

Esercizio 1

Trovare estremo superiore ed estremo inferiore dei seguenti insiemi

$$\begin{aligned}\mathcal{A} &= \left\{ \frac{2n}{n^2 + 1} : n \in \mathbb{Z} \right\}, & \mathcal{B} &= \left\{ (-1)^n n + \frac{1}{n} : n \in \mathbb{N} \setminus \{0\} \right\}, \\ \mathcal{C} &= \left\{ \frac{n+1}{n} : n \in \mathbb{N} \setminus \{0\} \right\}, & \mathcal{D} &= \left\{ (-1)^n \frac{n-1}{n} : n \in \mathbb{N} \setminus \{0\} \right\}, \\ \mathcal{E} &= \left\{ \sqrt{7n} - \sqrt{7(n+1)} : n \in \mathbb{N} \right\}, & \mathcal{F} &= \left\{ \frac{1}{n} - n : n \in \mathbb{N} \setminus \{0\} \right\}, \\ \mathcal{G} &= \left\{ \frac{1}{2^{n^2-4n+2}} : n \in \mathbb{N} \right\}.\end{aligned}$$

indicando in quali casi essi sono anche rispettivamente massimo e minimo.

Esercizio 2

Risolvere le seguenti disequazioni

$$\begin{aligned}x^3 - x^{3/2} - 6 &> 0, & 4^x &> 2, \\ x + 3 &< \sqrt[3]{x^3 + 27}, & x + \sqrt{x^2 - 10x + 9} &\geq \sqrt{x + \sqrt{x^2 - 10x + 9}}, \\ \sqrt[3]{x^2 - 19} &\geq \sqrt[3]{x + 1}, & \sqrt[4]{3x + 1} &< \sqrt[4]{x + 3}, \\ \sqrt{x + 1} + \sqrt{x - 1} &\leq 1, & \sqrt{x + 1} - \sqrt{x - 1} &\leq 1, \\ x - 8 &< \sqrt{x^2 - 9x + 14}, & x &< \sqrt{|x^2 - 5x + 6|}, \\ x - x^{4/5} - 2x^{3/5} &\geq 0, & x^{7/3} - 4x^{5/3} &> 0, \\ (x^2 - 3)^x &< x^2 - 3, & 8^{x+1} &> 2^{x^2}, \\ 2^{3x} - 5 \cdot 2^{3x/2} + 6 &> 0, & \log_{x+9} \left(\frac{x+7}{x-5} \right) &< 0, \\ 2^{x^2-3x+2} &> 1, & \left(\frac{1}{10} \right)^x &< 2^5, \\ \log_{1/3} \left(\frac{x+8}{x-3} \right) &< 0, & \log_2(x^2 - x) &< 1, \\ \log_2 (\log_3(2x-5)) &< 0, & \log_{1/5}(x^2 + 4x) &> -1, \\ \log_3 \left(1 + \frac{1}{x} \right) &\leq 1, & \left(\frac{1}{3} \right)^{(1-12x)x} &< 3.\end{aligned}$$