$$f(x) = \exp\left\{ \frac{|x+2|}{x+3} \right\}$$

$\mathrm{dom}(f) = \mathbb{R} \setminus \{-3\}$ però posso estendere f a $\mathbb{R}$

poiché $\lim_{x \to -3^-} f(x) = 0$

$$\underline{f : \mathbb{R} \longrightarrow \mathbb{R}}$$

$$f'(x) = e^{\frac{|x+2|}{x+3}} \quad \frac{|x+2|}{(x+3)^2} \quad \frac{1}{x+2}$$

$x \neq -2$

$$\mathrm{dom}\, f' = \mathbb{R} \setminus \{-3, -2\}$$

posso estendere f' in -3

$$\lim_{x \to -3^-} f'(x) = 0$$

ma non in -2

$$\lim_{x \to -2^-} f'(x) = -1 \qquad \lim_{x \to -2^+} f'(x) = 1$$

$$f'(x) = e^{\frac{|x+2|}{x+3}} \quad \frac{|x+2|}{(x+3)^2} \quad \frac{1}{x+2}$$

$$f'(x) = \begin{cases} -e^{\frac{|x+2|}{x+3}} \dfrac{1}{(x+3)^2} & \text{se } x < -2 \\ e^{\frac{|x+2|}{x+3}} \dfrac{1}{(x+3)^2} & \text{se } x > -2 \end{cases}$$

segno di f'

$f' > 0$ per $x \in (-2, +\infty)$

$f' < 0$ per $x \in (-\infty, -3) \cup (-3, -2)$

f' si annulla in -3

(f ha derivata sinistra in -3)

$$f''(x) \overset{Ex}{=} \dots = \frac{f(x)}{(x+3)^2}\left[\frac{1}{(x+3)^2} - \frac{2|x+2|}{(x+2)(x+3)}\right]$$

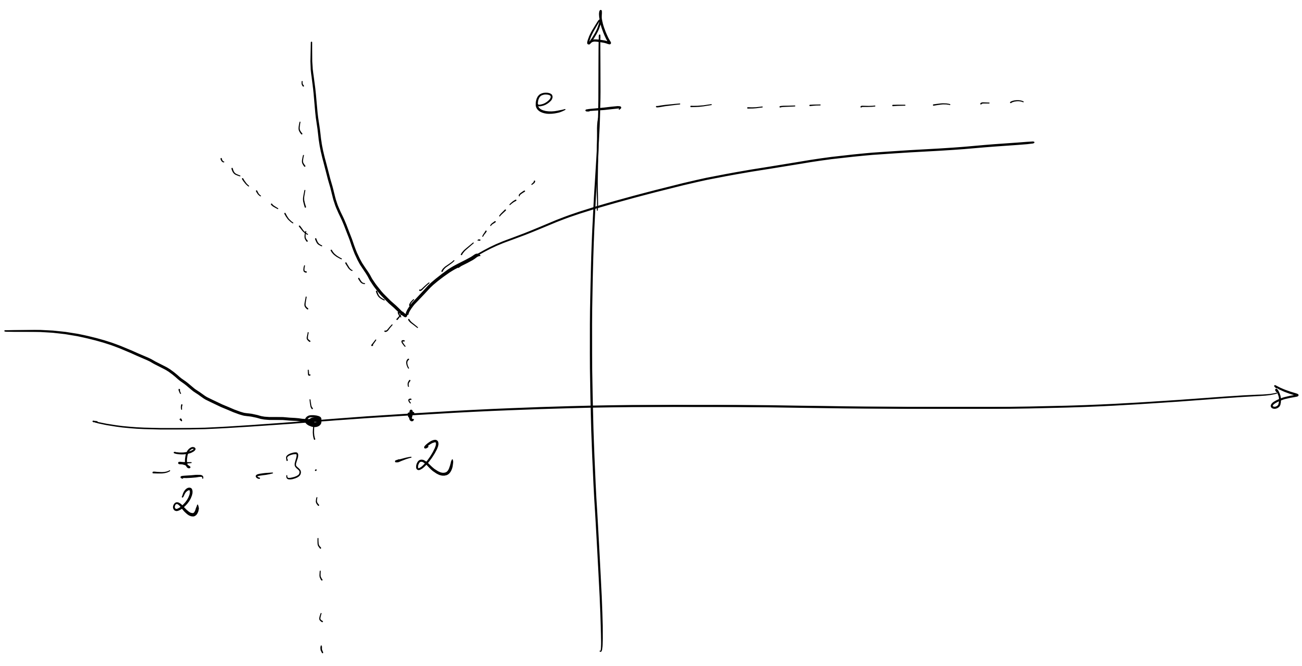
$f'' < 0$ in $\left(-\infty, -\frac{7}{2}\right) \cup (-2, +\infty)$

$f''\left(-\frac{7}{2}\right) = 0$

$f'' > 0$ in $\left(-\frac{7}{2}, -3\right) \cup (-3, -2)$

$$f'(x) = e^{\frac{|x+2|}{x+3}} \; \frac{|x+2|}{(x+3)^2} \; \frac{1}{x+2}$$

$$f'(x) = \begin{cases} -e^{\frac{|x+2|}{x+3}} \dfrac{1}{(x+3)^2} & \text{se } x < -2 \\ e^{\frac{|x+2|}{x+3}} \dfrac{1}{(x+3)^2} & \text{se } x > -2 \end{cases}$$



Si studi $f(x) = \left| -x - \operatorname{arctg} \frac{x+1}{x} \right|$

$\operatorname{dom}(f) = \mathbb{R} \setminus \{0\}$

$g(x) = -x - \operatorname{arctg} \frac{x+1}{x}$

$\lim_{x \to -\infty} g(x) = +\infty$ $\lim_{x \to +\infty} g(x) = -\infty$

$\lim_{x \to 0^-} g(x) = \pi/2$ $\lim_{x \to 0^+} g(x) = -\pi/2$

$$g'(x) = -\frac{2x(x+1)}{x^2 + (x+1)^2}$$

$$g' \begin{cases} > 0 & \text{in } (-1, 0) \\ = 0 & \text{in } -1 \\ < 0 & \text{in } (-\infty, -1) \cup (0, +\infty) \end{cases}$$

-1 punto di minimo locale per g

$g(-1) = 1 > 0 \implies g > 0$ in $(-\infty, 0)$

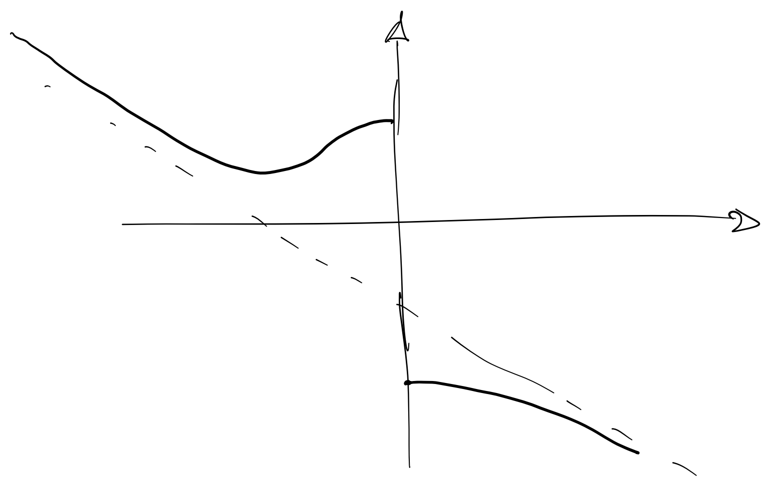
$g < 0$ in $(0, +\infty)$

$$g''(x) = \left(\frac{1}{x^2 + (x+1)^2}\right)^2 (-4x-2)$$

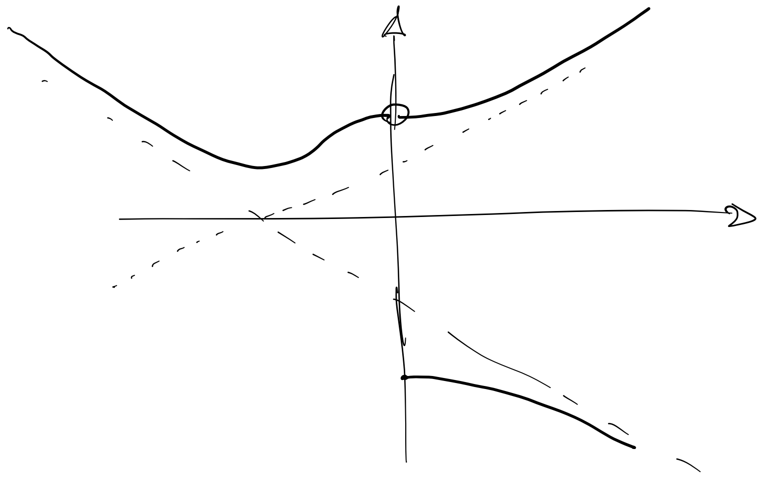
$$g'' \begin{cases} > 0 & \text{in } (-\infty, -1/2) \\ = 0 & \text{in } -\frac{1}{2} \\ < 0 & \text{in } \left(-\frac{1}{2}, 0\right) \cup (0, +\infty) \end{cases}$$

Asintoti: a $-\infty$ e a $+\infty$ $\quad -x - \pi/4$

g



f



f può essere estesa ad una funzione continua su $\mathbb{R}$

$C^0(\mathbb{R})$

$\left(f(0) = \pi/2 \right)$

$$\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^+} f'(x) = 0$$

f può essere estesa ad una funzione di classe $C^1(\mathbb{R})$

$\left(f(0) = \pi/2 \right)$

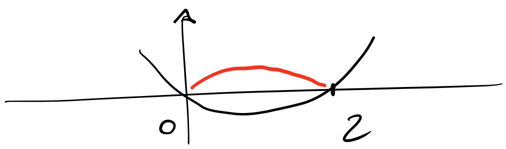
$$\lim_{x \to 0^-} f''(x) = -2 \qquad \lim_{x \to 0^+} f''(x) = +2$$

$$f \notin C^2(\mathbb{R})$$

Si studi $f(x) = |x^2 - 2x| \, e^x$

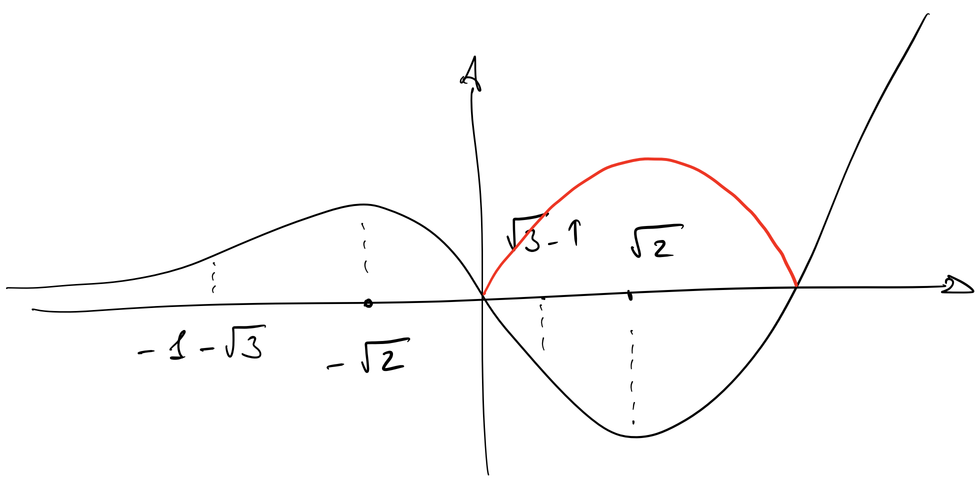
Anziché f posso studiare $g(x) = (x^2 - 2x)\, e^x$

$x^2 - 2x$



$|x^2 - 2x|$

g



f

Si studi $f(x) = (x \log|x| - x)^2$

OSS: $f(x) = x^2 (\log|x| - 1)$ è pari

$dom(f) = \mathbb{R} \setminus \{0\}$

$\lim_{x \to +\infty} f(x) = +\infty$ $\lim_{x \to 0+} f(x) = 0$

estendo f: $f: \mathbb{R} \to \mathbb{R}$, $f(0) = 0$

$$f' \begin{cases} > 0 & \text{in } (0,1) \cup (e, +\infty) \\ = 0 & \text{in } 1 \text{ e in } e \\ < 0 & \text{in } (1, e) \end{cases}$$

$$f'' \begin{cases} > 0 & \text{in } \left(0, e^{\frac{-\sqrt{5}-1}{2}}\right) \cup \left(e^{\frac{\sqrt{5}-1}{2}}, +\infty\right) \\ = 0 & \text{in } e^{\frac{-\sqrt{5}-1}{2}}, \text{ in } e^{\frac{\sqrt{5}-1}{2}} \\ < 0 & \text{in } \left(e^{\frac{-\sqrt{5}-1}{2}}, e^{\frac{\sqrt{5}-1}{2}}\right) \end{cases}$$

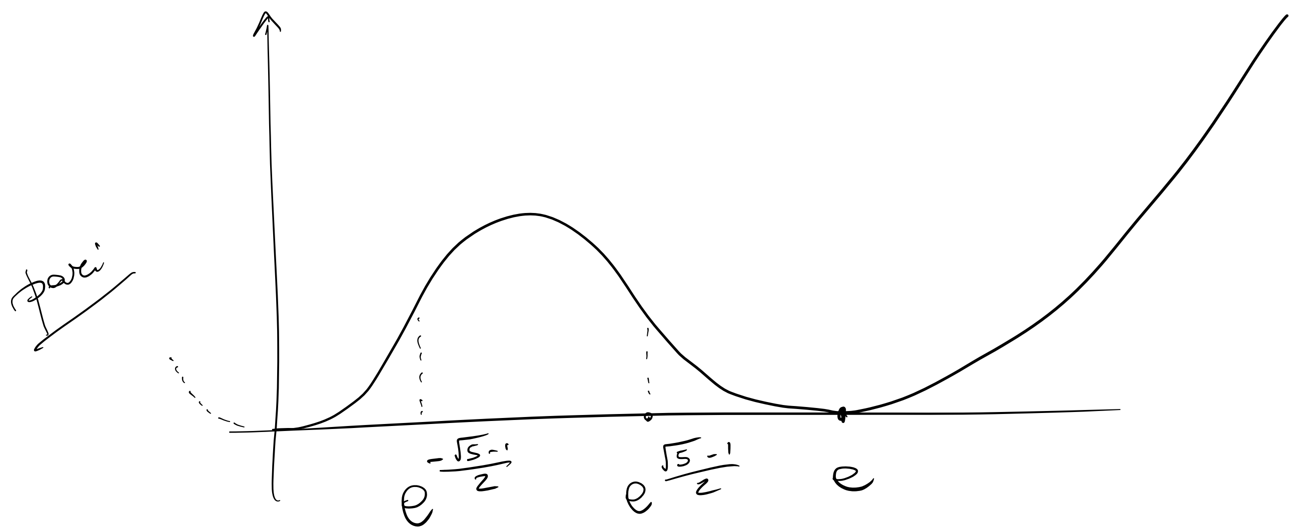
$$\lim_{x \to 0} f'(x) = 0$$

possiamo estendere non solo f ma anche f'

f' continua in $\mathbb{R}$

$$\boxed{f \in C^1(\mathbb{R})}$$

$$\lim_{x \to 0} f''(x) = +\infty \Rightarrow \boxed{f \notin C^2(\mathbb{R})}$$



Si studi $f(x) = \log|x-1| - \sqrt{|x|}$

f definita in $(-\infty, 1) \cup (1, +\infty)$

$$f'(x) = \frac{1}{x-1} - \frac{1}{2\sqrt{|x|}} \frac{|x|}{x} \qquad \mathrm{dom}(f') = \mathbb{R} \setminus \{0, 1\}$$

$$f'(x) = \begin{cases} \dfrac{2\sqrt{x} - x + 1}{2(x-1)\sqrt{x}} & \text{se } x > 0 \\[2ex] \dfrac{2\sqrt{-x} + x - 1}{2(x-1)\sqrt{-x}} & \text{se } x < 0 \end{cases}$$

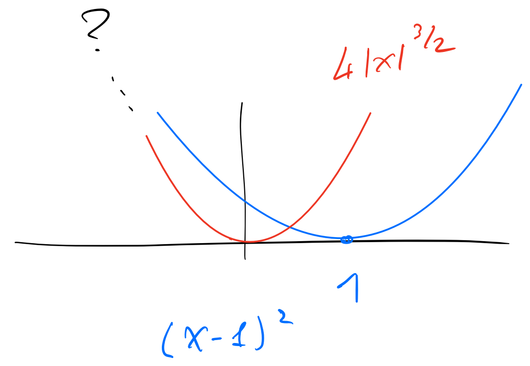
$$f' \begin{cases} < 0 & \text{in } (0,1) \cup (3+2\sqrt{2}, +\infty) \\ > 0 & \text{in } (-\infty, -1) \cup (-1, 0) \cup (1, 3+2\sqrt{2}) \\ = 0 & \text{in } -1 \text{ e in } 3+2\sqrt{2} \end{cases}$$

$$f''(x) = \frac{(x-1)^2 - 4|x|^{3/2}}{4|x|^{3/2}(x-1)^2}$$

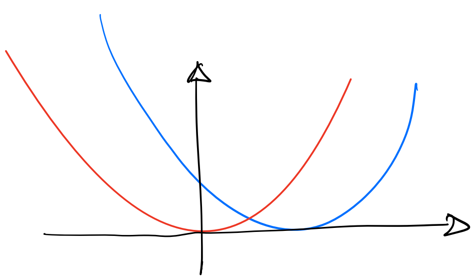
$x < 0$

segno di f'':

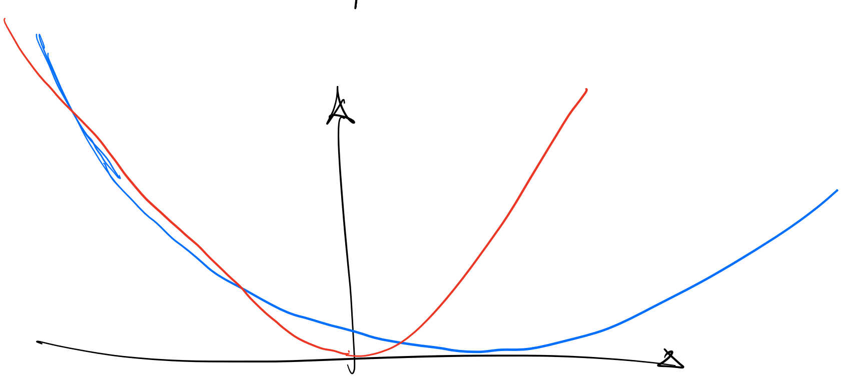
$$g(x) = \underbrace{(x-1)^2}_{g_1} - \underbrace{4|x|^{3/2}}_{g_2}$$



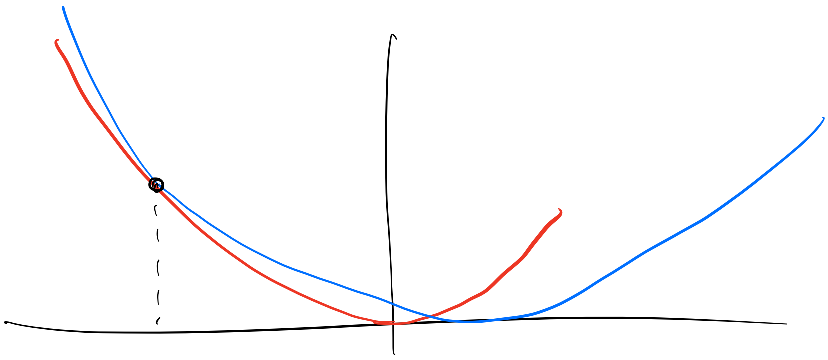
tre possibilità (per la convessità di g_1 e g_2)



g_1 e g_2 non sono mai uguali per $x<0$

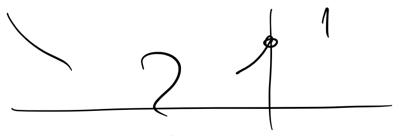


g_1 e g_2 sono uguali su due punti



$g_1 = g_2$ in un solo punto

Studio g in $(-\infty, 0)$.



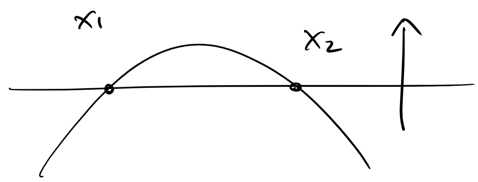
$$g'(x) = 2(x-1) + 6(-x)^{1/2}$$

$t = \sqrt{-x}$ $\qquad 2(t^2 - 3t + 1) = 0$

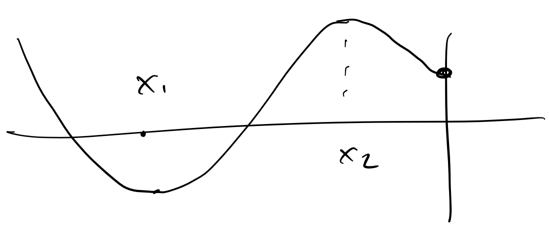
$\Updownarrow$

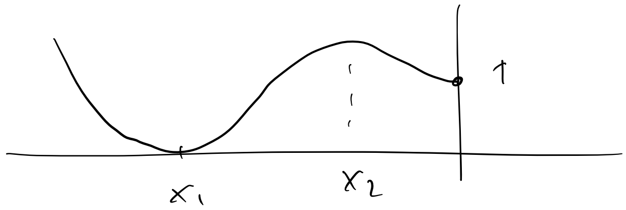
$x = -t^2$ $\qquad t = \frac{3}{2} \pm \frac{\sqrt{5}}{2}$

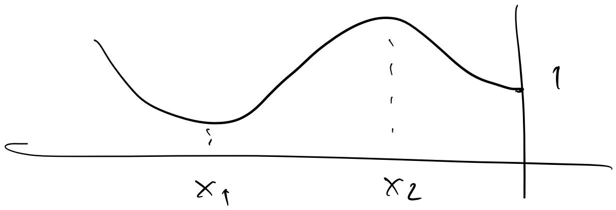
g'



g







Valutiamo g in $x_1 = -\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right)^2$

$$\boxed{g(x_1) < 0}$$

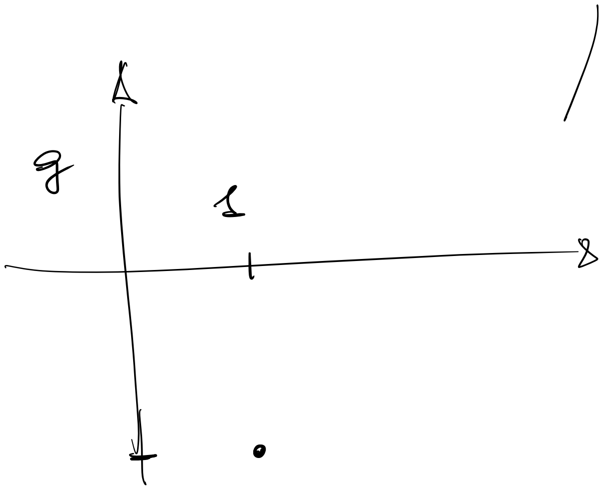
Conclusione:

due cambi di concavità per $x < 0$

$\boxed{x > 0}$ $\quad f''(x) = 0$ ha una sola soluzione per $x \in (0, 1)$

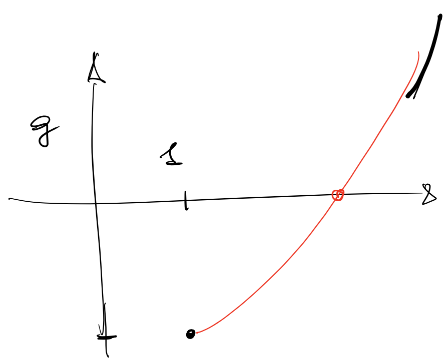
studiamo f'' (in realtà g) in $[1, +\infty)$

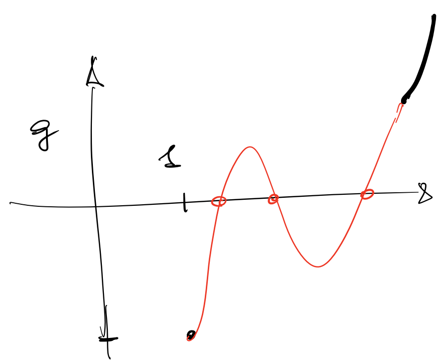
$$g'(x) = 2\left(x - 3\sqrt{x} - 1\right)$$

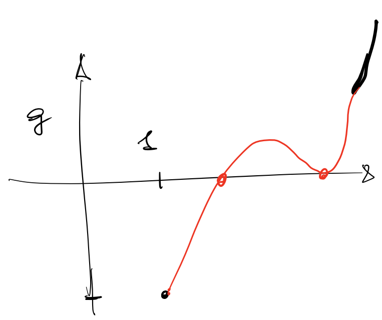


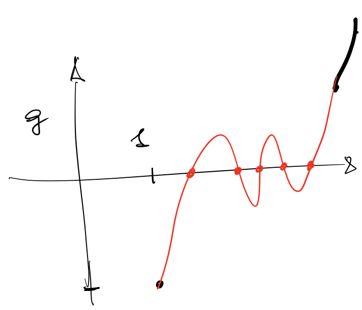
Dobbiamo scoprire quanti zeri ha g

che potrebbe essere fatta









$g(1) = -4$

$\lim_{x \to +\infty} g(x) = +\infty$

$\sqrt{x}^{-4} = t$

$x = t^2$

$2(t^2 - 3t - 1) = 0$

zeri $t_1 = \frac{3 - \sqrt{13}}{2} < 0$ $t_2 = \frac{3 + \sqrt{13}}{2}$

g ha un solo punto stazionario in $[1, +\infty)$

$\bar{x} = t_2^2$ può essere
- massimo locale ? No!
- minimo locale
- flesso

siccome $g'(1) < 0$ $\bar{x}$ è di minimo

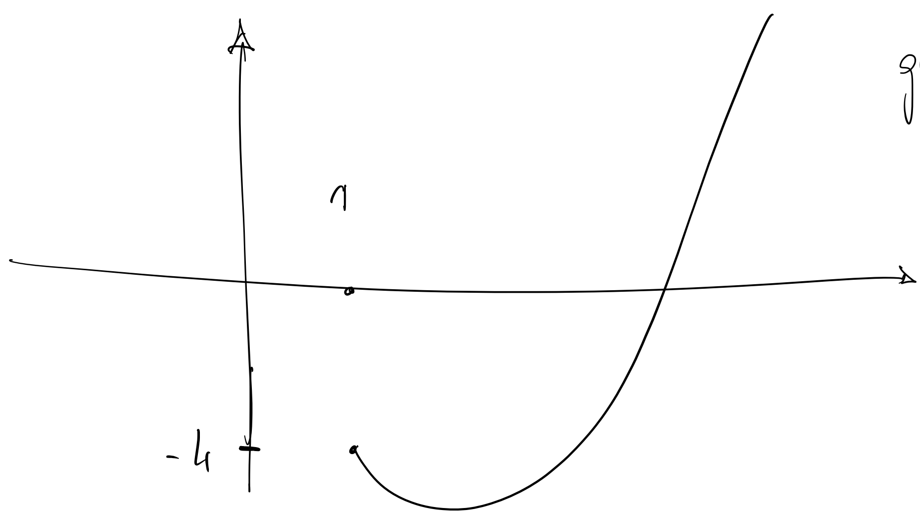
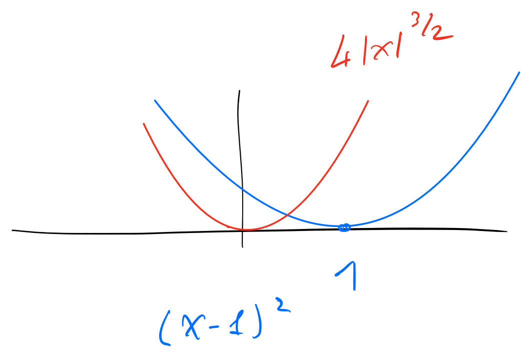


grafico di g

$\Rightarrow$ g (e quindi f'') si annulla in un solo punto $\bar{x} > 1$

(oltre che in un punto in $(0,1)$)

$x > 0$



sicuramente c'è un punto in cui f'' si annulla in $(0,1)$

ce ne deve essere anche un altro per $x \geq 1$

$f(-1) = 0$ $\qquad$ $f(3+2\sqrt{2}) < 0$

$\log(2+2\sqrt{2}) < \sqrt{3+2\sqrt{2}}$

$(\log(2+2\sqrt{2}))^2 < 3+2\sqrt{2}$

$\log(2+2\sqrt{2}) < \log 6 < \log e^2 = 2$

$$\lim_{x \to 0^-} f'(x) = +\infty \qquad \lim_{x \to 0^+} f'(x) = -\infty$$