CONFRONTI ASINTOTICI ESERCIZI

LUNEDÌ 12/11

Esercizio

Calcolare $\lim_{x\to 0} \dfrac{\operatorname{sen} x^2 + \ln(1+2x)}{x\cos x + \operatorname{sen}(3x)}$

Vogliamo riportarci ad una formulazione che ci permetta di utilizzare il Principio di Sostituzione

$\operatorname{sen} x^2 + \ln(1+2x) = f_1(x) + o(f_1(x))$

$x\cos x + \operatorname{sen}(3x) = g_1(x) + o(g_1(x))$

per $x \to 0$

Numeratore

$$\ln(1+y) = y + o(y) \quad \text{per } y \to 0$$

$$\Rightarrow \ln(1+2x) = 2x + o(x) \quad \text{per } x \to 0$$

$\downarrow 0$

$$\operatorname{sen} y = y + o(y) \quad \text{per } y \to 0$$

$$\operatorname{sen} x^2 = x^2 + o(x^2) = o(x) \quad \text{per } x \to 0$$

$$\operatorname{sen} x^2 + \ln(1+2x) = 2x + o(x) + o(x) = 2x + o(x)$$

Denominatore

$$x \cos x = x(1 + o(1)) = x + o(x) \quad \text{per } x \to 0$$

$$\operatorname{sen}(3x) = 3x + o(x) \quad \text{per } x \to 0$$

$$x \cos x + \operatorname{sen} 3x = 4x + o(x) \quad \text{per } \to 0$$

$$\lim_{x \to 0} \frac{\operatorname{sen} x^2 + \ln(1+2x)}{x \cos x + \operatorname{sen}(3x)} =$$

$$= \lim_{x \to 0} \frac{2x + o(x)}{4x + o(x)} \overset{\text{PdS}}{=} \lim_{x \to 0} \frac{2x}{4x} = \frac{1}{2}$$

$\to o(2x)$

$\to o(4x)$

Nota Bene: $c \neq 0$, numero reale

$$\varphi(x) = o(f(x)) \quad \text{per } x \to x_0$$

Allora $\varphi(x) = o(c\,f(x))$ per $\to x_0$

perché $\displaystyle\lim_{x \to x_0} \frac{\varphi(x)}{c f(x)} =$

$$= \lim_{x \to x_0} \frac{1}{c} \cdot \frac{\varphi(x)}{f(x)} = 0$$

0 perché $\varphi = o(f)$

Es.: . Se $\varphi(x) = o(4x)$ per $x \to 0$

$$\Rightarrow \varphi(x) = o\left(\frac{1}{4} \cdot 4x\right) = o(x) \quad \text{per } x \to 0$$

. Se $\varphi(x) = o(x)$ per $x \to 0$

$$\Rightarrow \varphi(x) = o(2x) \quad \text{per } x \to 0$$

Esercizio: Calcolare

$$\lim_{x \to 0^+} \frac{4x^2 \operatorname{sen}\sqrt{x} + (1-\cos x)^2}{\sqrt{x}\,\operatorname{senh} x^2 + (e^x - 1)^3}$$

Cerchiamo f_1 e g_1 tali che

$$4x^2 \operatorname{sen}\sqrt{x} + (1-\cos x)^2 = f_1(x) + o(f_1(x))$$

$$\sqrt{x} \operatorname{senh} x^2 + (e^x - 1)^3 = g_1(x) + o(g_1(x))$$

per $x \to 0$

Numeratore

$\operatorname{sen} y = y + o(y)$ per $y \to 0$

$$4x^2 \operatorname{sen}\sqrt{x} = 4x^2 \cdot \left(\sqrt{x} + o(\sqrt{x})\right) =$$

$$= 4x^{5/2} + o(x^{5/2}) \quad \text{per } x \to 0$$

$$(1-\cos x)^2 = \left(\frac{x^2}{2} + o(x^2)\right)^2 =$$

$\cos x = 1 - \frac{x^2}{2} + o(x^2)$

$$= \left(\frac{x^2}{2} + o(x^2)\right)^2 = \frac{x^4}{4} + 2\,\frac{x^2}{2}\cdot o(x^2) + \left(o(x^2)\right)^2 =$$

$$= \frac{x^4}{4} + o(x^4)$$

$$4x^2 \operatorname{sen}\sqrt{x} + (1-\cos x)^2 = 4x^{5/2} + o(x^{5/2}) + \underbrace{\frac{x^4}{4}}_{= o(x^{5/2})} + o(x^4) =$$

per $x \to 0$

$$= 4x^{5/2} + o(x^{5/2})$$

Denominatore

$\operatorname{senh} y = y + o(y)$ per $y \to 0$

$$\sqrt{x} \operatorname{senh} x^2 + (e^x - 1)^3$$

$$\operatorname{senh} x^2 = x^2 + o(x^2) \qquad \text{per } x \to 0$$

$$\sqrt{x}\, \operatorname{senh} x^2 = \sqrt{x}\,(x^2 + o(x^2)) = x^{5/2} + o(x^{5/2})$$

$$(e^x - 1)^3 = (x + o(x))^3 = x^3 + o(x^3)$$

$$\hookrightarrow \lim_{x\to 0} \frac{(e^x-1)^3}{x^3} = \lim_{x\to 0} \left(\frac{e^x-1}{x}\right)^3 = 1$$

$$\sqrt{x}\, \operatorname{sen} x^2 = x^{5/2} + o(x^{5/2}) + \underbrace{x^3}_{o(x^{5/2}) \text{ per } x\to 0} + o(x^3) =$$

$$= x^{5/2} + o(x^{5/2})$$

$$\lim_{x\to 0} \frac{4x^2 \operatorname{sen}\sqrt{x} + (1-\cos x)^2}{\sqrt{x}\,\operatorname{senh} x^2 + (e^x-1)^3} = \lim_{x\to 0} \frac{4x^{5/2} + o(x^{5/2})}{x^{5/2} + o(x^{5/2})} \overset{PdS}{=}$$

$$= \lim_{x\to 0} \frac{4x^{5/2}}{x^{5/2}} = 4$$

Esercizio Calcolare

$$\lim_{x\to 0} \frac{\operatorname{sen}(\cosh x - 1) + \operatorname{sen}(\cos x - 1)}{\cosh\left(\operatorname{senh}\left(\frac{x^2}{2}\right)\right) - \cosh(\operatorname{sen} x^2)}$$

Numeratore

$$\operatorname{sen}(\cosh x - 1) = \underbrace{\cosh x - 1}_{x^2/2 + o(x^2)} + \underbrace{o(\cosh x - 1)}_{o(x^2)}$$

$$\operatorname{sen}(\cos x - 1) = \underbrace{\cos x - 1}_{-x^2/2 + o(x^2)} + \underbrace{o(\cos x - 1)}_{o(x^2)}$$

Riportiamo!

$$\operatorname{sen}(\cosh x - 1) = \overbrace{\cosh x - 1}^{\frac{x^2}{2}+\frac{x^4}{4!}+o(x^4)} - \frac{1}{6}(\cosh x - 1)^3 +$$

$$+ o\left((\cosh x - 1)^3\right) =$$

$$= \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4) - \frac{1}{6}\left(\frac{x^2}{2} + o(x^2)\right)^3 +$$

$$+ o\left(\left(\frac{x^2}{2} + o(x^2)\right)^3\right) =$$

$$= \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4) - \frac{1}{6}\left(\underbrace{\frac{x^6}{8}}_{=o(x^4)} + o(x^6)\right) +$$

$$+ o\left(\left(\frac{x^6}{8} + o(x^6)\right)\right) =$$

$$= \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4)$$

$$\operatorname{sen}(\cos x - 1) = \overbrace{\cos x - 1}^{-\frac{x^2}{2}+\frac{x^4}{4!}+o(x^4)} - \frac{1}{6}(\cos x - 1)^3 +$$

$$+ o\left((\cos x - 1)^3\right) =$$

$$= -\frac{x^2}{2} + \frac{x^4}{4!} + o(x^4) - \frac{1}{6}\left(-\frac{x^2}{2} + o(x^2)\right)^3 +$$

$$+ o\left(\left(-\frac{x^2}{2} + o(x^2)\right)^3\right) =$$

$$= -\frac{x^2}{2} + \frac{x^4}{4!} + o(x^4) - \frac{1}{6}\left(-\frac{x^6}{8} + o(x^6)\right) +$$

$$+ o\left(-\frac{x^6}{8} + o(x^6)\right) =$$

$$= -\frac{x^2}{2} + \frac{x^4}{4!} + o(x^4)$$

$$\operatorname{sen}(\cosh x - 1) + \operatorname{sen}(\cos x - 1) =$$

$$= \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4) - \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4) =$$

$$= 2 \frac{x^4}{4!} + o(x^4) = \frac{x^4}{12} + o(x^4)$$

per $x \to 0$

Denominatore

$$\cosh\left(\operatorname{senh}\left(\frac{x^2}{2}\right)\right) = 1 + \frac{1}{2}\left(\operatorname{senh}\left(\frac{x^2}{2}\right)\right)^2 + o\left(\left(\operatorname{senh}\left(\frac{x^2}{2}\right)\right)^2\right)$$

$$\cosh(\operatorname{sen} x^2) = 1 + \frac{1}{2}(\operatorname{sen} x^2)^2 + o((\operatorname{sen} x^2)^2)$$

$$\cosh\left(\operatorname{senh}\left(\frac{x^2}{2}\right)\right) = 1 + \frac{1}{2}\left(\underbrace{\operatorname{senh}\frac{x^2}{2}}_{= \frac{x^2}{2} + o(x^2)}\right)^2 + o\left(\left(\operatorname{senh}\frac{x^2}{2}\right)^2\right)$$

$\operatorname{senh} y = y + o(y)$ per $y \to 0$

$$\operatorname{senh}\left(\frac{x^2}{2}\right) = \frac{x^2}{2} + o(x^2)$$

$$= 1 + \frac{1}{2}\left(\frac{x^2}{2} + o(x^2)\right)^2 + o\left(\left(\frac{x^2}{2} + o(x^2)\right)^2\right) =$$

$$= 1 + \frac{1}{2}\left(\frac{x^4}{4} + \underbrace{2 \cdot \frac{x^2}{2} \cdot o(x^2)}_{o(x^4)} + \underbrace{(o(x^2))^2}_{o(x^4)}\right) +$$

per $x \to 0$

$$+ o\left(\frac{x^4}{4} + 2 \cdot \frac{x^2}{2} o(x^2) + (o(x^2))^2\right) =$$

$$= 1 + \frac{1}{8} x^4 + o(x^4)$$

$$\cosh(\operatorname{sen} x^2) = 1 + \frac{1}{2}(\overbrace{\operatorname{sen} x^2}^{x^2+o(x^2)})^2 + o\left((\operatorname{sen} x^2)^2\right) =$$

$$= 1 + \frac{1}{2}\left(x^2 + o(x^2)\right)^2 + o\left((x^2+o(x^2))^2\right)$$

$$\overset{!}{=} 1 + \frac{1}{2}\left(x^4 + o(x^4)\right) + o\left((x^4+o(x^4))\right)$$

$$\overset{!}{=} 1 + \frac{1}{2}x^4 + o(x^4)$$

$$\cosh\left(\operatorname{senh}\left(\frac{x^2}{2}\right)\right) - \cosh(\operatorname{sen} x^2) =$$

$$= 1 + \frac{1}{8}x^4 + o(x^4) - 1 - \frac{1}{2}x^4 - o(x^4) =$$

$$= -\frac{3}{8}x^4 + o(x^4)$$

$$\lim_{x\to 0} \frac{\operatorname{sen}(\cosh x - 1) + \operatorname{sen}(\cos x - 1)}{\cosh\left(\operatorname{senh}\left(\frac{x^2}{2}\right)\right) - \cosh(\operatorname{sen} x^2)} =$$

$$= \lim_{x\to 0} \frac{\frac{x^4}{12} + o(x^4)}{-\frac{3}{8}x^4 + o(x^4)} \overset{\text{PdS}}{=}$$

$$= \lim_{x\to 0} \frac{\frac{x^4}{12}}{-\frac{3}{8}x^4} = \frac{1}{12}\cdot\left(-\frac{8}{3}\right) = -\frac{2}{9}$$

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Esercizio Calcolare

$$\lim_{x\to 0} \frac{\operatorname{sen}(\tan x)+\tan(\operatorname{sen} x)}{\operatorname{arcsen}(\arctan x)+\arctan(3\operatorname{arcsen} x)}$$

$$\operatorname{sen}(\tan x) = \tan x + o(\tan x)$$

per $x \to 0$

$$\lim_{x\to 0} \frac{\tan(\operatorname{sen} x)}{\underbrace{\tan x}_{\frac{\operatorname{sen} x}{\cos x}}} = \lim_{x\to 0} \underbrace{\frac{\tan(\operatorname{sen} x)}{\operatorname{sen} x}}_{\to 1} \cdot \cos x = 1$$

$$\lim_{x\to 0} \frac{\tan(\operatorname{sen} x)}{\operatorname{sen} x} \underset{y = \operatorname{sen} x}{=} \lim_{y\to 0} \frac{\tan y}{y} = 1$$

$$\Rightarrow \tan(\operatorname{sen} x) = \tan x + o(\tan x)$$

per $x \to 0$

Numeratore

$$\operatorname{sen}(\tan x) + \tan(\operatorname{sen} x) = \tan x + o(\tan x) +$$
$$+ \tan x + o(\tan x) = 2\tan x + \underbrace{o(\tan x)}_{o(2\tan x)}$$

Denominatore

$$\arctan y = y + o(y) \quad \text{per } y \to 0$$

$$\Rightarrow \arctan(3 \operatorname{arcsen} x) = 3 \operatorname{arcsen} x + o(\operatorname{arcsen} x)$$

$$\lim_{x\to 0} \frac{\operatorname{arcsen}(\arctan x)}{\operatorname{arcsen} x} =$$

$$= \lim_{x \to 0} \underbrace{\frac{\operatorname{arcsen}(\arctan x)}{\arctan x}}_{\to 1} \cdot \underbrace{\frac{\arctan x}{x}}_{\to 1} \cdot \underbrace{\frac{x}{\operatorname{arcsen} x}}_{\to 1} = 1$$

$$\lim_{y \to 0} \frac{\operatorname{arcsen} y}{y} = 1$$

$$\Rightarrow \operatorname{arcsen}(\arctan x) = \operatorname{arcsen} x + o(\operatorname{arcsen} x)$$

$$\operatorname{arcsen}(\arctan x) + \arctan(3 \operatorname{arcsen} x) =$$

$$= \operatorname{arcsen} x + o(\operatorname{arcsen} x) + 3 \operatorname{arcsen} x + o(\operatorname{arcsen} x)$$

$$\overset{!}{=} 4 \operatorname{arcsen} x + o(\operatorname{arcsen} x)$$

$$\lim_{x \to 0} \frac{\operatorname{sen}(\tan x) + \tan(\operatorname{sen} x)}{\operatorname{arcsen}(\arctan x) + \arctan(3 \operatorname{arcsen} x)} =$$

$$= \lim_{x \to 0} \frac{2 \tan x + o(\tan x)}{4 \operatorname{arcsen} x + o(\operatorname{arcsen} x)} \overset{\text{PdS}}{=}$$

$$= \lim_{x\to 0} \frac{2\tan x}{4 \arcsin x} = \lim_{x \to 0} \frac{1}{2} \frac{\tan x}{x} \cdot \frac{x}{\arcsin x} =$$

$$= \frac{1}{2}$$

Esercizio

Stabilire per quali $\alpha \in \mathbb{R}$ esiste finito (che non è il simbolo $\pm\infty$) e diverso da 0 il

$$\lim_{x\to 0} \frac{(1+x^{\alpha-1})^{\frac{1}{\sin^2 x}} - 1}{\sinh x}$$

$$\lim_{x\to 0^+} \frac{(1+x^{\alpha-1})^{\frac{1}{\sin^2 x}} - 1}{\underbrace{\sinh x}_{= x + o(x)}} \qquad \frac{1}{\sin^2 x} \to +\infty$$

- $\alpha - 1 < 0 \qquad (\alpha < 1) \qquad \underbrace{x^{\alpha-1}}_{= \frac{1}{x^{1-\alpha}}} \to +\infty$ per $x \to 0^+$

$$\Rightarrow \lim_{x\to 0^+} \frac{(1+x^{\alpha-1})^{\frac{1}{\sin^2 x}} - 1}{\sinh x} = +\infty$$

- $\alpha - 1 = 0 \qquad (\alpha = 1) \qquad x^{\alpha - 1} = 1 \quad \forall x > 0$

$$\Rightarrow \lim_{x\to 0^+} \frac{2^{\frac{1}{\sin^2 x}}}{\sinh x} = +\infty$$

• $\boxed{\alpha - 1 > 0}$ $(\alpha > 1)$

$\left(1 + x^{\alpha-1}\right)^{\frac{1}{\sin^2 x}}$ è una forma indeterminata 1^∞ per $x \to 0$

$x^{\alpha-1} \to 0$

$$\left(1 + x^{\alpha-1}\right)^{\frac{1}{\sin^2 x}} = e^{\frac{1}{\sin^2 x}\ln\left(1+x^{\alpha-1}\right)}$$

$$\lim_{x\to 0} \frac{\ln\left(1+x^{\alpha-1}\right)}{\sin^2 x} = \lim_{x\to 0} \frac{x^{\alpha-1}+o\left(x^{\alpha-1}\right)}{x^2+o(x^2)} \stackrel{PdS}{=}$$

$$= \lim_{x\to 0^+} \frac{x^{\alpha-1}}{x^2} = \begin{cases} +\infty & \text{se } 0<\alpha-1<2 \quad (1<\alpha<3) \\ 1 & \text{se } \alpha-1=2 \quad (\alpha=3) \\ 0 & \text{se } \alpha-1>2 \quad (\alpha>3) \end{cases}$$

$$\lim_{x\to 0^+} \left(1+x^{\alpha-1}\right)^{\frac{1}{\sin^2 x}} = \begin{cases} +\infty & \text{se } 1<\alpha<3 \\ e & \text{se } \alpha=3 \\ 1 & \text{se } \alpha>3 \end{cases}$$

$$\lim_{x\to 0^+} \frac{\left(1+x^{\alpha-1}\right)^{\frac{1}{\sin^2 x}} - 1}{\sinh x} = \begin{cases} +\infty & 1<\alpha<3 \\ +\infty & \alpha=3 \\ \text{forma indeterminata } \frac{0}{0} & \alpha>3 \end{cases}$$

Caso $\alpha > 3$

$$\lim_{x\to 0} \frac{\ln(1+x^{\alpha-1})}{\operatorname{sen}^2 x} = 0$$

$$(1+x^{\alpha-1})^{\frac{1}{\operatorname{sen}^2 x}} = e^{\frac{\ln(1+x^{\alpha-1})}{\operatorname{sen}^2 x}} \quad \to 0$$

$$e^y = 1 + y + o(y)$$

$$(1+x^{\alpha-1})^{\frac{1}{\operatorname{sen}^2 x}} = 1 + \frac{\ln(1+x^{\alpha-1})}{\operatorname{sen}^2 x} + o\left(\frac{\ln(1+x^{\alpha-1})}{\operatorname{sen}^2 x}\right)$$

$$(1+x^{\alpha-1})^{\frac{1}{\operatorname{sen}^2 x}} - 1 = \frac{\ln(1+x^{\alpha-1})}{\operatorname{sen}^2 x} + o\left(\frac{\ln(1+x^{\alpha-1})}{\operatorname{sen}^2 x}\right)$$

$$\lim_{x\to 0} \frac{(1+x^{\alpha-1})^{\frac{1}{\operatorname{sen}^2 x}} - 1}{\operatorname{senh} x} \overset{\text{PdS}}{=}$$

$x + o(x)$

$$= \lim_{x\to 0} \frac{\ln(1+x^{\alpha-1})}{x \operatorname{sen}^2 x} =$$

$$= \lim_{x\to 0} \frac{x^{\alpha-1} + o(x^{\alpha-1})}{x(x^2+o(x^2))} \overset{\text{PdS}}{=} \lim_{x\to 0} \frac{x^{\alpha-1}}{x^3},$$

che è finito e $\neq 0 \iff \boxed{\alpha = 4}$

Esercizio Calcolare

$$\lim_{x\to+\infty} \frac{\underbrace{x^5}_{=o(e^x)} + e^x + \underbrace{x \operatorname{sen} x}_{=o(e^x)}}{3e^x + \underbrace{x^{15}\ln x}_{=o(x^{15}\cdot x)=o(x^{16})=o(e^x)}}$$

$$\lim_{x\to+\infty} \frac{x^5+e^x+x\operatorname{sen} x}{3e^x+x^{15}\ln x} =$$

$$= \lim_{x\to+\infty} \frac{e^x+o(e^x)}{3e^x+o(e^x)} \overset{PdS}{=} \frac{1}{3}$$

Esercizio Calcolare

$$\lim_{x\to+\infty} \frac{xe^x + \underbrace{x^2}_{=o(e^x)=o(xe^x)} + \underbrace{\operatorname{senh} x}_{=o(xe^x)}}{e^{2x} + \underbrace{\cosh x}_{=o(e^{2x})} + \underbrace{x\ln x}_{=o(\cosh x)}}$$

$$\operatorname{senh} x = \frac{e^x-e^{-x}}{2} = \frac{e^x}{2}+o(e^x) \quad \text{per } x\to+\infty$$

$$\lim_{x\to+\infty} \frac{\operatorname{senh} x}{xe^x} \overset{PdS}{=} \lim_{x\to+\infty} \frac{e^x/2}{xe^x} = 0$$

$$\operatorname{senh} x = o(xe^x)$$

$$\cosh x = \frac{e^x+e^{-x}}{2} = \frac{e^x}{2}+o(e^x)$$

$$\lim_{x\to+\infty} \frac{\cosh x}{e^{2x}} \overset{PdS}{=} \lim_{x\to+\infty} \frac{1}{2}\frac{e^x}{e^{2x}} = 0$$

$$\lim_{x\to+\infty} \frac{xe^x + x^2 + \operatorname{senh} x}{e^{2x} + \cosh x + x\ln x} =$$

$$= \lim_{x\to+\infty} \frac{xe^x + o(xe^x)}{e^{2x} + o(e^{2x})} \underset{=}{\text{PdS}}$$

$$= \lim_{x\to+\infty} \frac{xe^x}{e^{2x}} = \lim_{x\to+\infty} x e^{-x} = 0$$

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Esercizio Calcolare

$$\lim_{x\to+\infty} \frac{x^4 \ln(x^4+3) + x^3 + (\ln(\cosh x))^4}{x^{20}e^{-x} + x^4 \ln(2+x^5) + x^2 \cos x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^x}{2} + o(e^x) \quad \text{per } x \to +\infty$$

$$x^4 \ln(x^4+3) \gg x^3$$

$$\lim_{x\to+\infty} \frac{\ln(\cosh x)}{x} = \lim_{x\to+\infty} \frac{\ln\left(e^x\left(\frac{1+e^{-2x}}{2}\right)\right)}{x} =$$

$$= \lim_{x\to+\infty} \frac{\ln(e^x) + \ln\left(\frac{1+e^{-2x}}{2}\right)}{x} =$$

$$= \lim_{x\to+\infty} \frac{x + \underbrace{\ln\left(\frac{1+e^{-2x}}{2}\right)}_{\to \ln\frac{1}{2}\ = o(x)}}{x} = 1$$

$$\Rightarrow \ln(\cosh x) = x + o(x) \quad \text{per } x\to+\infty$$

$$\Rightarrow (\ln(\cosh x))^4 = x^4 + o(x^4) \quad \text{per } x\to+\infty$$

e poiché $x^4 = o(x^4 \ln(x^4+3))$ per $x\to+\infty$

perché $\lim_{x\to+\infty} \frac{x^4}{x^4\ln(x^4+3)} = \lim_{x\to+\infty} \frac{1}{\ln(x^4+3)} = 0$

$$\Rightarrow (\ln(\cosh x))^4 = o(x^4\ln(x^4+3))$$

Numeratore

$$x^4\ln(x^4+3) + x^3 + (\ln(\cosh x))^4 =$$
$$= x^4\ln(x^4+3) + o(x^4\ln(x^4+3))$$

Denominatore

$$\underbrace{x^{20}e^{-x}}_{\xrightarrow{x\to+\infty} 0} + x^4\ln(2+x^5) + \underbrace{x^2\cos x}_{= o(x^4\ln(2+x^5))}$$

$$\lim_{x\to+\infty} \frac{x^2\cos x}{x^4\ln(2+x^5)} =$$
$$= \lim_{x\to+\infty} \frac{\cos x}{x^2\ln(2+x^5)} = 0$$

$$x^{20}e^{-x} + x^4 \ln(7+x^5) + x^7 \cos x =$$
$$= x^4 \ln(7+x^5) + o(x^4 \ln(7+x^5))$$

$$\lim_{x\to+\infty} \frac{x^4 \ln(x^4+3) + x^3 + (\ln(\cosh x))^4}{x^{20}e^{-x} + x^4 \ln(7+x^5) + x^7 \cos x} =$$

$$= \lim_{x\to+\infty} \frac{x^4 \ln(x^4+3) + o(x^4 \ln(x^4+3))}{x^4 \ln(7+x^5) + o(x^4 + \ln(7+x^5))} \overset{\text{PdS}}{=}$$

$$= \lim_{x\to+\infty} \frac{x^4 \ln(x^4+3)}{x^4 \ln(7+x^5)} = \lim_{x\to+\infty} \frac{\ln(x^4+3)}{\ln(7+x^5)} =$$

$$\frac{\ln x^4}{\ln x^5} = \frac{4\ln x}{5\ln x}$$

$$= \lim_{x\to+\infty} \frac{\ln\left(x^4\left(1+\frac{3}{x^4}\right)\right)}{\ln\left(x^5\left(1+\frac{7}{x^5}\right)\right)} =$$

$$= \lim_{x\to+\infty} \frac{\ln x^4 + \ln\left(1+\frac{3}{x^4}\right)}{\ln x^5 + \ln\left(1+\frac{7}{x^5}\right)}$$

$$\ln\left(1+\frac{3}{x^4}\right) \to 0, \quad \ln\left(1+\frac{3}{x^4}\right) = o(\ln x^4) = o(4\ln x)$$

$$\ln\left(1+\frac{7}{x^5}\right) \to 0, \quad \ln\left(1+\frac{7}{x^5}\right) = o(\ln(x^5)) = o(5\ln x)$$

$$= \lim_{x\to+\infty} \frac{4\ln x}{5\ln x} = \frac{4}{5}$$

Esercizio Calcolare

$$\lim_{x\to 0^+} \frac{\overset{\to +\infty}{e^{1/x}} + \overset{\to -\infty}{\frac{\ln x}{x^4}}}{\underset{\to +\infty}{\operatorname{senh}\frac{1}{x}} + \underset{\to +\infty}{\ln\left(e^{1/x}+1\right)}}$$

$$\lim_{x\to 0^+} \frac{e^{1/x}}{\frac{\ln x}{x^4}} = \lim_{x\to 0^+} \frac{x^4 e^{1/x}}{\ln x} = \quad t = \frac{1}{x}$$

$$= \lim_{t\to+\infty} \frac{e^t}{t^4 \ln\frac{1}{t}} = -\lim_{t\to+\infty}\frac{e^t}{t^4\ln t} = -\infty$$

$$\frac{\ln x}{x^4} = o\left(e^{1/x}\right) \text{ per } x\to 0^+$$

perché $\displaystyle\lim_{x\to 0^+} \frac{\frac{\ln x}{x^4}}{e^{1/x}} = 0$

Numeratore

$$e^{1/x} + \frac{\ln x}{x^4} = e^{1/x} + o\left(e^{1/x}\right)$$

Denominatore

$$\operatorname{senh}\frac{1}{x} + \ln\left(e^{1/x}+1\right)$$

$$\operatorname{senh}\frac{1}{x} = \frac{1}{2}\left(e^{1/x} - e^{-1/x}\right) = \frac{1}{2}e^{1/x} + o\left(e^{1/x}\right)$$

per $x\to 0^+$

$e^{-1/x} \to 0$ per $x\to 0^+$

$$\ln\left(e^{1/x}+1\right)=\ln\left(e^{1/x}\left(1+e^{-1/x}\right)\right)=$$

$$=\ln e^{1/x}+\ln\left(1+e^{-1/x}\right)=$$

$$=\frac{1}{x}+\ln\left(1+e^{-1/x}\right)=\frac{1}{x}+o\left(\frac{1}{x}\right)=o\left(e^{1/x}\right)$$

per $x\to 0^+$

$$\operatorname{senh}\frac{1}{x}+\ln\left(e^{1/x}+1\right)=\frac{1}{2}e^{1/x}+o\left(e^{1/x}\right)$$

per $x\to 0^+$

$$\lim_{x\to 0^+}\frac{e^{1/x}+\frac{\ln x}{x^4}}{\operatorname{senh}\frac{1}{x}+\ln\left(e^{1/x}+1\right)}\overset{\text{PdS}}{=}\lim_{x\to 0^+}\frac{e^{1/x}}{\frac{1}{2}e^{1/x}}=2$$

Esercizio Calcolare

$$\lim_{x\to 0^+}\frac{x^{\ln x}+\cot^2 x}{e^{1/x}+\frac{1}{\tanh^2 x}}\qquad\frac{\infty}{\infty}$$

$$\lim_{x\to 0^+}x^{\ln x}=+\infty=\lim_{x\to 0^+}\cot^2 x$$

Numeratore

$$\lim_{x\to 0}\frac{\frac{1}{\operatorname{sen}^2 x}}{1/x^2}=1$$

$$\cot^2 x=\left(\frac{\cos x}{\operatorname{sen} x}\right)^2\sim\frac{1}{\operatorname{sen}^2 x}\sim\frac{1}{x^2}\quad\text{per } x\to 0$$

[$f\sim g$ per $x\to x_0$ se $\lim_{x\to x_0}\frac{f(x)}{g(x)}=l\in\mathbb{R}\setminus\{0\}$
f è asintotica a g per $x\to x_0$]

$$x^{\ln x} = e^{(\ln x)^2}$$

$$\lim_{x\to 0^+} \frac{x^{\ln x}}{1/x^2} = \lim_{x\to 0^+} x^2 \, x^{\ln x} =$$

$$= \lim_{x\to 0^+} x^{\overbrace{2+\ln x}^{\to -\infty}} = +\infty$$

$$\frac{1}{x^2} = o\left(x^{\ln x}\right) \Rightarrow \cot^2 x = o\left(x^{\ln x}\right)$$

$$x^{\ln x} + \cot^2 x = x^{\ln x} + o\left(x^{\ln x}\right)$$

Denominatore

$$e^{1/x} + \frac{1}{\tanh^2 x} = e^{1/x} + o\left(e^{1/x}\right)$$

$$\frac{1}{\tanh^2 x} = \left(\frac{\cosh x}{\sinh x}\right)^2 \sim \frac{1}{(\sinh x)^2} \sim \frac{1}{x^2}$$

$$\lim_{x\to 0} \frac{\sinh x}{x} = 1$$

$$\Rightarrow \frac{1}{\tanh^2 x} = o\left(e^{1/x}\right)$$

$$\lim_{x\to 0^+} \frac{x^{\ln x} + \cot^2 x}{e^{1/x} + \frac{1}{\tanh^2 x}} = \lim_{x\to 0^+} \frac{x^{\ln x} + o\left(x^{\ln x}\right)}{e^{1/x} + o\left(e^{1/x}\right)} \overset{\text{PoLS}}{=}$$

$$= \lim_{x \to 0^+} \frac{x^{\ln x}}{e^{1/x}} = \lim_{x \to 0^+} \frac{e^{(\ln x)^2}}{e^{1/x}} =$$

$$= \lim_{x \to 0^+} e^{(\ln x)^2 - \frac{1}{x}} = 0$$

$$\lim_{x \to 0^+} \left[(\ln x)^2 - \frac{1}{x} \right] = -\infty$$