}, \quad m = 0,$$

$$\sigma_m(V_\varphi) = \{0\} \cup \{\hat{\varphi}(k) : k = 0, \dots, m-1\} \cup \{\hat{\varphi}(\lambda) : \operatorname{Re} \lambda \geq m\}, \quad m = 1, 2, \dots,$$

where $\hat{\varphi}(\lambda) = \int_0^1 \varphi(s)s^\lambda ds$. Note that $\sup\{|\hat{\varphi}(\lambda)| : \operatorname{Re} \lambda \geq m\} \rightarrow 0$ as $m \rightarrow \infty$. In particular, we localise the spectra of Diogo's, Lighthill's and some other noncompact Volterra integral operators occurring in the practice.

We also treat the Volterra integral operators of a more general form

$$(V_{\varphi,a}u)(t) = \int_0^t t^{-1}\varphi(s/t)a(t, s)u(s)ds, \quad 0 \leq t \leq T,$$

where $\varphi \in L^1(0, 1)$, $a \in C^m(0 \leq s \leq t \leq T)$, $m \geq 0$. It occurs that $\sigma_m(V_{\varphi,a}) = a(0, 0)\sigma_m(V_\varphi)$. In particular, if $a(0, 0) = 0$ then $\sigma_m(V_{\varphi,a}) = \{0\}$ and $V_{\varphi,a}$ as an operator in $C^m[0, T]$ occurs to be compact.

We prove the convergence and establish error estimates of the polynomial collocation methods for the Volterra integral equation $\mu u = V_{\varphi,a}u + f$ assuming that $\mu \neq 0$, $\mu \neq a(0, 0)\hat{\varphi}(k)$, $k = 0, 1, \dots$, and that $f \in C^m[0, T]$ where m is sufficiently large so that $\mu \notin a(0, 0)\sigma_m(V_\varphi)$; in particular, $m = 0$ suits if $\mu \notin a(0, 0)\sigma_0(V_\varphi)$, i.e., $\mu \neq a(0, 0)\hat{\varphi}(\lambda)$ for $\operatorname{Re} \lambda \geq 0$.

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Contributed talks: Session D

Approximation methods in numerical linear algebra

Organizers: C. Brezinski and M. Eiermann

Error estimates for choosing the best parameter in Tikhonov regularization

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The optimal choice of the parameter in Tikhonov's regularization method for ill-conditioned systems of linear equations is an important problem.

In this talk, we present a new procedure for that purpose. It is based on estimations of the error of the solution of a system of linear equations.

Such estimates are obtained via an extrapolation method that will be described. They are based on the SVD of the matrix of the system, but do not require the knowledge of its singular values. They are valid for any method, direct or iterative.

Then, they will be adapted to the particular case of Tikhonov regularization.

Numerical experiments show the efficiency of the procedure.

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A numerical solution of the constrained weighted energy problem

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A numerical algorithm is proposed to solve the constrained weighted energy problem from potential theory. As one of possible applications of this algorithm we study the convergence properties of the rational Lanczos iteration method for the symmetric eigenvalue problem. The constrained weighted energy problem characterizes the region containing those eigenvalues that are well approximated by the Ritz values. The region depends on the distribution of the eigenvalues, on the distribution of the poles of rational functions and on the ratio between the size of the matrix and the number of iterations. Our algorithm gives the possibility to find the boundary of this region in an effective way.

We give numerical examples for different distributions of poles and eigenvalues and compare the results of our algorithm with the convergence behavior of the explicitly performed rational Lanczos algorithm.

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Spectral Methods for Parameterized Matrix Equations*

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We examine the problem of a parameterized linear system of equations

$$A(s)x(s) = b(s)$$

where the elements of the matrix $A(s)$ and the right hand side $b(s)$ depend on a set of parameters s . Such systems arise in control problems, electronic circuit design, image deblurring, and computational methods for PDEs with random input parameters.

We analyze both a spectral Galerkin method and a pseudospectral method based on a tensor-product of univariate orthogonal polynomials to approximate the components of the solution vector $x(s)$. We provide asymptotic error estimates for these methods that depend on a measure of singularity of the parameterized matrix, and we offer an in-depth discussion of the connections between the Galerkin and pseudospectral approaches. These connections lead to efficient computational strategies for large-scale problems with high dimensional parameter dependence.

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Shift optimization for solution of large scale evolutionary problems by means of Galerkin approach on rational Krylov subspaces

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We consider the computation of the parameterized vector

$$u(t) = \exp(-tA)\varphi, \quad t \geq 0,$$

where

$$u(t), \varphi \in \mathbb{R}^N \quad \text{and} \quad A = A^* \in \mathbb{R}^{N \times N}, \quad 0 < \lambda_{\min} I \leq A \leq \lambda_{\max} I.$$

To this end, we use Galerkin approximation on m -dimensional rational Krylov subspaces

$$\text{span}\{(A + s_1^{(m)}I)^{-1}\varphi, \dots, (A + s_m^{(m)}I)^{-1}\varphi\}, \quad m \geq 1, \quad s_j^{(m)} \in \mathbb{C}.$$

By means of Fourier transformation, the problem of optimization of shifts $s_j^{(m)}$ is reduced to a partial case of the third Zolotaryov problem in the extended complex plane. We show, however, that there exists a sequence of almost optimal m -tuples of *real positive* shifts (which is computationally advantageous). These shifts $s_j^{(m)} \in [\lambda_{\min}, \lambda_{\max}]$ and the corresponding convergence factor ρ are expressed in terms of the principal and complementary full elliptic integrals K and K' and the Jacobi elliptic function dn by the formulae

$$s_j^{(m)} = \lambda_{\max} \text{dn} \left(\frac{2(m-j)+1}{2m} K'(\delta), \sqrt{1-\delta^2} \right) \quad (1 \leq j \leq m), \quad \rho = \exp \left[-\frac{\pi}{4} \cdot \frac{K'(\mu)}{K(\mu)} \right],$$

where $\delta = \frac{\lambda_{\min}}{\lambda_{\max}}$ and $\mu = \left(\frac{1-\sqrt{\delta}}{1+\sqrt{\delta}} \right)^2$.

We also construct an infinite sequence of shifts $s_j \in [\lambda_{\min}, \lambda_{\max}]$, $j \in \mathbb{N}$, independent of m , yielding a *nested sequence* of rational Krylov subspaces with the same (best possible) Cauchy–Hadamard convergence factor ρ .

As illustration, we consider solving the 3D diffusion problem for Maxwell's system appearing in electromagnetic geophysical oil exploration.

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On convergence of Krylov subspace approximations of time-invariant Hermitian dynamical systems

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We extend a priori convergence estimates of Krylov (including rational Krylov) subspace approximations of the exponential of Hermitian matrices to the solution of stable dynamical systems

$$\sum_{i=0}^m A_i \left(\frac{d}{dt} + sI \right)^i u(t) = b(t), \quad u|_{t<0} = 0,$$

where $m \leq \infty$, $A_i = A_i^* \in \mathbf{R}^{N \times N}$, $s \leq 0$, and $u(t), b(t) \in \mathbf{R}^N$, $b|_{t<0} = 0$ (not assuming that evolution of $b(t)$ to be described by a low-dimensional subspace of \mathbf{R}^N). We show that the reduced equation is stable, estimate boundary of its spectral domain in the left complex half-plane and derive an a priori error bound via rational approximation of the exponential on that boundary. The obtained results applied to a fractional order problem arising in the solution of the dispersive Maxwell's system.

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Deflated Restarting of Krylov Subspace Approximations for Matrix Functions

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The approximation of $f(A)b$ for a square matrix A , a vector b and a function f arises in many computational tasks, most notably linear systems of equations and initial value problems. When A is non-Hermitian and large, Krylov subspace approximations lead to expensive long recurrences for generating a basis. Restarting the iteration periodically typically delays convergence, and acceleration techniques for compensating this delay have been developed for solving linear systems which make use of invariant subspaces extracted from the sequence of Krylov subspaces generated in the course of the iteration.

In this talk we describe how such deflated restarting techniques can be generalized to matrix functions. In this case the restarting process itself as well as the deflation technique are most easily understood based on an interpretation of the Krylov subspace approximation as polynomial interpolation at Ritz values, where the deflation method aims to force some interpolation nodes to eigenvalues of A close to singularities or maxima of the function f , thus deflating the approximation of the associated error components. Besides a derivation of the technique, we present a numerical implementation and several examples.

This is joint work with Michael Eiermann and Stefan Güttel.

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Verified Computation of Square Roots of a Matrix^{*}

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We present methods to compute verified square roots of a square matrix A . Given an approximation X to the square root, obtained by a classical floating point algorithm, we use interval arithmetic to find an interval matrix which is guaranteed to contain the error of X . Our approach is based on the Krawczyk method which we modify in two different ways in such a manner that the computational complexity for an $n \times n$ matrix is reduced from n^5 to n^3 . The methods are based on the spectral decomposition or, in the case that the eigenvector matrix is ill conditioned, on a similarity transformation to block diagonal form.

The methods work for large matrices, and their computational overhead is bounded by a factor of 3 to 7, independently of the dimension, compared to the time needed to get the approximation X via a standard algorithm like Higham's method based on the Schur decomposition, implemented in MATLAB as `rootm.m`.

Numerical experiments using INTLAB, a MATLAB toolbox providing an efficient realization of interval arithmetic, prove that our methods are computationally efficient and that they yield narrow enclosures provided X is a good approximation. This is particularly true for symmetric matrices, since their eigenvector matrix is perfectly conditioned.

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On rational Krylov approximations to matrix functions^{*}

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We consider the problem of extracting approximations to $f(A)\mathbf{b}$ from rational Krylov spaces $q_{m-1}(A)^{-1}\mathcal{K}_m(A, \mathbf{b})$ (the zeros of the polynomial q_{m-1} are referred to as the *poles* of the rational Krylov space). Two common methods, namely the Rayleigh-Ritz method and the shift-invert method, are characterized as rational interpolation procedures where the interpolation nodes are rational Ritz values or rank-one perturbations of them. A constrained extremal problem from logarithmic potential theory is used to describe the convergence of rational Ritz values to the eigenvalues of A (see [1]). Here an external field of a special form is required for taking into account the poles of the rational Krylov space.

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On solution of linear problems by the extrapolated Tikhonov method

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We consider extrapolation of regularization methods for solving linear problem $Au = f$, $f \in \mathcal{R}(A)$, where A is either a compact operator between infinite-dimensional Hilbert spaces H and F or $Au = f$ is a system of linear equations with ill-conditioned matrix A . For approximation of the solution u_* of this ill-posed problem we use in general case Tikhonov method $u_\alpha = (\alpha I + A^*A)^{-1}A^*f_\delta$ and in case $F = H$, $A = A^* \geq 0$ the Lavrentiev method $u_\alpha = (\alpha I + A)^{-1}f_\delta$. Here $\alpha > 0$ and f_δ is known approximation to exact data f . For a posteriori choice of the regularization parameter α often approximations with different parameters α_i are computed but a single approximation u_α is used. In case of smooth solution u_* the accuracy of these methods may be increased by various extrapolation algorithms [1, 2]. We propose, as in [1], to use for extrapolated approximation the linear combination

$$v_{n,\alpha} = \sum_{i=1}^n d_i u_{\alpha_i}, \quad d_i = \prod_{j=1, j \neq i}^n \left(1 - \frac{\alpha_i}{\alpha_j}\right)^{-1}.$$

Here $\alpha = \alpha_i$ with some fixed i , for example $\alpha = \alpha_1$. In $v_{n,\alpha}$ both parameters n and α can be viewed as regularization parameters.

1) If exact noise level δ with $\|f_\delta - f\| \leq \delta$ is known and sequence $\alpha_1, \alpha_2, \dots$ is given, we propose in extrapolated Tikhonov method the rule for choice of $n = n_{ME}$ guaranteeing $\|v_{n,\alpha} - u_*\| \leq \|v_{n-1,\alpha} - u_*\|$ for $n = 1, 2, \dots, n_{ME}$ and $\|v_{n,\alpha} - u_*\| \rightarrow 0$ ($\delta \rightarrow 0$). If $n \geq 2$ and q_1, \dots, q_{n+1} are fixed we propose rules for choice of $\alpha_i = q_i \alpha$.

2) If approximate noise level δ is known with $\|f_\delta - f\|/\delta \leq c$ as $\delta \rightarrow 0$, where c is an unknown constant, then our rules for choice of n and α guarantee convergence $\|v_{n,\alpha} - u_*\| \rightarrow 0$ ($\delta \rightarrow 0$) and in case $\|f_\delta - f\| \leq \delta$ also order optimal error estimates.

3) If no information about noise level is available, we propose to choose parameters that minimize some functionals.

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Approximation theoretic aspects of variable Hilbert scales

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Variable Hilbert scales are important for the analysis of inverse problems in Hilbert spaces, as these constitute a way to describe smoothness of objects other than functions on domains. As for the classical approximation theory of functions belonging to some smoothness class, here we also may study properties like interpolation, approximation,...

Within the context of discretization of inverse problems, first results on approximation theoretic properties appeared in [2]. Later, and more systematically, such aspects were presented within the context of approximation theory in [1].

In this talk we shall introduce the concept of variable Hilbert scales, and we shall highlight the known results concerning approximation theoretic aspects.

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Implementation of Lawson-Adams multistep methods for large stiff problems

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In recent years there has been a great interest in the construction of exponential integrators for large stiff semi-linear problems of the type

$$y'(t) = f(y(t)) = Ly(t) + N(y(t)). \quad (1)$$

These integrators use the exponential function (and related functions) of L (typically the Jacobian of f or an approximation to it) inside the numerical method.

For the solution of (1) we consider the change of variables, also called the *Lawson transformation* $v(t) = e^{(t_n-t)L}y(t)$ that leads to the transformed (hopefully non-stiff) problem

$$v'(t) = e^{(t_n-t)L}N(e^{(t-t_n)L}v(t)). \quad (2)$$

Using a classical integrator for (2) yields a so-called Lawson method. Here we consider the explicit Adams methods for (2) that lead to methods of the type

$$y_{n+1} = e^{hL}y_n + h \sum_{k=0}^q \beta_k e^{h(k+1)L}N(y_{n-k}), \quad (3)$$

where h is the time step. For the computation of the matrix exponentials in (3) we use the Restricted-Denominator Rational Arnoldi algorithm ([1, 2]). If $A(= jhL)$ is a sectorial operator such that its numerical range is contained in the left half complex plane, the basic idea for the computation of $\exp(A)v$ is to consider the transform

$$Z = (I - \delta A)^{-1}, \quad (4)$$

where δ is a suitable parameters, and to approximate the corresponding $g(Z)v = \exp(A)v$ working with the standard Arnoldi method based on the Krylov subspaces generated by Z and v .

Using this approach inside (3), a variable-stepsize variable-order implementation is discussed and tested on some classical stiff equations arising from the semidiscretization of parabolic problems.

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On approximating matrix functions in the solution of fractional differential equations

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In this talk we address the problem to approximate the Mittag-Leffler function when applied to matrix arguments.

Situations of this kind arise in the numerical solution of large systems of differential equations of fractional order (FDEs) which are difficult to solve by standard approaches. Indeed, classical methods for time-integration of FDEs (e.g., fractional linear multistep methods or product integration rules) are extremely slow and expensive for systems of very large dimension or when integration is required on long-time intervals.

Only recently Moret and Novati [2] have applied the matrix function approach in this context, showing its good performance.

In this work, based on the results in [1], we present promising methods based on matrix functions, obtained by combining approximation theory with Krylov subspace methods.

The features of different approaches are discussed and compared with results provided by classical convolution quadrature rules.

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Matrix computations in the numerical solution of the nonlinear Schrödinger equation^{*}

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The nonlinear Schrödinger equation

$$i \frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial x^2} + 2|q|^2 q, \quad q = q(x, t),$$

where the initial potential $q(x, 0)$ is known, plays an important role in many applicative areas, like light propagation in optical fibers and, in general, wave propagation in nonlinear media.

Besides the finite difference approach, one of the more commonly used numerical method is the so called *split-step Fourier* method. Recently, a new method, based on the inverse scattering transform, was introduced in [1].

We will discuss the linear algebra techniques employed in this algorithm, which take advantage of the structure of the matrices involved, and compare its performance with the *split-step Fourier* method.

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Edge-preserving multilevel methods for deblurring, denoising, and segmentation^{*}

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We discuss a cascadic multilevel image restoration method for reducing blur and noise in contaminated images that allows both spatially variant and spatially invariant point-spread functions (PSF). The method requires the solution of a linear system of equations on each level. These systems are solved by an iterative method, the choice of which depends on properties of the PSF. We introduce a thresholding updating strategy in order to suppress “ringing.” Constraints on the solution can be added in the cascadic environment. The restriction operators are defined by solving local weighted least-squares problems, and the prolongation operators are determined by piecewise linear prolongation followed by integrating a discretized nonlinear Perona-Malik diffusion equation for a few time-steps. The purpose of the integration is to reduce noise. The cascadic multilevel method so obtained shares the computational efficiency and simplicity of truncated iteration for the solution of linear discrete ill-posed problems with the edge-preserving property of nonlinear models. The multilevel method proceeds from coarser to finer levels, and regularizes by truncated iteration on each level. For many image restoration problems, the multilevel method demands fewer matrix-vector product evaluations on the finest level than the corresponding 1-level truncated iterative method, and often determines restorations of higher quality. A benefit of our multilevel approach to image restoration is that it easily can be combined with image segmentation, as is illustrated in the present work.

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Poster session

Two interpolation operators on irregularly distributed data in inner product spaces

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Two interpolation operators in inner product spaces for irregularly distributed data are compared. The first is a well-known polynomial operator, which in a certain sense generalizes the classical Lagrange interpolation polynomial (see [2]). The second can be obtained by modifying the first so to get a partition-of-unity interpolant. Numerical tests and theoretical considerations on errors show that the second operator gives better approximations, working in particular from \mathbb{R}^m to \mathbb{R}^n and from $\mathcal{C}[-\pi, \pi]$ to \mathbb{R} . The operators could represent interesting tools to face nonlinear system modelling and system identification problems with a black-box approach.

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An accelerated algorithm for Navier–Stokes equations

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The numerical approximation of the steady-state Navier–Stokes equations by fractional step methods typically requires a large amount of computational time which is related to the total number of iterations, the size of the problem and the used algorithm. As a consequence, the convergence rate may become slow and it is interesting to apply acceleration techniques to improve the overall iterative scheme.

In this work, the Characteristic–Based–Split (CBS) scheme [3] is accelerated with the Minimum Polynomial Extrapolation (MPE) method [1, 2] obtaining a new algorithm to solve steady state Navier–Stokes equations both for compressible and incompressible problems. In particular, in the CBS scheme, the semi-implicit version with global time-stepping is used for non-isothermal incompressible problems. The fully-explicit version of the code is used for compressible flows. The MPE method is a vector extrapolation method that transforms the original sequence into another sequence converging to the same limit faster than the original CBS scheme.

The proposed algorithm is tested on several two-dimensional benchmark problems to demonstrate the new computational features arising from the introduction of the extrapolation procedure showing a reduction of the computational cost of the simulations.

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Approximate Fekete Points

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Fekete points for polynomial interpolation are points that maximize the Vandermonde determinant (in any polynomial basis) on a given compact set, and thus ensure that the corresponding Lebesgue constant is bounded by the dimension of the polynomial space. We have developed a greedy algorithm, that computes multivariate approximate Fekete points by extracting maximum volume submatrices from rectangular Vandermonde matrices on suitable discretization meshes. It works on arbitrary geometries and uses only optimized tools of numerical linear algebra (essentially QR-like factorizations). There is a strong connection with the theory of admissible meshes for multivariate polynomial approximation, recently developed by J.P. Calvi and N. Levenberg. There are also good perspectives in the application to numerical cubature, and to the numerical solution of PDEs by collocation and discrete least squares methods.

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Polynomial interpolation and algebraic cubature at the Padua points

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The problem of choosing “good” nodes on a given compact set is a central one in multivariate polynomial interpolation. Besides unisolvence, which is by no means an easy problem, for practical purposes one needs slow growth of the Lebesgue constant and computational efficiency. A new set of points for polynomial interpolation on the square $[-1, 1]^2$, called “Padua points”, was introduced and experimentally studied. It has been proved that they are unisolvent in the full polynomial space Π_n^2 , n the polynomial degree, and that they give the first known example of non tensor-product optimal interpolation in two variables, since their Lebesgue constant has minimal order of growth of $\mathcal{O}((\log n)^2)$. Moreover, an explicit representation of their fundamental Lagrange polynomials was given, which is based on the reproducing kernel of the space $\Pi_n^2([-1, 1]^2)$ equipped with the bivariate Chebyshev inner product. We give here an overview of the theoretical and computational issues concerning interpolation and Clenshaw–Curtis-like cubature at the Padua points.

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Newton's methods for a class of nonlinear hypersingular integral equations

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In this work we consider the following nonlinear hypersingular integral equation

$$-\frac{\varepsilon}{\pi} \int_{-1}^1 \frac{g(y)}{(y-x)^2} dy + \gamma(x, g(x)) = f(x), |x| < 1, \quad (1)$$

where $0 < \varepsilon \leq 1$, and the unknown function g satisfies the boundary conditions $g(\pm 1) = 0$. The integral has to be understood as the "finite part" of the strongly singular integral in the sense of Hadamard, who introduced this concept in relation to Cauchy principal value. Equations of this type (NHIEs) can be used to model many problems in fracture mechanics (see [1, 2, 3, 4, 5] and the references given there). In the present work we apply Newton and modified Newton's methods to the discrete equation obtained by applying a collocation method to equation (1) and, we prove the convergence of these methods in opportune Sobolev spaces.

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Spectral analysis and preconditioning techniques for RBF-collocation matrices

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Meshless collocation methods based on radial basis functions lead to structured linear systems which, for equispaced grid points, have almost a multilevel Toeplitz structure. In particular, for a 2-dimensional PDE we find almost two-level Toeplitz matrices that is we encounter Toeplitz structures with Toeplitz blocks, where both the number of blocks and the block-size grow with the number of collocation points. For simplicity we consider a Cartesian $n \times n$ grid with n^2 internal points. Hence the complete collocation matrix A_n is the sum of a symmetric two-level Toeplitz matrix of size n^2 with coefficients $c_{i,j}$, $c_{i,j} = c_{|i|,|j|}$, and a matrix of rank $2n$ which collects the boundary conditions. In [1] upper bounds for the condition numbers of the Toeplitz matrices approximating a one dimensional model problem were proved, where the correction to the one-level Toeplitz structure has rank at most equal to 2. In this note we refine the one-dimensional results, by explaining in a mathematically rigorous manner some numerics reported in [1], and we show a preliminary analysis concerning conditioning, extremal spectral behavior, and global spectral results in the two-dimensional case for the structured part. By exploiting recent tools in the literature [4, 5], a global distribution theorem in the sense of Weyl (see e.g. [2]) is proved also for the complete matrix-sequence A_n . Moreover, we test several well-known preconditioners [3] and design preconditioning techniques in order to overcome the ill-conditioning of the matrices. A spectral analysis of the preconditioned matrices is also given to support the conclusion of a wide numerical experimentation.

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Non stationary interpolatory subdivision schemes arising from Hurwitz symmetric approximating symbols

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In this work we present a general strategy to deduce a family of non stationary interpolatory subdivision schemes from a symmetric non-interpolatory one whose symbol is of Hurwitz type.

At each step of the subdivision recursion, the strategy here proposed consists in solving a polynomial equation involving the symbol of the non-interpolatory scheme we start with. For several type of Hurwitz symbols, we are able to tailor the solution of the polynomial equation and to provide an efficient procedure for the computation of the coefficients of the corresponding family of interpolatory masks.

In this general context, our attention will be focused on the derivation of non stationary interpolatory subdivision schemes generating exponential polynomials, since they appear to be of great interest in curve design for their capability of reproducing important analytical shapes.

Several examples, giving evidence of the effectiveness of the proposed method, will be illustrated.

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A Meshless Method for Image Reconstruction

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Many image processing techniques work with scattered data distribution usually employing grid based methods leading to several numerical problems. To address this issue, a numerical method avoiding mesh generation can be used. Such a method performs an integral representation by means of a smoothing kernel function and, in the discrete formulation, involves domain particles. In this work, the Smoothed Particle Hydrodynamics methodology is proposed in Image Processing context and applied to reconstruct corrupted digital images. In order to show the capability of this approach, simulations with even and uneven particles distributions are performed. Appreciable reconstruction quality improvements are obtained by iteratively applying the method.

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Modeling oscillatory time-series with physical insights and Kernel-based methods^{*}

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Recently we discussed the modeling of a class of acoustic systems through physical knowledge and kernel-based methods, showing the effectiveness of this approach for some computability and model inversion problems [1]. In this communication, we report further investigations on this approach, focusing in particular on the interaction between the parameters of the physically-inspired part of the model and the parameters of the kernel-based part, and on how the inclusion of a kernel-based component affects the overall stability of physically-based systems which exhibit oscillatory dynamics. The experimental section will present results for the inversion problem applied to a class of models of phonation characterized by self-sustained oscillatory behavior, and that can be adapted to recorded voice signals.

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On a class of C^2 Hermite interpolatory subdivision schemes

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Subdivision schemes, which consist in repeated applications of some simple rules determining successive refinements of rough starting polygons or grids, are probably the most popular tools to build graphs of functions, curves and surfaces. Among the others, Hermite subdivision interpolating schemes have been proposed and analyzed from different points of view, including their ability to preserve some typical geometric properties of the starting data. Assuming a given set of data (position and derivatives up to a given order) is prescribed, an interpolatory Hermite subdivision scheme determines simultaneously, by successive refinements, a function interpolating the data and its derivatives.

An interesting family of C^1 Hermite (HC^1 for short) subdivision schemes has been proposed in [1] and analyzed by several authors. The region of convergence of the scheme is completely understood, and efficient strategies for the selection of the parameters describing the scheme have been identified. In this paper we focus on the family of 2-point C^2 Hermite (HC^2) subdivision schemes proposed by Merrien in [2] which produce a C^2 function and its first and second derivatives, whenever the associated parameters are selected to ensure convergence. Despite the Merrien HC^2 scheme is of great interest due the high smoothness of the resulting functions, it is not yet widely used mainly because its region of convergence is not completely characterized, and the scheme depends on large number of parameters (basically four) whose selection is a very hard task.

We provide some progresses concerning both the previous points. More precisely, first, we extend the sufficient conditions for convergence given in [2] by using some recent techniques to determine the joint spectral radius of a related pair of matrices.

Then, we present two one parameter families of convergent Merrien HC^2 schemes providing an analytic interpretation of them. The reduction of the number of the free parameters is of salient interest in practical applications because the only remaining parameter allows some control of the shape of the produced interpolant.

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Inexact rational Krylov approximations to matrix functions*

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We consider the problem of extracting approximations to $f(A)\mathbf{b}$ from rational Krylov spaces $q_{m-1}(A)^{-1}\mathcal{K}_m(A, \mathbf{b})$ (the zeros of the polynomial q_{m-1} are referred to as the *poles* of the rational Krylov space). This approach requires the solution of shifted linear systems with A . In practice these systems are solved inexactly and it can be shown that the Rayleigh-Ritz approximation \mathbf{f}_m belongs to a disturbed problem $f(A + E_m)\mathbf{b}$. We decompose the error

$$\|f(A)\mathbf{b} - \mathbf{f}_m\| \leq \underbrace{\|f(A)\mathbf{b} - f(A + E_m)\mathbf{b}\|}_{\text{sensitivity error}} + \underbrace{\|f(A + E_m)\mathbf{b} - \mathbf{f}_m\|}_{\text{approximation error}},$$

and show how to obtain practical estimators for both error components. This results in a useful and effective stopping criterion.

Our error estimators are demonstrated at the problem of approximating the resolvent function $f^\tau(z) = (z - \tau)^{-1}$ for the 2D-Laplacian, with purely imaginary parameters $\tau \in [\tau_{min}, \tau_{max}]$, where we choose the poles of the rational Krylov space near-optimal on the negative real axis.

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Approximation by Surface Splines*

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We present an approximation scheme using *surface splines* (the fundamental solutions of polyharmonic equations in R^d) that treats functions, $f : \Omega \rightarrow R$, defined on smoothly bounded domains, $\Omega \subset R^d$, and delivers theoretically optimal rates of convergence. By exploiting an integral representation involving *boundary layer potentials*, this approach isolates the notorious boundary effects in easy-to-manage boundary integrals. The representation at the heart of this approach is discussed and is shown to coincide with the native space extension (i.e., the native space norm minimizing extension) of the function – initially defined only on Ω – to all of R^d .

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A stable collocation method for two-point boundary value problems with rapid growth solutions*

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The main purpose of the presented work is to investigate the efficiency and flexibility of the approximate Fekete points of Sommariva and Vianello [1] (see also [2]) in the collocation solution of two-point boundary value problems with the exact solutions which have small regions of rapid growth. Because of a rapid growth of the corresponding exact solutions exhibiting a narrow pike, interior or boundary layers, and a turning point, the global and local approximation schemes are considered. Whereas transformed orthogonal polynomials highly oscillating in the region of rapid growth are used in the global approach, the local approach is represented by the B-splines on equidistant and graded meshes. These trial functions together with the mentioned approximate Fekete points ensure stable solution of the corresponding systems of linear equations and high precision results for problems with moderately and highly rapid growth solutions as well. The presented numerical results confirm high flexibility of the approximate Fekete points in the collocation solution of difficult two-point boundary value problems by using different kinds of trial functions.

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More efficient approximations of functions in Clifford algebras *

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Many functions in Clifford algebras can be computed as functions of matrices, using standard representation theory. This poster examines some of the tradeoffs involved in computing such functions using matrix decompositions, versus using the matrix inverse and the usual algebra operations.

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Some computational methods of bivariate approximation by discrete Chebyshev polynomials

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Let $\Omega = \{0, 1, \dots, N-1\} \times \{0, 1, \dots, N-1\}$. We are considering problem of approximation of discrete functions $f = f(x, y)$ determined on Ω by Fourier sums on basis of $\tau_{nm}(x, y) = \tau_n(x)\tau_m(y)$ where $\tau_n(x)$ is discrete Chebyshev polynomial of n degree. For a set of functions we made a comparison of approximation characteristics of triangular, rectangular and hyperbolic partial sum that correspondingly:

$$S_n^{tr}(x, y) = \sum_{0 \leq k+l \leq n} \check{f}_{kl} \tau_{kl}(x, y); \quad S_n^{rect}(x, y) = \sum_{\substack{0 \leq k \leq n \\ 0 \leq l \leq n}} \check{f}_{kl} \tau_{kl}(x, y);$$

$$S_n^{hyp}(x, y) = \sum_{0 \leq k * l \leq n} \check{f}_{kl} \tau_{kl}(x, y) \quad (1)$$

For multivariate approximation based on trigonometric polynomials hyperbolic partial sums have the best approximative properties in terms of that the number of coefficients maintaining given precision of approximation using hyperbolic partial sums is lesser than the number of coefficients maintaining the same precision for other kind of partial sums [1]. For research of approximative characteristics of partial sums based on Chebyshev polynomials we developed computer program that realizes bivariate approximation by different kinds of partial sums.

The main problem that appears in computational realization of calculations (1) is developing of fast and robust algorithms for finding values of $\tau_n(x)$. For evaluating this values can be used recurrence procedure that was described in [2]. We rewrote this recurrence relation to avoid accumulation of numerical errors. In that way we avoided computational instabilities.

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Nonstationary Scaling Functions and Wavelets

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Nonstationary subdivision schemes (NSSs) [1, 3] use scale-dependent masks to generate limit functions with prescribed properties. For instance, the NSS associated with binomial masks gives rise to C^∞ limit functions [3] while NSSs associated with exponential masks give rise to limit functions which reproduce exponential polynomials [5].

Scale-dependent masks generate multiresolution analysis (MRAs) whose scaling functions, i.e. the basic limit functions, are scale-dependent, too. This means that the sequence of nested spaces generating the MRA possesses a different stable basis at each scale level and a different wavelet basis have to be constructed for each level [2].

Our goal is to construct wavelet bases with good approximation properties (smoothness, symmetry, vanishing moments) associated with the nonstationary scaling functions introduced in [4].

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Spinor Splines

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Pairs of counterrotating spinors may be basis for approximation splines for 3D movements. We show that this idea may be applicated for either free 3D movements, robotics, or even relativistic equations. Our idea is to use fluid dynamics techniques to be generalized. The kind of flow that is used as the starting idea is that due to a diffusing point vortex of strenght Γ that we will normalize by means of $\Gamma/2\pi$ and an arbitrary characteristic length l . Thus normalized, the problem is governed by the nondimensional vorticity equation

$$\omega_\tau = \omega_{xx} + \omega_{yy} - Re(u\omega_x + v\omega_y)$$

As we use two counterrotating point vortices we just choose l as the distance between them with an initial condition

$$\omega(x, y, 0) = 2\pi\delta(x - \frac{1}{2}, y) - 2\pi\delta(x + \frac{1}{2}, y)$$

An appropriate treatment of vorticity may lead us to trat such vortices as spinors in order to represent the whole phenomena in terms of spinor splines. We first show it for fluid dynamics and then other examples generalized interpreting what are the different coefficients in fluid mechanics to the case of the application

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Error formulas for multivariate rational interpolation

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We consider multivariate rational interpolation in a very practical context, in scattered data points and using quite general polynomials. The interpolation error $|f(x_1, \dots, x_d) - r(x_1, \dots, x_d)|$ is easy to estimate when the interpolation points are a subset of a large data cloud, because then the error in each data point that is not an interpolation point is also known. When $f(x_1, \dots, x_d)$ is only known through a (possibly complicated) function subprogram, then upper bounds for the error are more practical. Information on the interpolation error can be used to update the rational interpolant.

To derive useful error bounds we depart from multivariate polynomial error formulae using a multi-index notation, finite difference functions, Newton fundamental polynomials and directional derivatives, as in [1].

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Non-linear algebraic and differential properties of hypergeometric-type functions: Unified approach*

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A computational method for the explicit construction of general non-linear sum rules involving hypergeometric type functions and their derivatives of any order is presented. The method only requires the knowledge of the coefficients of the differential equation that these well known functions satisfy, which is the simplest Lamé equation also known as hypergeometric type differential equation, i.e. a second order linear differential equation:

$$\sigma(x)y''(x) + \tau(x)y'(x) + \lambda y(x) = 0$$

where σ and τ are polynomials of degree at most two and one, respectively, and λ is a constant.

Here we focus our attention in the quadratic case for which, as illustration of the method, a particularly simple sum rule is explicitly constructed in terms of the differential equation coefficients.

Main tool of our unified approach is an integral representation [1, 2] for hypergeometric-type functions which can be obtained in terms of the differential equation coefficients.

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A stable collocation solution of the Poisson problems on planar domains*

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The aim of this work is to show how the collocation method may be used for the approximate solution of elliptic partial differential equations in a stable and efficient way. Extensive numerical experiments concerning the collocation solution of the Poisson problems with the homogeneous Dirichlet boundary conditions, different right-hand side functions and different trial functions are described. These problems are considered on the unit disc and the trial functions are selected to be algebraic polynomials, Bessel functions, B-splines and radial basis functions satisfying and not satisfying the prescribed boundary conditions as well. The most important aspect of this work consists in the use of approximate Fekete points recently developed by Sommariva and Vianello [1]. As the presented results show, these easily computable collocation points, giving well-conditioned collocation matrices, open new horizons for the collocation solution of elliptic partial differential equations on planar and higher-dimensional domains.

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