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MATHEMATISCHE NACHRICHTEN

Holomorphic symmetric differentials and a birational characterization of abelian varieties

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Abstract
A generically generated vector bundle on a smooth projective variety yields a rational map to a Grassmannian, called Kodaira map. We answer a previous question, raised by the asymptotic behaviour of such maps, giving rise to a birational characterization of abelian varieties. In particular we prove that, under the conjectures of the Minimal Model Program, a smooth projective variety is birational to an abelian variety if and only if it has Kodaira dimension 0 and some symmetric power of its cotangent sheaf is generically generated by its global sections.

KEYWORDS
abelian varieties, positivity of vector bundles, projective varieties

MSC (2010)
14E30, 14J15, 14J60, 14K99

1 | INTRODUCTION

The aim of this work is to answer positively a question raised in the framework of the investigation on stable base loci for vector bundles started in [1].

In the recent work [17] we extended the construction of the Iitaka fibration to the case of higher rank vector bundles, and the natural setting to do so is to consider asymptotically generically generated (AGG) vector bundles. A vector bundle E on a projective variety X is said to be *asymptotically generically generated* when some symmetric power $\mathrm{Sym}^m E$ is generated over a nonempty open subset $U \subseteq X$ by its global sections $H^0(X, \mathrm{Sym}^m E)$.

In the same work we proved that if the cotangent bundle Ω_X of a smooth projective variety X with Kodaira dimension 0 is strongly semiample, then the variety must be isomorphic to an abelian Variety (we say that a vector bundle E on a projective variety X is *strongly semiample* when some symmetric power $\mathrm{Sym}^m E$ is globally generated). That led to consider the question whether the generic condition AGG can give a birational characterisation of abelian varieties.

In the present work we give a positive answer to this question under the hypothesis that the main conjecture of the minimal model program be satisfied:

Theorem 1.1. *Let X be a smooth variety of Kodaira dimension 0. Suppose that X admits a minimal model Y and that the abundance conjecture holds for Y. If the vector bundle Ω_X is asymptotically generically generated, then X is birational to an abelian variety.*

In particular in dimension at most 3 we obtain:

Corollary 1.2. *Let X be a smooth projective variety of dimension $n \leqslant 3$. The following conditions are equivalent:*

1. *The variety X is birational to an abelian variety, in particular the Albanese map $a_X : X \to \mathrm{Alb}(X)$ is surjective and birational.*
2. *The variety X has Kodaira dimension $\mathrm{kod}(X, K_X) = 0$ and the cotangent bundle Ω_X is asymptotically generically generated.*

In order to prove the theorem we first prove that within the hypotheses we can apply a criterion due to Greb–Kebekus–Peternell (cf. [8]) to show that the minimal model Y of X is a quotient of an abelian variety, then we show that on such a quotient the cotangent bundle of a resolution cannot be asymptotically generically generated unless the quotient is an abelian variety itself.

2 | NOTATION, PREVIOUS RESULTS, LEMMATA

We will work with projective varieties over the field $\mathbb{C}$ of complex numbers. When the variety X is smooth, we will denote Ω_X the cotangent bundle, and identify it with the sheaf of Kähler differentials on X. As usual we will denote $\Omega_X^p = \bigwedge^p \Omega_X$ the higher exterior powers, identified with the sheaf of holomorphic p-forms.

We give in this section the main definitions and results for base loci and Kodaira maps for vector bundles, most of the definitions can be found in [17].

2.1 | Stable base locus

Definition 2.1. Let E be a vector bundle on a projective variety X.

1. We call *base locus* of E the closed subset
$$\mathrm{Bs}(E) := \left\{ x \in X \mid ev_x : H^0(X, E) \to E(x) \text{ is not surjective} \right\} \subseteq X$$
and *stable base locus* the closed subset
$$\mathbb{B}(E) := \bigcap_{m>0} \mathrm{Bs}\left(\mathrm{Sym}^m E\right) \subseteq X.$$
2. We say that a vector bundle E on X is *strongly semiample* if $\mathbb{B}(E) = \emptyset$, i.e. if some symmetric power of E is globally generated.
3. We say that a vector bundle E on X is *generically generated* if $Bs(E) \neq X$, i.e. if E is generated over a nonempty open subset $U \subseteq X$ by its global sections $H^0(X, E)$.
4. We say that a vector bundle E on X is *asymptotically generically generated* if $\mathbb{B}(E) \neq X$.

Remark 2.2. In general the definition of strong semiampleness is not equivalent to the usual definition of semiampleness: it is stronger and not equivalent to the fact that $\mathcal{O}_{\mathbb{P}(E)}(1)$ is semiample. A simple counterexample to the equivalence can be found in [17] (Example 3.2).

The following theorems are proved in [17] (Theorems 4.14 and 4.17):

Theorem 2.3. *Let X be a smooth projective variety. Then X is isomorphic to an abelian variety if and only if the cotangent bundle is strongly semiample and X has Kodaira dimension* 0.

Theorem 2.4. *Let X be a smooth projective surface. Then X is birational to an abelian variety if and only if the cotangent bundle of X is asymptotically generically generated and X has Kodaira dimension* 0.

Those theorems lead to ask whether the following holds:

Question 2.5. Let X be a smooth projective variety of Kodaira dimension 0. Suppose that the cotangent bundle Ω_X is asymptotically globally generated. Is X birational to an abelian variety?

The main result in this work is an affirmative answer to this question, at least in the case where Minimal Model Program and Abundance Conjecture can be applied.

In order to prove these results, we apply our previous results in order to show that the theorem holds in the case a minimal model of X is smooth, then we show that within the hypotheses a singular minimal model cannot occur.

2.2 | Automorphisms of abelian varieties and complex tori

We will use repeatedly the following lemmas:

Lemma 2.6. *Let V be a complex vector space and $A = V/\Lambda$ a complex torus, let $Aut(A)$ be the automorphism group of A as a complex manifold, let $T_A \subset Aut(A)$ be the subgroup of translations, ant let $Aut_0(A)$ be the subgroup of automorphisms fixing $0 \in A$.*

Then the morphisms in $Aut_0(A)$ are also group automorphisms of A, they are induced by linear maps $V \to V$ and there is an exact sequence $0 \to T_A \to Aut(A) \to Aut_0(A) \to 0$ making $Aut(A)$ a semidirect product of the subgroup T_A with $Aut_0(A)$.

Proof. These are classical properties of complex tori. To prove them, observe that there is a sujective homomorphism:

$$Aut(A) \twoheadrightarrow Aut_0(A)$$

$$g \mapsto \big(t_{-g(0)} \circ g \, : \, x \mapsto g(x) - g(0)\big).$$

This map has a section, and its kernel are exactly translations. To prove that automorphisms in $Aut_0(A)$ are induced by linear map on V lift the maps to the universal cover of A which is V, then use periodicity properties (every partial derivative is holomorphic and periodic modulo Λ so it is constant), cf. [3, Thm. 2.3] for more details. □

Corollary 2.7. *Let $g : A \to A$ be an automorphism of a complex torus $A = V/\Lambda$. Then g is a translation if and only if it induces the identity $g^* : H^0(A, \Omega_A) \to H^0(A, \Omega_A)$ on holomorphic 1-forms.*

Proof. Recall that the tangent and cotangent bundles of A are trivial, therefore there are canonical identifications:

$$V \otimes \mathcal{O}_A \cong T_A \quad \text{and} \quad V^* \otimes \mathcal{O}_A \cong \Omega_A\,.$$

So a map $g \in Aut_0$ is the identity if and only if it is induced by the identity map $G = id_V : V \xrightarrow{\sim} V$, if and only if the map $g^* = G^t : V^* \to V^*$ is the identity. Then apply Lemma 2.6. □

2.3 | Birational properties of holomorphic forms

We recall that having a globally generated cotangent bundle, or a globally generated symmetric power of a cotangent bundle are not birationally invariant properties. To observe this, consider the blowing up $\widetilde{A}$ of an abelian variety on a point. Then the cotangent bundle on A and all its symmetric powers are trivial, then globally generated, but the cotangent bundle and its symmetric powers on $\widetilde{A}$ are not generated on the exceptional divisor.

However, we will use some birationally invariant properties of these bundles that we sum up here:

Lemma 2.8. *Let X and Y be smooth projective varieties, suppose that there exists a birational map $X \dashrightarrow Y$. Then:*

1. $\dim H^0(X, \mathrm{Sym}^m \Omega_X) = \dim H^0(Y, \mathrm{Sym}^m \Omega_Y)$;
2. *A symmetric power $\mathrm{Sym}^m \Omega_X$ is generically generated if and only if $\mathrm{Sym}^m \Omega_Y$ is generically generated;*

i.e. the property of having an asymptotically generically generated cotangent bundle is a birational property, and the dimension of a symmetric m-power of the cotangent bundle is a birational invariant of smooth projective varieties.

Proof. By resolution of the indeterminacy locus we can suppose that there exists a regular, surjective, birational map $f : X \to Y$, with center $Z \subset Y$ and exceptional divisor $E \subset X$.

The pull back $f^* : H^0(Y, \mathrm{Sym}^m \Omega_Y) \to H^0(X, \mathrm{Sym}^m \Omega_X)$ is an injective morphism. Its inverse can be constructed as follows: restrict global sections of $\mathrm{Sym}^m \Omega_X$ to $X \setminus E \cong Y \setminus Z$, then as $\mathrm{codim}_Y Z \geqslant 2$ these sections can be extended to Y, and this provides with an inverse of f^*, therefore the two spaces are isomorphic.

There is an exact sequence

$$0 \to f^*\Omega_Y \to \Omega_X \to \mathcal{F} \to 0$$

where $\mathcal{F}$ is a sheaf supported on the exceptional divisor E. Therefore out of the loci $Z \subset Y$ and $E \subset X$ the vector bundles $\mathrm{Sym}^m \Omega_Y$ and $\mathrm{Sym}^m \Omega_X$ are isomorphic, this isomorphism induces isomorphic spaces of global sections over Y and X, so also the second property is statisfied. □

Remark 2.9. The proof above applies as well to show that the space of global sections of the m-tensor power of Ω_X, and all of its natural direct summands, are isomorphic.

3 | QUOTIENTS OF ABELIAN VARIETIES

In this section we show that a smooth projective variety of Kodaira dimension 0 with asymptotically generically generated cotangent bundle is birational to a quotient of an abelian variety.

We will use the following theorem to show that a minimal model of the variety we are dealing with is a quotient of an abelian variety:

Theorem 3.1 (Greb–Kebekus–Peternell, [8])**.** *Let Y be a normal, complex, projective variety of dimension n, with at worst KLT singularities. Assume that Y is smooth in codimension 2, and assume that the canonical divisor of Y is numerically trivial, $K_Y \equiv 0$. Further, assume that there exist ample divisors $H_1, \dots, H_{n-2}$ on Y and a desingularization $\pi : \widetilde{Y} \to Y$ such that $c_2(\Omega_{\widetilde{Y}}).\pi^*H_1 \dots \pi^*H_{n-2} = 0$. Then, there exist an abelian variety A and a finite, surjective, Galois morphism $A \to Y$ that is étale in codimension 2.*

Remark 3.2. We remark that by Kawamata [12] the condition that $K_Y \equiv 0$ is equivalent to the fact that K_Y is torsion (therefore K_Y is semiample and Y has Kodaira dimension 0). The abundance conjecture predicts that if K_Y is nef then it is semiample, which for Kodaira dimension 0 varieties Y is the same as the equivalence between "K_Y nef" and "$K_Y \equiv 0$".

Remark 3.3. We observed above that for a smooth variety, the property of having an asymptotically generically generated cotangent bundle is a birationally invariant property (Lemma 2.8), contrarily to having a strongly semiample cotangent bundle.

Let us use the result above in order to prove that the variety X of Kodaira dimension 0 with asymptotically generically generated cotangent bundle is birational to a quotient of an Abelian variety.

Proposition 3.4. *Let X be a smooth projective variety with $\mathrm{kod}(X) = 0$. Suppose that the cotangent bundle Ω_X is asymptotically generically generated, and that X admits a minimal model Y that satisfies the abundance conjecture (i.e. Y has terminal singularities and K_Y is numerically trivial). Then, there exists an abelian variety A and a finite, surjective, Galois morphism $A \to Y$ that is étale in codimension 2. In particular X is birational to a quotient of an abelian variety by a finite group, and the quotient map is ètale in codimension 2.*

Proof. Consider a minimal model $Y \sim_{bir} X$ such that K_Y is numerically trivial. Let us choose a resolution of singularities $\pi : \widetilde{Y} \to Y$, and show that, for some $H_1, \dots, H_{2n-2}$ ample divisors on Y, we have $c_2(\Omega_{\widetilde{Y}}).\pi^*H_1 \dots \pi^*H_{n-2} = 0$. As Y has terminal singularities, Y^{sing} has codimension at least 3 in Y, so we can suppose that $H_1 \cap \dots \cap H_{n-2} \cap Y^{sing} = \emptyset$. As $\widetilde{Y}$ is smooth and birational to X, for some $k > 0$ the symmetric power $\mathrm{Sym}^k \Omega_{\widetilde{Y}}$ is generically generated (Lemma 2.8).

Now let us consider the base locus of $\mathrm{Sym}^k\Omega_{\widetilde{Y}}$: this is contained in a divisor in $\widetilde{Y}$ linearly equivalent to a multiple of the canonical divisor $K_{\widetilde{Y}}$. In fact, let R be the rank of $\mathrm{Sym}^k\Omega_{\widetilde{Y}}$, choose R global sections $\sigma_1, \dots, \sigma_R$ that generate $\mathrm{Sym}^k\Omega_{\widetilde{Y}}$ at one point y, then $\sigma_1 \wedge \dots \wedge \sigma_R$ is a global section of $\det\left(\mathrm{Sym}^k\Omega_{\widetilde{Y}}\right)$, therefore vanishes on a multiple of the canonical divisor. So the sections $\sigma_1, \dots, \sigma_R$ generate $\mathrm{Sym}^k\Omega_{\widetilde{Y}}$ outside of that divisor. As the Kodaira dimension of $\widetilde{Y}$ is 0 and Y is terminal, any multiple of the canonical divisor does not move and its support is contained in the exceptional locus of π. So $\mathrm{Bs}\left(\mathrm{Sym}^k\Omega_{\widetilde{Y}}\right) \subset Exc(\pi)$. In particular $\left(\mathrm{Sym}^k\Omega_{\widetilde{Y}}\right)_{|\pi^{-1}(H_1\cap\dots\cap H_{n-2})}$ is generated by its global sections, its determinant is trivial, and therefore it is trivial.

So $\left(\Omega_{\widetilde{Y}}\right)_{|\pi^{-1}(H_1\cap\dots\cap H_{n-2})}$ is a vector bundle on a smooth projective variety, whose symmetric k-power is trivial. It can be shown that such a vector bundle is a direct sum of torsion line bundles (cf. [16, Thm. 3.2 and Rem. 5.3]), therefore its Chern classes are 0.

Then $c_2\left(\Omega_{\widetilde{Y}}\right).\pi^*H_1 \dots \pi^*H_{n-2} = 0$, and we can apply Proposition 3.1. □

So we have shown that, for a Kodaira dimension 0 variety X (satisfying the MMP conjectures), if the cotangent is asymptotically generically generated then a model of X is a quotient of an abelian variety. Let us distinguish two cases according to this quotient being smooth or singular.

4 | THE SMOOTH CASE

In this section we show that a smooth projective variety X with Kodaira dimension 0 and asymptotically generically generated cotangent bundle is birational to an abelian variety if a minimal model of X is smooth.

The results of this section make use of very similar constructions to some of the ones in [17], we will recall anyway the constructions needed.

We have proven above that a smooth projective variety X with Kodaira dimension 0 (satisfying the MMP conjectures) and asymptotically generically generated cotangent bundle Ω_X has a minimal model Y which is a quotient of an abelian variety.

Let us show that if the minimal model Y is smooth then Y is an abelian variety itself:

Theorem 4.1. *Let X be a smooth projective variety, suppose that the cotangent bundle Ω_X is asymptotically generated by global sections, and that X is birational to a smooth quotient $Y = A/G$ of an abelian variety by a finite group. Then Y is an abelian variety.*

Proof. In order to prove that $Y = A/G$ is an abelian variety we will prove that G acts on A by translations, i.e. we will prove that the action of G on $H^0(A, \Omega_A)$ is trivial (cf. Corollary 2.7).

Since the quotient $f : A \to A/G = Y$ is smooth and Y has Kodaira dimension 0, then f is an étale map: if there were ramification, there would be an effective divisor R such that $0 = K_A = f^*K_Y + R$, therefore K_Y would not be pseudoeffective, and Y would be uniruled by Theorem 2.6 in [2].

Therefore $f^*\Omega_Y \cong \Omega_A$, and $H^0\left(Y, \mathrm{Sym}^m\Omega_Y\right) \cong H^0\left(A, \mathrm{Sym}^m\Omega_A\right)^G$ for all $m > 0$. As Y is a smooth variety birational to X, then Ω_Y is asymptotically generically generated (by Lemma 2.8 above), therefore $\dim H^0\left(Y, \mathrm{Sym}^k\Omega_Y\right) \geqslant \mathrm{rk}\left(\mathrm{Sym}^k\Omega_Y\right)$, for some $k > 0$. So the inclusion

$$H^0\left(Y, \mathrm{Sym}^k\Omega_Y\right) \cong H^0\left(A, \mathrm{Sym}^k\Omega_A\right)^G \subseteq H^0\left(A, \mathrm{Sym}^k\Omega_A\right)$$

must be an equality, as $\dim H^0\left(A, \mathrm{Sym}^k\Omega_A\right) = \mathrm{rk}\left(\mathrm{Sym}^k\Omega_A\right)$ for all $k > 0$.

So the action of G on $H^0\left(A, \mathrm{Sym}^k\Omega_A\right)$ is trivial, and this can happen if and only if G acts on $H^0(A, \Omega_A)$ through homothethies (*i.e.* multiplication by roots of 1): in fact as G is a finite group, then the action of an element $g \in G$ on $H^0(A, \Omega)$ is diagonalizable. Given two eigenvectors $v, w \in H^0(A, \Omega_A)$, with eigenvalues λ and μ, choosing $k > 0$ such that $\lambda^k = \mu^k = 1$ and such that G acts trivially on $H^0\left(A, \mathrm{Sym}^k\Omega_A\right) = \mathrm{Sym}^k H^0(A, \Omega_A)$, we have

$$g \cdot \left(v^{k-1}.w\right) = \lambda^{k-1}v^{k-1}.\mu w = \left(\frac{\mu}{\lambda}\right) v^{k-1}.w = v^{k-1}.w$$

so $\lambda = \mu$.

But in this case the action of G is trivial on $H^0(A,\Omega_A)$, otherwise there would be points with non-trivial stabilizer, and this cannot occur as we showed that the quotient is étale: if the action of an element $g \in G$ on $H^0(X,\Omega_A)$ is given by multiplication by $\lambda_g \neq 1$, then g acts on A by $x \mapsto \lambda_g.x + \tau_g$ for some $\tau_g \in A$ (cf. Lemma 2.6), therefore there is a point $y = (1-\lambda_g)^{-1}\tau_g \in A$ fixed by g.

Hence, by Corollary 2.7, G acts by translations on the variety A, so the quotient Y is an abelian variety. □

Remark 4.2. The proof of the last theorem is very similar to the proof that a Kodaira dimension 0 variety with strongly semiample cotangent bundle is isomorphic to an abelian variety given in [17], we just observe that strong semiampleness is not needed here, and that the characterisation of abelian varieties appearing in [17] is a corollary of Theorem 4.1.

5 THE SINGULAR CASE

In this section we show that the cotangent bundle of a resolution of singularities of a singular quotient $Y = A/G$ cannot be asymptotically generically generated.

We will use the following result on extensions of symmetric differentials:

Theorem 5.1 (Greb–Kebekus–Kovacs, [6, Cor. 3.2])**.** *Let Y be a normal variety. Fix an integer $k > 0$. Suppose that there exists a normal variety W and a finite surjective morphism $\gamma : W \to Y$, such that for any resolution of singularities $p : \widetilde{W} \to W$ with simple normal crossing (snc) exceptional locus $F = Exc(p)$ the sheaf*

$$p_*\mathrm{Sym}^k\Omega^1_{\widetilde{W}}(log F)$$

is reflexive. Then for any resolution of singularities $\pi : \widetilde{Y} \to Y$ with snc exceptional locus $E = Exc(\pi)$ the sheaf $\pi_*\mathrm{Sym}^k\Omega^1_{\widetilde{Y}}(log\, E)$ is reflexive.

Remark 5.2. In [6] the property of the sheaf $\pi_*\mathrm{Sym}^k\Omega^1_{\widetilde{Y}}(log\, E)$ being reflexive for any such resolution of singularities of Y is stated by saying that "Extension theorem holds for Sym^k-forms on Y", and is considered in greater generality for *reflexive tensor operations* on differential 1-forms on logarithmic pairs (Y,Δ).

Theorem 5.3. *Let X be a smooth projective variety, and suppose that X is birational to a quotient of an abelian variety A by a finite group G, such that the quotient map is étale in codimension 1. If $Y = A/G$ is singular, then the cotangent bundle Ω_X is not asymptotically generically generated.*

Proof. We apply Theorem 5.1 to Y and the finite map

$$\gamma : A \to Y = A/G\,.$$

As A is smooth, the hypothesis trivially apply. Let us call $\pi : \widetilde{Y} \to Y$ a resolution of singularities of Y, with snc exceptional divisor E, then for any $m > 0$ the sheaf $\pi_*\mathrm{Sym}^m\Omega^1_{\widetilde{Y}}(log\, E)$ is reflexive. Let $F \subset Y$ be the locus where the map γ is not étale, this locus having codimension at least 2. So

$$H^0\Big(Y,\pi_*\mathrm{Sym}^m\Omega^1_{\widetilde{Y}}(log\, E)\Big) = H^0\Big(Y \setminus (Y^{sing} \cup F),\pi_*\mathrm{Sym}^m\Omega^1_{\widetilde{Y}}(log\, E)\Big)$$

and we have the following inclusions for all $m > 0$:

$$\begin{aligned} H^0(\widetilde{Y},\mathrm{Sym}^m\Omega_{\widetilde{Y}}) &\subseteq H^0(\widetilde{Y},\mathrm{Sym}^m\Omega_{\widetilde{Y}}(log\, E)) = H^0\Big(Y,\pi_*\mathrm{Sym}^m\Omega^1_{\widetilde{Y}}(log\, E)\Big) \\ &= H^0\Big(Y \setminus (Y^{sing} \cup F),\pi_*\mathrm{Sym}^m\Omega^1_{\widetilde{Y}}(log\, E)\Big) \subseteq H^0(A,\mathrm{Sym}^m\Omega_A)^G \subseteq H^0(A,\mathrm{Sym}^m\Omega_A) \end{aligned}$$

(the inclusion $H^0\Big(Y \setminus (Y^{sing} \cup F),\pi_*\mathrm{Sym}^m\Omega^1_{\widetilde{Y}}(log\, E)\Big) \subseteq H^0(A,\mathrm{Sym}^m\Omega_A)^G$ holding by pulling back symmetric forms on $Y \setminus (Y^{sing} \cup F)$ to G-invariant symmetric forms on $A \setminus \gamma^{-1}(Y^{sing} \cup F)$ and extending them to A).

Now, suppose by contradiction that X has asymptotically generically generated cotangent bundle Ω_X, then the same holds for the cotangent bundle $\Omega_{\widetilde{Y}}$ as $\widetilde{Y}$ is smooth and birational to X, so for some $k > 0$:

$$\mathrm{rk}\big(\mathrm{Sym}^k\Omega_{\widetilde{Y}}\big) \leqslant \dim H^0\big(\widetilde{Y}, \mathrm{Sym}^k\Omega_{\widetilde{Y}}\big) \leqslant \dim H^0\big(A, \mathrm{Sym}^k\Omega_A\big) = \mathrm{rk}\big(\mathrm{Sym}^k\Omega_A\big),$$

but the two vector bundles have the same rank, so all the inclusions above are equalities, in particular

$$H^0\big(A, \mathrm{Sym}^k\Omega_A\big)^G = H^0\big(A, \mathrm{Sym}^k\Omega_A\big) \tag{5.1}$$

and

$$H^0\big(A, \mathrm{Sym}^k\Omega_A\big) \cong H^0\big(\widetilde{Y}, \mathrm{Sym}^k\Omega_{\widetilde{Y}}\big) = H^0\big(\widetilde{Y}, \mathrm{Sym}^k\Omega_{\widetilde{Y}}(log\, E)\big). \tag{5.2}$$

Let us remark that by Theorem 4.1 above (and its proof), we know that if $Y = A/G$ is smooth then G acts by translations on A and indeed the inclusions above are equalities in this case.

Let us show that if $Y = A/G$ is singular the equalities above cannot hold.

First, notice that we can apply the same arguments as in the proof of Theorem 4.1 and show that $H^0\big(A, \mathrm{Sym}^k\Omega_A\big)^G = H^0\big(A, \mathrm{Sym}^k\Omega_A\big)$ implies that G acts on $H^0\big(A, \Omega_A\big)$ by homothethies, i.e. the action $G \to GL\big(H^0\big(A, \Omega_A\big)\big)$ is given by $g \mapsto \chi_g Id_{H^0(A,\Omega_A)}$ for some character $\chi : G \to \mathbb{C}^*$.

Therefore we can study in detail the local structure of the singularities and their resolutions. Let us observe that the singular points of $Y = A/G$ are isolated, and can be resolved by one blowing-up each singular point.

In fact if a point $a \in A$ has a non trivial stabilizer $\mathrm{Stab}_a \subseteq G$, the action $\mathrm{Stab}_a \to GL\big(H^0\big(A, \Omega_A\big)\big)$ is faithful, as we can assume that the action of G on A is faithful, and a non-trivial element $g \in G$ that acts trivially on $GL\big(H^0\big(A, \Omega_A\big)\big)$ would be a translation and could not be in Stab_a. Then the stabilizer Stab_a is a cyclic group, as it injects in $\mathbb{C}^* \cdot Id_{H^0(A,\Omega_A)}$.

So the stabilizer is a cyclic group that acts on a sufficiently small coordinate neighborhood of a as mutiplication on the coordinates by roots of unity,

$$g : \big(u_1, \dots, u_n\big) \mapsto \big(\chi_g u_1, \dots, \chi_g u_n\big)$$

therefore the point a is the only point in the neighborhood that is stabilized by an element of G, and the image of a in Y is an isolated cyclic quotient singularity of type $\frac{1}{r}(1, \dots, 1)$, with r being the order of the stabilizer (cf. [18, 4.2]).

This kind of singularities are are known to be isomorphic to cones on a Veronese variety and to be resolved by a single blowing up, furthermore if we write this quotient singularity as $\mathbb{C}^n \to \mathbb{C}^n/\mathbb{Z}_r$, then the quotient map lifts to a map from the blowing up of $\mathbb{C}^n$ in 0 to the resolution of the singularity of $\mathbb{C}^n/\mathbb{Z}_r$. We carry out the proof of these facts, which are well known to experts, in Remark 5.4 below.

Now we can study the differentials on the resolution of singularities by considering a commutative diagram of the following form:

$$\begin{array}{ccc} \widetilde{A} & \xrightarrow{f} & \widetilde{Y} \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle \pi} \\ A & \xrightarrow{\gamma} & Y \end{array} \tag{5.3}$$

where π is the resolution of Y by blowing up the finite number of points in Y^{sing}, γ is the quotient map, $\widetilde{A}$ is the blowing-up of A in the points $\gamma^{-1}(Y^{sing})$, and f a lift of the quotient map γ to a map $\widetilde{A} \to \widetilde{Y}$. Furthermore f is a covering of degree $|G|$ ramified along the exceptional locus of p, and the action of G on A extends to an action of G on $\widetilde{A}$, whose quotient is $f : \widetilde{A} \to \widetilde{Y}$.

To construct the diagram and prove the statements above, we can provide first a local description and then check that the action of G extends globally to $\widetilde{A}$ and that its quotient is $\widetilde{Y}$.

First notice that since a lift of the map γ exists locally around the points in $\gamma^{-1}(Y^{sing})$ (according to Remark 5.4 below), then it exists globally such a map $f : \widetilde{A} \to \widetilde{Y}$.

Then observe that, by the slice theorem (cf. [4, paragraph 2], for the use we make of Luna's étale slice theorem proven in [15]), we can choose a local neighborhood U of a point $x \in A$ such that the quotient U/Stab_x is a neighborhood of $\gamma(x)$.

Let us construct this explicitly for a point $a \in A$ with non-trivial stabilizer: we can choose a sufficiently small coordinate neighborhood $U_a \subset A$ around a point $a \in A$ with nontrivial stabilizer $\mathrm{Stab}_a \subseteq G$, in such a way that U_a is stable for the action of Stab_a, and that all of its translates by other elements of G are pairwise disjoint. Then for an element $g \in G \setminus \mathrm{Stab}_a$, the other point $g \cdot a \in A$ has the conjugated of Stab_a as stabilizer, the quotient of the neighborhood U_a/Stab_a is isomorphic to the quotient $(g \cdot U_a)/g \cdot \mathrm{Stab}_a \cdot g^{-1}$, and these quotients are identified to neighborhoods of the point $\gamma(a) \in Y$.

Now an element in the stabilizer acts on U_a by multiplying all coordinates by the same root of the unity, so we can lift the action of Stab_a on U_a to the blowing-up $\widetilde{U_a}$ of the point a, and remark that the exceptional divisor is fixed by this action, so the quotient $\widetilde{U_a}/\mathrm{Stab}_a$ is isomorphic to the resolution of the singularity of the neighborhood $\gamma(U_a)$ obtained by blowing up the singular point. If an element g is not in the stabilizer Stab_a, it will give an isomorphism between $\widetilde{U_a}$ and $\widetilde{g \cdot U_a}$. Repeating this argument with all points having a non-trivial stabilizer and shrinking the open neighborhoods if needed, we can see that the action of G on A extends to an action on $\widetilde{A}$ (in fact, we are lifting the action of G to the blowing up of a G-invariant subset).

The quotient $\widetilde{A}/G$ is normal, the map $f : \widetilde{A} \to \widetilde{Y}$ is clearly G-invariant so it factors through $\widetilde{A}/G$, and the map $\widetilde{A}/G \to \widetilde{Y}$ is a bijective map between normal varieties (locally it is described above and in Remark 5.4) therefore it is an isomorphism. So quotient map for the action of G on $\widetilde{A}$ is $f : \widetilde{A} \to \widetilde{Y}$.

In order to describe the map f, let us observe that we can choose local coordinates $(u_1, \dots, u_n) \in A$ around a point $a \in A$ with non trivial stabilizer, in such a way that the action of Stab_a on A is given by

$$g \cdot (u_1, \dots, u_n) = (\chi_g u_1, \dots, \chi_g u_n).$$

And we can choose local coordinates in the blowing-up $(x_1, w_2 \dots, w_n) \in \widetilde{A}$ around a point $x \in \widetilde{A}$ in the exceptional divisor, with $p(x) = a \in A$, in such a way that the blowing-up map $p : \widetilde{A} \to A$ is given in local coordinates by

$$(x_1, w_2 \dots, w_n) \mapsto (x_1, x_1 w_2 \dots, x_1 w_n),$$

i.e. the exceptional divisor $Exc(p)$ is given locally by the equation $x_1 = 0$, and the blowing up is defined locally by $w_i = u_i/u_1$.

Then the action of $g \in \mathrm{Stab}_a$ on A lifts to an action on $\widetilde{A}$ which is given in local coordinates by:

$$g \cdot (x_1, w_2 \dots, w_n) = (\chi_g x_1, w_2 \dots, w_n),$$

with fixed divisor $x_1 = 0$. Therefore we have a covering $f : \widetilde{A} \to \widetilde{Y}$ ramified along the divisor $x_1 = 0$, that in those local on $\widetilde{A}$ and local coordinates $(y_1, \dots, y_n)$ on $\widetilde{Y}$ is given by:

$$(x_1, w_2 \dots, w_n) \mapsto (x_1^m, w_2 \dots, w_n).$$

Now, by Lemma 2.8, as p is birational and A and $\widetilde{A}$ are smooth, p^* is an isomorphism. Finally, the equalities (5.2) above show that in the following diagram the other vertical map and γ^* are isomorphisms:

$$\begin{tikzcd}
H^0(\widetilde{A}, \mathrm{Sym}^k \Omega_{\widetilde{A}}) & H^0(\widetilde{Y}, \mathrm{Sym}^k \Omega_{\widetilde{Y}}) \arrow[l, hook', "f^*"'] \arrow[d, "\cong"] \\
H^0(A, \mathrm{Sym}^k \Omega_A) \arrow[u, "p^*"] & H^0(Y, \pi_* \mathrm{Sym}^k \Omega_{\widetilde{Y}}(log E)) \arrow[l, hook', "\gamma^*"']
\end{tikzcd}$$

However we see that

$$f^* : H^0(\widetilde{Y}, \mathrm{Sym}^k \Omega_{\widetilde{Y}}) \to H^0(\widetilde{A}, \mathrm{Sym}^k \Omega_{\widetilde{A}})$$

cannot be an isomorphism: in fact, given the local coordinates above, we can choose a base $du_1, \dots, du_n$ of $H^0(A, \Omega_A)$. Now the holomorphic symmetric differential $(du_1)^k$ is pulled back to $(dx_1)^k \in H^0(\widetilde{A}, \mathrm{Sym}^k \Omega_{\widetilde{A}})$, and this cannot be the pull-back of a holomorphic symmetric differential in $H^0(\widetilde{Y}, \mathrm{Sym}^k \Omega_{\widetilde{Y}})$, in fact $f^* dy_1 = m x_1^{m-1} dx_1$ vanishes along the divisor $x_1 = 0$, and $f^* dy_j = dw_j$ for $j = 2, \dots, n$ so no holomorphic symmetric differential on $\widetilde{Y}$ can pull back to $(dx_1)^k$. $\square$

Remark 5.4. Let us show here the properties of the singularity of type $\frac{1}{r}(1, \dots, 1)$ stated in the proof of the theorem above. A general reference for these matters is Reid's Young Person's Guide [18].

First let us remark that the singularity is a cone on a Veronese variety: the action is that of the group G of rth roots of unity on $\mathbb{C}^n$ where the generator $\mu = \exp(2\pi i/r)$ acts diagonally by $\mu.(x_1, \dots, x_n) = (\mu x_1, \dots, \mu x_n)$. The invariant polynomials for this action are generated by all monomials X_I of degree r in $X_1, \dots, X_n$. Therefore, the quotient $\mathbb{C}^n \to \mathbb{C}^n/G$ is given by the ring injection

$$\mathbb{C}[X_1, \dots, X_n]^G = \mathbb{C}[X_I \mid I = (i_1, \dots, i_n), i_1 + \dots + i_n = r] \subset \mathbb{C}[X_1, \dots, X_n].$$

Now the affine variety corresponding to the algebra generated by all degree r monomials is a cone on the degree r Veronese image of $\mathbb{P}^{n-1}$ in $\mathbb{P}^{N-1}$, in other words it is the affine cone in $\mathbb{C}^N$ corresponding to the homogeneous ideal of the image of $\mathbb{P}^{n-1}$ in $\mathbb{P}^{N-1}$, where N is the number of monomials of degree r in n variables. Therefore, to resolve the singularity it is enough to blow up the vertex $(0, \dots, 0) \in \mathbb{C}^N$.

Let us consider the blowing up of the point $(0, \dots, 0) \in \mathbb{C}^N$, the blowing up of the point $(0, \dots, 0) \in \mathbb{C}^n$, and the following diagram:

$$\begin{tikzcd} \widetilde{\mathbb{C}^n} \arrow[d, "p"'] & \widetilde{\mathbb{C}^N} \arrow[d, "\pi"] \\ \mathbb{C}^n \arrow[r, "\gamma"] & \mathbb{C}^N \end{tikzcd}$$

The maximal ideal corresponding to the point $(0, \dots, 0) \in \mathbb{C}^N$ has inverse image ideal by γ the power $(X_1, \dots, X_n)^r \subset \mathbb{C}[X_1, \dots, X_n]$, and inverse ideal sheaf by $\gamma \circ p$ the invertible sheaf $\mathcal{O}_{\mathbb{C}}(-rE)$, where E is the exceptional divisor of p: in fact in local coordinates $(x_1, w_2, \dots, w_n)$ around a point in E, where E is given by the equation $x_1 = 0$, the map p is given by $(x_1, w_2 \dots, w_n) \mapsto (x_1, x_1 w_2 \dots, x_1 w_n)$, the quotient map γ is given by $(x_1, \dots x_n) \mapsto (x_I)$, so the inverse ideal sheaf is given locally by $x_1^r = 0$.

Therefore by the universal property of blowing up (cf. [9, Prop. 7.14, Ch. II]) the map $\gamma \circ p$ factors through the blowing up π, and in particular gives a map $\widetilde{\mathbb{C}^n} \to \widetilde{\mathbb{C}^n/G}$, which is a cyclic r-cover ramified over E. This local construction can be extended globally to construct the commutative diagram (5.3) above, as detailed in the proof of Theorem 5.3.

This completes the proof of Theorem 1.1. Corollary 1.2 follows as the Minimal Model Program holds in dimension 3, together with abundance conjecture (cf. [13]).

6 | REMARKS AND EXAMPLES

6.1 | Compact Kähler case

We remark that most of the techniques used hold in the projective case, in particular Theorem 3.1, and this is the reason for stating the main results for smooth projective varieties.

However the theorem proved in [17], characterizing abelian varieties as those varieties of Kodaira dimension 0 having some symmetric power of the cotangent bundle globally generated, works as well to characterize complex tori (biholomorphically) among compact Kähler manifolds. This construction has been carried out explicitely in the recent work [16] together with some considerations on compact complex manifolds, so the main question of having a *bimeromorphic* characterisation through symmetric differentials makes sense also for compact Kähler or compact complex manifolds.

According to [11] the MMP should work for Kähler manifolds, and does work in dimension 3, with a suitable characterization of torus quotients, so in dimension 3 the same characterization for compact Kähler manifolds bimeromorphic to complex tori should hold. Furthermore a characterisation of 3-dimensional compact complex manifolds which arise as quotients of complex tori by finite groups has been given in [5], and it would be interesting to prove a bimeromorphic characterisation in dimension 3 using these results.

In any case it is worth asking whether a similar bimeromorphic characterisation of complex tori holds in any dimension. We leave this to future investigations.

6.2 | Higher Kodaira dimension

In [10], Höring gives a classification of varieties X of any Kodaira dimension $k(X)$ having (strongly) semiample cotangent bundle Ω_X, in particular he proves that these varieties are finite étale quotients of a product of an abelian variety and a variety Y with ample canonical bundle and same Kodaira dimension as X.

In case of Kodaira dimension 0 we proved in [17] that we do not need to look at étale quotients, and here we prove that we have a birational characterization as well. It would be interesting to look at varieties with higher Kodaira dimension and AGG cotangent bundle as well.

6.3 | Kummer varieties

The Kummer surface, being a $K3$-surface, in known not to have any holomorphic symmetric differentials (cf. [14]). In fact Kummer surfaces are defined as resolution of singular quotients of abelian surfaces by the group $G = \{\pm 1\}$, therefore Theorem 5.3 applies: the cotangent bundle and its symmetric powers are not generically generated. In this case there is the advantage that the action of the finite group $G = \{\pm 1\}$ is very easy to describe, in particular it is trivial on the holomorphic sections of all even symmetric powers $\mathrm{Sym}^{2k}\Omega_A$, therefore many of the equalities appearing in the proof of Theorem 5.3 hold. We think it may be useful to follow explicitly the constructions studied above, and observe how the various spaces of holomorphic differentials behave in this case. In particular we can understand clearly the reason why there is an isomorphism of the spaces of logarithmic symmetric differentials (of even degree) on the Kummer surface and holomorphic symmetric differentials on the abelian surface, while there are no holomorphic symmetric differentials on the Kummer surface.

Furthermore this example can be studied in the higher dimensional case as well, considering quotients of abelian varieties of any dimension by the group $G = \{\pm 1\}$.

Example 6.1. Let A be an abelian surface, $Y = A/\{\pm 1\}$, and $\pi : \widetilde{X} \to Y$ the resolution of singularities, where $\widetilde{X}$ is a K3 surface. The singular surface Y is not terminal, however it does have klt singularities. Then we can apply Theorem 5.1 to the map $\pi : \widetilde{X} \to Y$. We have a diagram as in the proof of Theorem 5.3:

$$\begin{array}{ccc} \widetilde{A} & \xrightarrow{f} & \widetilde{X} \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle \pi} \\ A & \xrightarrow{\gamma} & Y \end{array}$$

with $p : \widetilde{A} \to A$ the blowing up of A in the 16 points of order 2, f covering of degree 2, ramified over the exceptional locus of p. For $m = 2$ we have:

$$H^0\big(\widetilde{X}, \mathrm{Sym}^2\Omega_{\widetilde{X}}(log\, E)\big) = H^0\big(Y \setminus Y^{sing}, \mathrm{Sym}^2\Omega_Y\big)$$

$$= H^0\big(A, \mathrm{Sym}^2\Omega_A\big)^{\pm 1} = H^0\big(A, \mathrm{Sym}^2\Omega_A\big) = H^0\big(\widetilde{A}, \mathrm{Sym}^2\Omega_{\widetilde{A}}\big).$$

Now, following the same notations notations as in the proof Theorem 5.3, we see that a basis du_1, du_2 of $H^0\big(A, \Omega_A\big)$ pulls back to the basis of $H^0\big(\widetilde{A}, \Omega_{\widetilde{A}}\big)$ that locally looks like $dx_1, w_2 dx_1 + x_1 dw_2$.

The basis $du_1^2, du_1 du_2, du_2^2$ of $H^0\big(A, \mathrm{Sym}^2\Omega_A\big)$ is invariant for the action of $\{\pm 1\}$, and pulls back to a basis of $H^0\big(\widetilde{A}, \mathrm{Sym}^2\Omega_{\widetilde{A}}\big)$.

Now clearly dx_1^2 is not coming from a symmetric differential in $H^0\big(\widetilde{X}, \mathrm{Sym}^2\Omega_{\widetilde{X}}\big)$ as $f^* dy_1 = 2x_1 dx_1$. However $f^*\left(\frac{1}{y_1} dy_1^2\right) = 4dx_1^2$, so we can see the reason why

$$p^* H^0\big(A, \mathrm{Sym}^2\Omega_A\big) \cong f^* H^0\big(\widetilde{X}, \mathrm{Sym}^2\Omega_{\widetilde{X}}(log\, E)\big)$$

Example 6.2. In the same way, given an abelian variety A of dimension at least 3, a Kummer variety $Y = A/\{\pm 1\}$ has isolated singularities, and is terminal, so we can follow the same constructions as above. In that case the second symmetric power of the cotangent bundle cannot be generically generated, however we have that

$$p^*H^0(A, \mathrm{Sym}^2\Omega_A) \cong f^*H^0(\widetilde{X}, \mathrm{Sym}^2\Omega_{\widetilde{X}}(log\, E))$$

as in the 2-dimensional case.

Remark 6.3. If we consider exterior powers instead of symmetric differentials, then we do not need to look at the logarithmic complex: in [7] it is proven that for any KLT variety X with log resolution $\pi : \widetilde{X} \to X$ and for any $1 \leqslant p \leqslant \dim X$ the sheaf $\pi_*\Omega^p_{\widetilde{X}}$ is reflexive, therefore:

$$H^0(X \setminus X^{sing}, \Omega^p_X) \cong H^0\left(\widetilde{X}, \Omega^p_{\widetilde{X}}\right).$$

This is not the case for symmetric powers, as it is shown in Examples 6.1 and 6.2 above: given an abelian surface A and the corresponding Kummer surface $\widetilde{X}$, resolution of the quotient $Y = A/\{\pm 1\}$, we have:

$$H^0(A, \Omega^2_A) \cong H^0(A, \Omega^2_A)^{\{\pm 1\}} \cong H^0(Y \setminus Y^{sing}, \Omega^2_Y) \cong \mathbb{C} \cong H^0\left(\widetilde{X}, \Omega^2_{\widetilde{X}}\right),$$

however

$$\mathbb{C}^3 \cong H^0(Y \setminus Y^{sing}, \mathrm{Sym}^2\Omega^1_Y) \cong H^0(A, \mathrm{Sym}^2\Omega^1_A) \neq H^0(X, \mathrm{Sym}^2\Omega^1_X) = 0,$$

and similarly in the higher dimensional case.

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