EQUAZIONI E DISEQUAZIONI TRIGONOMETRICHE

$\operatorname{sen} x = a$

se $a > 1$ non c'è soluzione

se $a < -1$ non c'è soluzione

se $a = 1$ $x = \frac{\pi}{2}$ è soluzione

$x = \frac{\pi}{2} + 2k\pi \quad k \in \mathbb{Z}$

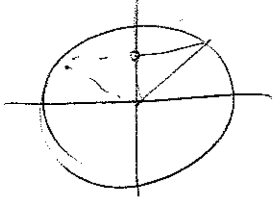
sono tutte e sole le soluzioni

se $a = -1$ $x = \frac{3}{2}\pi + 2k\pi$

se $-1 < a < 1$ $x = \arcsin a + 2k\pi$

$x = \pi - \arcsin a + 2k\pi$

$= -\arcsin a + (2k+1)\pi$



~~esempio~~

Disequazione: $\sin x > a \quad (\geq, <, \leq)$

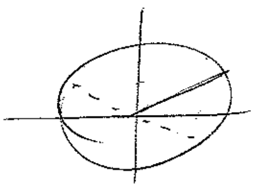
idem (più o meno)

Esempio: $\sin x = 1/2$

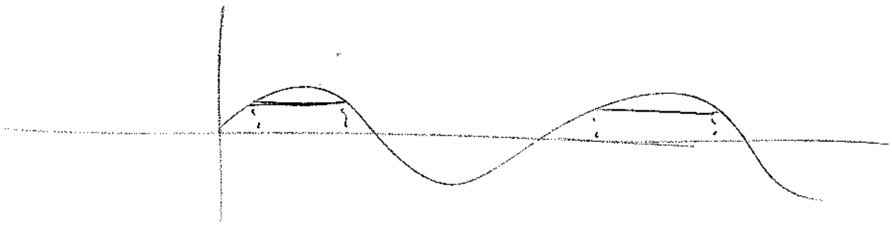
le soluzioni sono

$$x_k = \frac{\pi}{6} + 2k\pi \qquad k \in \mathbb{Z}$$

$$y_k = \frac{5}{6} + 2k\pi = -\frac{\pi}{6} + (2k+1)\pi$$



$\sin x > \frac{1}{2}$



$$x_k < x < y_k \qquad k \in \mathbb{Z}$$

~~$\frac{\pi}{6} + 2k\pi < x < \frac{5}{6} + 2k\pi$~~

$$\bigcup_{k \in \mathbb{Z}} \left(\frac{\pi}{6} + 2k\pi, \frac{5}{6}\pi + 2k\pi \right) =$$

$$= \left\{ x \in \mathbb{R} \;\middle|\; \frac{\pi}{6} + 2k\pi < x < \frac{5\pi}{6} + 2k\pi, \; k \in \mathbb{Z} \right\}$$

In maniera semplice $\quad \cos x = a$

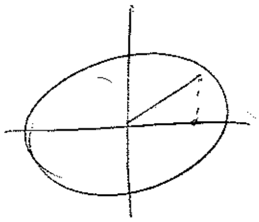
se $a > 1$ non c'è soluzione

se $a < -1$ " " "

se $a = 1$ $\quad x_k = 2k\pi$ soluzioni

se $a = -1$ $\quad y_k = (2k+1)\pi$ soluzioni

se $-1 < a < 1$ $\quad \boxed{x_k = \arccos a + 2k\pi} \quad k \in \mathbb{Z}$



$$y_k = 2\pi - \arccos a + 2k\pi =$$
$$= 2(k+1)\pi - \arccos a$$

$$\boxed{y_k = 2k\pi - \arccos a}$$

disequazioni : $\quad \cos x > a$

$\operatorname{tg} x = a \quad a \in \mathbb{R}$

soluzioni $\quad \operatorname{arctg} a + k\pi \quad k \in \mathbb{Z}$

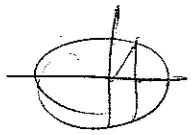
$$2\cos^2 x - 5\cos x + 2 < 0$$

$$2t^2 - 5t + 2 < 0$$

$$t = \frac{5 \pm \sqrt{25-16}}{4} = \begin{matrix} \nearrow 2 \\ \searrow 1/2 \end{matrix}$$

$$\frac{1}{2} < t < 2$$

$$\frac{1}{2} < \cos x < 2 \qquad \cos x > 1/2$$



$$\cos x = \frac{1}{2} \qquad x = \frac{\pi}{3} + \dots$$

$$2k\pi - \frac{\pi}{3} < x < \frac{\pi}{3} + 2k\pi$$

$$tg^2 x - (\sqrt{3}+1)\, tg\, x < -\sqrt{3}$$

$$t^2 - (\sqrt{3}+1)t + \sqrt{3} = 0$$

$$t = \frac{\sqrt{3}+1 \pm \sqrt{3+1+2\sqrt{3}-4\sqrt{3}}}{2} =$$

$$= \frac{\sqrt{3}+1 \pm (\sqrt{3}-1)}{2} = \begin{matrix} \nearrow \sqrt{3} \\ \searrow 1 \end{matrix}$$

$1 < \operatorname{tg} x < \sqrt{3}$

$\frac{\pi}{4} < x < \frac{\pi}{3} \quad + \pi$

$\left\{ x \in \mathbb{R} \;\middle|\; \frac{\pi}{4} + k\pi < x < \frac{\pi}{3} + k\pi, \; k \in \mathbb{Z} \right\}$

$\operatorname{sen}^2 x < \operatorname{sen} x$

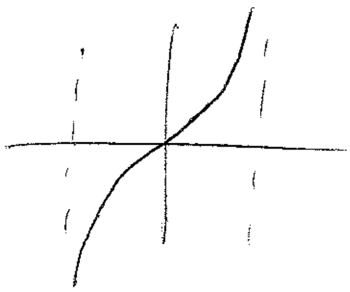
$t^2 - t < 0 \qquad (t-1)t < 0$

$t = 0 \quad t = 1 \quad 0 < t < 1$

$0 < \operatorname{sen} x < 1$

$\left\{ x \in \mathbb{R} \;\middle|\; 2k\pi < x < 2k\pi + \frac{\pi}{2}, \; 2k\pi + \pi/2 < x < (2k+1)\pi, \; k \in \mathbb{Z} \right\}$

$\operatorname{sen} x < \cos x$



se $\cos x > 0 \qquad \operatorname{tg} x < 1$

se $\cos x < 0 \qquad \operatorname{tg} x > 1$

1 $\begin{cases} \cos x > 0 \\ \operatorname{tg} x < 1 \end{cases}$ $\quad x \in \left(-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right)$, $\quad x \in \left(-\frac{\pi}{2} + k\pi, \frac{\pi}{4} + k\pi\right)$

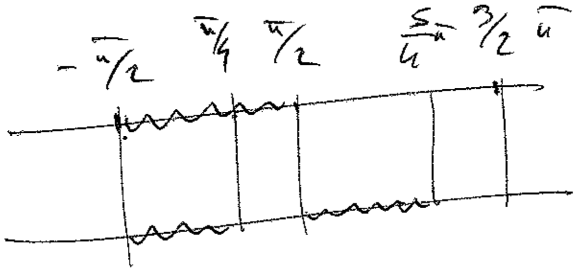
2 $\begin{cases} \cos x < 0 \\ \operatorname{tg} x > 1 \end{cases}$ $\quad x \in \left(\frac{\pi}{2} + 2k\pi, \frac{3}{2}\pi + 2k\pi\right)$, $\quad x \in \left(\frac{\pi}{4} + k\pi, \frac{\pi}{2} + k\pi\right)$

3 $\cos x = 0$ per $x = \frac{\pi}{2} + k\pi$ $\qquad x = \frac{\pi}{2} + 2k\pi$

$x = \frac{3}{2}\pi + 2k\pi$

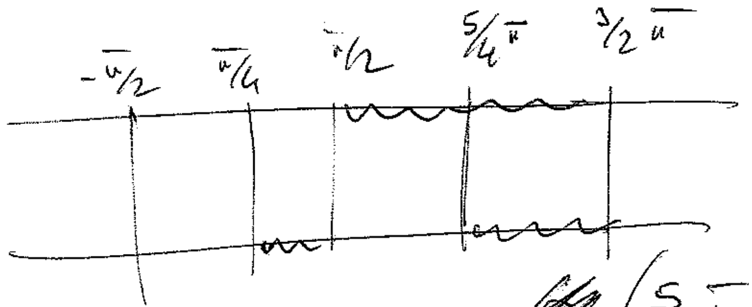
Se $x = \frac{3}{2}\pi + 2k\pi$ $\quad \operatorname{sen} x < \cos x$

~~Conclusione~~



1.

$\left(-\frac{\pi}{2} + 2k\pi, \frac{\pi}{4} + 2k\pi\right)$



2

$\left(\frac{5}{4}\pi + 2k\pi, \frac{3}{2}\pi + 2k\pi\right)$

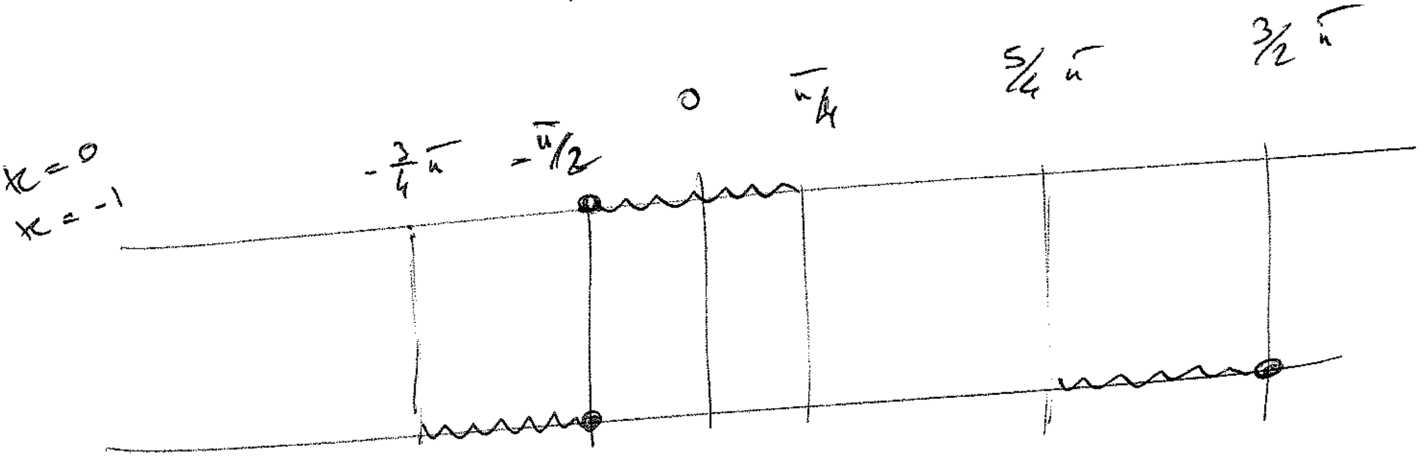
Conclusione: $\quad \operatorname{sen} x < \cos x$ se

$$x \in \bigcup_{k \in \mathbb{Z}} \left[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{4} + 2k\pi\right) \cup \left(\frac{5}{4}\pi + 2k\pi, \frac{3}{2}\pi + 2k\pi\right]$$

~~Vediamo le soluzioni in $[0, 2\pi]$~~

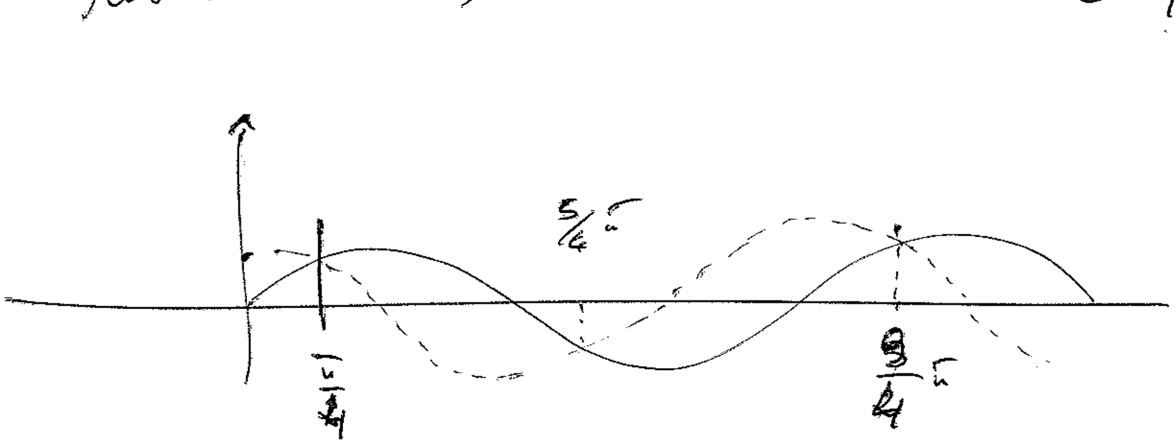
~~$\operatorname{sen} x < \cos x$ in $\left[0, \frac{\pi}{4}\right) \cup$~~

cioè $x \in \bigcup_{k \in \mathbb{Z}} \left(-\frac{3}{4}\pi + 2k\pi, \frac{\pi}{4} + 2k\pi\right)$



Più semplicemente: graficamente

risolvere $\operatorname{sen} x < \cos x$ in $\left[\frac{\pi}{4}, \frac{9}{4}\pi\right]$



$\operatorname{sen} x = \cos x$? $\left(\text{in } \left[\frac{\pi}{4}, \frac{9}{4}\pi\right]\right)$

$x = \frac{\pi}{4}$, $x = \frac{5}{4}\pi$, $x = \frac{9}{4}\pi$

quindi in $\left(\frac{5}{4}\pi, \frac{9}{4}\pi\right)$

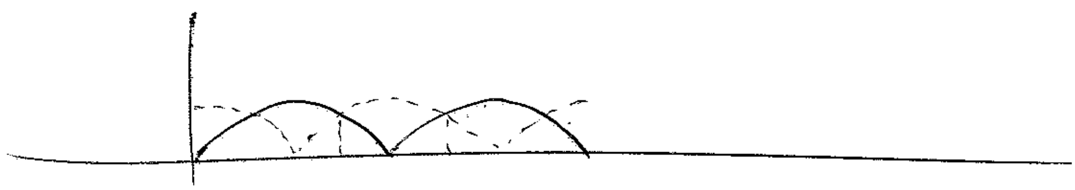
$\operatorname{sen}^2 x < \cos^2 x$

Oss: 1) $a^2 = |a|^2$

2) $x \mapsto x^2$ definita per $x \geq 0$ è monotona crescente

$$f : [0, +\infty) \to \mathbb{R}$$
$$x \mapsto x^2$$

$|\operatorname{sen} x|^2 < |\cos x|^2 \iff |\operatorname{sen} x| < |\cos x|$



vediamo prima in $\left[\frac{\pi}{2}, \frac{3}{2}\pi\right]$

$|\operatorname{sen} x| = |\cos x|$ quando $\operatorname{sen} x = \cos x$

$\operatorname{sen} x = -\cos x$

$x = \frac{3}{4}\pi$, $x = \frac{5}{4}\pi$

quindi $x \in \left(\frac{3}{4}\pi, \frac{5}{4}\pi\right)$

su $\mathbb{R}$? $\left(\frac{3}{4}\pi + k\pi, \frac{5}{4}\pi + k\pi\right)$

$k \in \mathbb{Z}$

perché? $|\operatorname{sen} x|$ e $|\cos x|$

sono periodiche di periodo π! Ex

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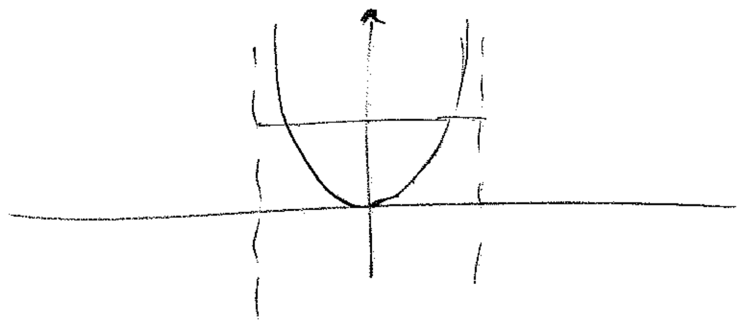
oppure: se $\cos x \neq 0$ $\quad tg^2 x < 1$



$\Updownarrow$

$|tg\, x| < 1$

$|tg\, x| < 1$ in $(-\pi/2, \pi/2)$

sse $x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

su $\mathbb{R}$? $\left(-\frac{\pi}{4} + k\pi, \frac{\pi}{4} + k\pi\right)$

$k \in \mathbb{Z}$

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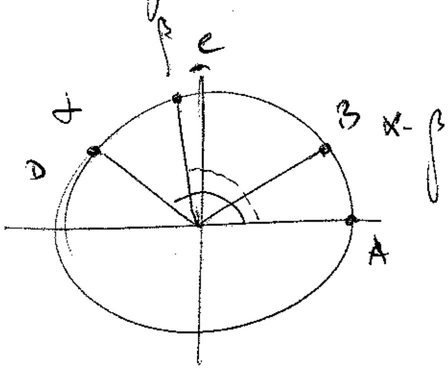
$\operatorname{sen} x < \cos 2x$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$$

anche le altre

$$(*)\quad \cos(\alpha-\beta) = \cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta$$



la distanza tra C e D è uguale a quella tra A e B

$C = (\cos\beta, \operatorname{sen}\beta)$

$D = (\cos\alpha, \operatorname{sen}\alpha)$

$$(\cos\alpha - \cos\beta)^2 + (\operatorname{sen}\alpha - \operatorname{sen}\beta)^2 =$$

$$= (\cos(\alpha-\beta) - 1)^2 + \operatorname{sen}^2(\alpha-\beta)$$

$$= 2 - 2\cos(\alpha-\beta)$$

$$\cos^2\alpha + \cos^2\beta + \operatorname{sen}^2\alpha + \operatorname{sen}^2\beta - 2\cos\alpha\cos\beta +$$
$$-2\operatorname{sen}\alpha\operatorname{sen}\beta = 2 - 2\cos(\alpha-\beta).$$

da cui $(*)$

$\operatorname{sen} x < \cos 2x$

$$\cos 2x = \cos^2 x - \operatorname{sen}^2 x =$$

$$= 1 - (\cos^2 x + \operatorname{sen}^2 x) + \cos^2 x - \operatorname{sen}^2 x$$

$$= 1 - 2 \operatorname{sen}^2 x$$

$$\operatorname{sen} x < 1 - 2 \operatorname{sen}^2 x$$

$$2 \operatorname{sen}^2 x + \operatorname{sen} x - 1 < 0$$

$$2t^2 + t - 1 < 0 \qquad = 0 \quad t = -1, \quad t = 1/2$$

$[0, 2\pi) \qquad -1 < \operatorname{sen} x < \frac{1}{2}$

$$x \in \left[0, \frac{\pi}{6}\right) \cup \left(\frac{5}{6}\pi, \frac{3}{2}\pi\right) \cup \left(\frac{3}{2}\pi, 2\pi\right)$$

$$\operatorname{sen}^2 x = \frac{1}{2}\left(2 \operatorname{sen}^2 x - \cos^2 x + \cos^2 x\right) = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}\left(2 \cos^2 x - \operatorname{sen}^2 x + \operatorname{sen}^2 x\right) = \frac{1}{2}(1 + \cos 2x)$$

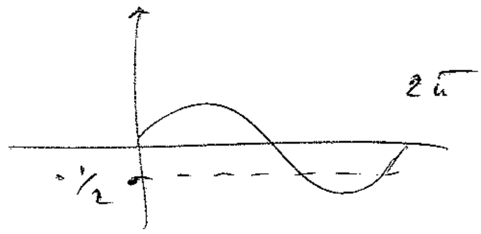
$4 \operatorname{sen} x \cos x + 1 < 0$

$\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha \cos\beta + \cos\alpha \operatorname{sen}\beta$

$\alpha = \beta = x \qquad \operatorname{sen} 2x = 2 \operatorname{sen} x \cos x$

$2 \operatorname{sen} 2x + 1 < 0$

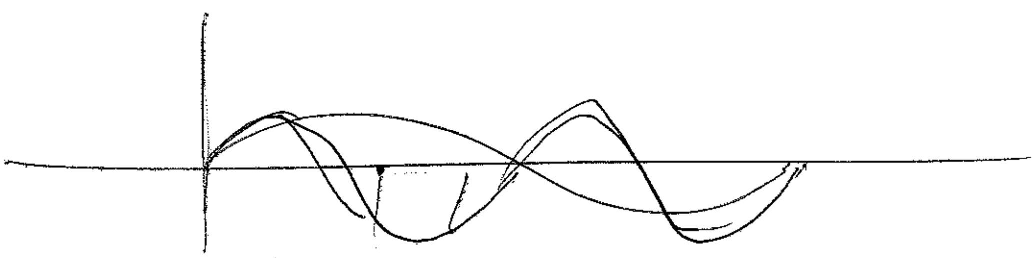
$\operatorname{sen} 2x < -\frac{1}{2}$



$\frac{7}{6}\pi < 2x < \frac{11}{6}\pi$

$\frac{7}{6}\pi + 2k\pi < 2x < \frac{11}{6}\pi + 2k\pi$

$\frac{7}{12}\pi + k\pi < x < \frac{11}{12}\pi + k\pi$



$$2\cos^2 x + 3\operatorname{sen} x - 3 > 0$$

$$\cos^2 x = 1 - \operatorname{sen}^2 x$$

$$2 - 2\operatorname{sen}^2 x + 3\operatorname{sen} x - 3 > 0$$

per $\forall x$

$$\sqrt{1-2\operatorname{sen}^2 x} \geq \sqrt{2}\operatorname{sen} x + 1 \qquad [0, 2\pi]$$

$$1^\circ \quad \begin{cases} 1 - 2\operatorname{sen}^2 x \geq 0 \\ \sqrt{2}\operatorname{sen} x + 1 < 0 \end{cases}$$

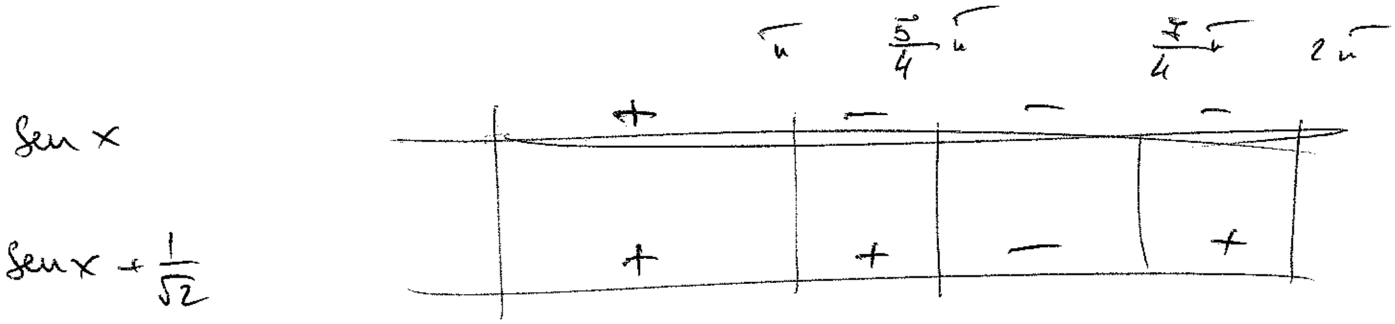
$$2^\circ \quad \begin{cases} 1 - 2\operatorname{sen}^2 x \geq 0 \\ 1 - 2\operatorname{sen}^2 x \geq 2\operatorname{sen}^2 x + 1 + 2\sqrt{2}\operatorname{sen} x \end{cases}$$

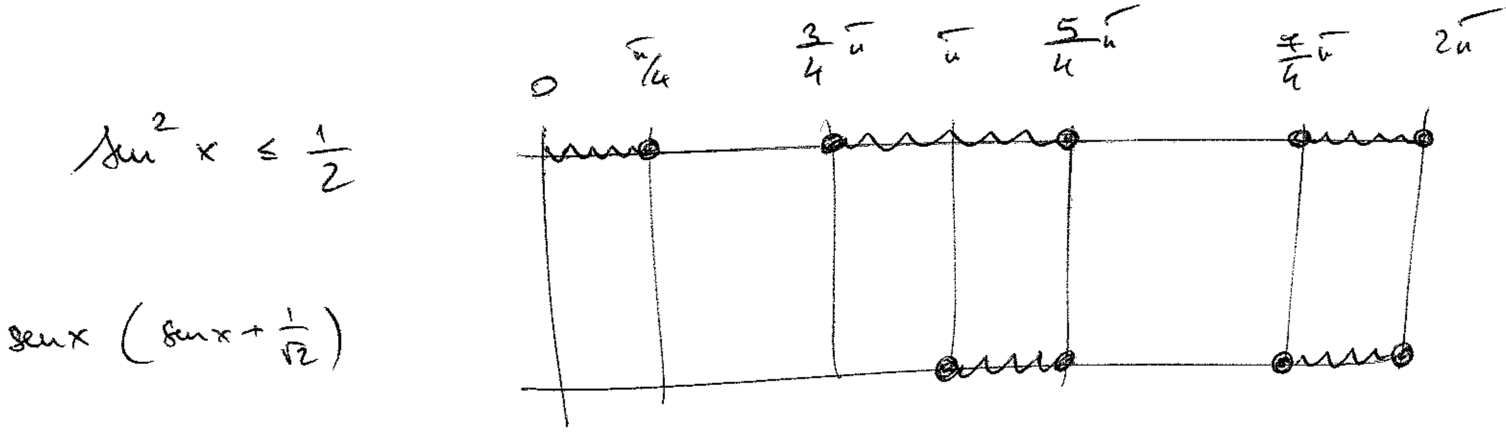
$$1^\circ \quad \begin{cases} \operatorname{sen}^2 x \leq \frac{1}{2} & 0 \leq x \leq \frac{\pi}{4},\ \frac{3}{4}\pi \leq x \leq \frac{5}{4}\pi,\ \frac{7}{4}\pi \leq x \leq 2\pi \\ \operatorname{sen} x < -\frac{1}{\sqrt{2}} & \frac{5}{4}\pi < x < \frac{7}{4}\pi \end{cases}$$

non ha soluzioni!

(7)

$$2^\circ \quad \begin{cases} \operatorname{sen}^2 x \le \dfrac{1}{2} \\[2ex] 4 \operatorname{sen} x \left(\operatorname{sen} x + \dfrac{1}{\sqrt{2}} \right) \le 0 \end{cases}$$





$$\left\{ x \in [0, 2\pi] \;\middle|\; \pi \le x \le \frac{5}{4}\pi \,,\quad \frac{7}{4}\pi \le x \le 2\pi \right\}$$

$$\left\{ x \in \mathbb{R} \;\middle|\; (2k+1)\pi \le x \le \frac{5}{4}\pi + 2k\pi \,, \quad \frac{7}{4}\pi + 2k\pi \le x \le (2k+2)\pi \quad k \in \mathbb{Z} \right\}$$

$$\frac{1}{2}\,\text{sen}\,2\vartheta + \frac{\sqrt{3}}{2}\cos 2\vartheta \geq \text{sen}\left(\vartheta + \pi/6\right)$$

$$\cos\frac{\pi}{3}\,\text{sen}\,2\vartheta + \text{sen}\,\frac{\pi}{3}\cos 2\vartheta \geq \text{sen}\left(\vartheta + \frac{\pi}{6}\right)$$

$$\text{sen}\left(\frac{\pi}{3} + 2\vartheta\right) \geq \text{sen}\left(\vartheta + \frac{\pi}{6}\right)$$

$$\text{sen}\left(2\left(\frac{\pi}{6} + \vartheta\right)\right) \geq \text{sen}\left(\vartheta + \frac{\pi}{6}\right)$$

$$2\,\text{sen}\left(\frac{\pi}{6} + \vartheta\right)\cos\left(\frac{\pi}{6} + \vartheta\right) \geq \text{sen}\left(\vartheta + \frac{\pi}{6}\right)$$

$$\text{sen}\left(\frac{\pi}{6} + \vartheta\right)\left[2\cos\left(\frac{\pi}{6} + \vartheta\right) - 1\right] \geq 0$$

$$\text{sen}\left(\frac{\pi}{6} + \vartheta\right) \geq 0 \qquad 0 \leq \frac{\pi}{6} + \vartheta \leq \pi$$

$$-\frac{\pi}{6} \leq \vartheta \leq \frac{5}{6}\pi$$

$$\cos\left(\frac{\pi}{6} + \vartheta\right) \geq \frac{1}{2} \qquad 0 \leq \frac{\pi}{6} + \vartheta \leq \pi/3$$

$$\frac{5}{3}\pi \leq \frac{\pi}{6} + \vartheta \leq 2\pi$$

$$-\frac{\pi}{6} \leq \vartheta \leq \frac{\pi}{6}\,, \quad \frac{3}{2}\pi \leq \vartheta \leq \frac{11}{6}\pi$$

$-\frac{\pi}{6}$		$\frac{\pi}{6}$		$\frac{5}{6}\pi$		$\frac{3}{2}\pi$		$\frac{11}{6}\pi$
	$+$		$+$		$-$		$-$	
	$+$		$-$		$-$		$+$	

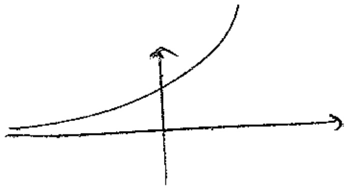
$$\left\{ x \in \left[-\frac{\pi}{6}, \frac{11}{6}\pi\right] \;\middle|\; -\frac{\pi}{6} \le x \le \frac{\pi}{6},\; \frac{5}{6}\pi \le x \le \frac{3}{2}\pi \right\}$$

DIS. ESPONENZIALI E LOGARITMICHE

$$8^{x+1} \geq 2^{x^2}$$

$$(2^3)^{x+1} = 2^{3x+3} \geq 2^{x^2}$$

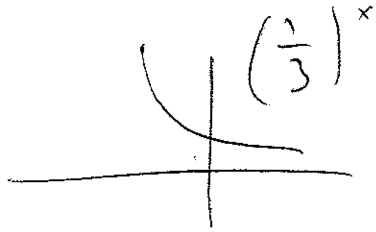
$f : \mathbb{R} \to \mathbb{R}$
$x \mapsto 2^x$



è monotona crescente

qdi $\Leftrightarrow$ $\quad 3x + 3 \geq x^2 \quad$ (E x)

$$\left(\frac{1}{3}\right)^{(1-12x)x} < 3$$



$$\left(\frac{1}{3}\right)^{(1-12x)x} < \left(\frac{1}{3}\right)^{-1}$$

$$\Leftrightarrow \quad (1-12x)x > -1$$

oppure $\quad \left(\frac{1}{3}\right)^{(1-12x)x} = \left(3^{-1}\right)^{(1-12x)x} = 3^{(12x-1)x}$

$$(12x-1)x < 1$$

$12x^2 - x - 1 < 0 \quad$ Ex

$$e^{4x^4 - 5x^2 + 1} < 1 = e^0$$

$$4x^4 - 5x^2 + 1 < 0$$

$$\pi^{4x^4 - 5x^2 + 1} < 1 \quad \text{idem}$$

$$\left(\frac{1}{4}\right)^{4x^4 - 5x^2 + 1} < 1 \iff 4x^4 - 5x^2 + 1 > 0$$

$$4^x + 2^x - 2 < 0 \qquad 4^x = (2^2)^x = 2^{2x}$$

$$2^{2x} + 2^x - 2 < 0 \qquad \text{boh? ...}$$

se però chiamo $t = 2^x$

$$\Rightarrow 2^{2x} = t^2$$

$$t^2 + t - 2 < 0$$

$t^2 + t - 2 < 0$ se $-2 < t < 1$

$-2 < 2^x < 1$

$-2 < 2^x$ sempre $\left(2^x > 0 \right)$

$2^x < 1 = 2^0 \iff x < 0$

$-2 < 2^x < 1 \iff x < 0$

$e^{|x-1|} < e^x \iff |x-1| < x$

$\begin{cases} x - 1 \geq 0 \\ x - 1 < x \end{cases}$ $\qquad$ $\begin{cases} x - 1 < 0 \\ 1 - x < x \end{cases}$

$\begin{cases} x \geq 1 \\ -1 < 0 \end{cases}$ $\qquad$ $\begin{cases} x < 1 \\ x > 1/2 \end{cases}$

$\left\{ x \in \mathbb{R} \mid \frac{1}{2} < x \right\} = \left(\frac{1}{2}, +\infty \right)$

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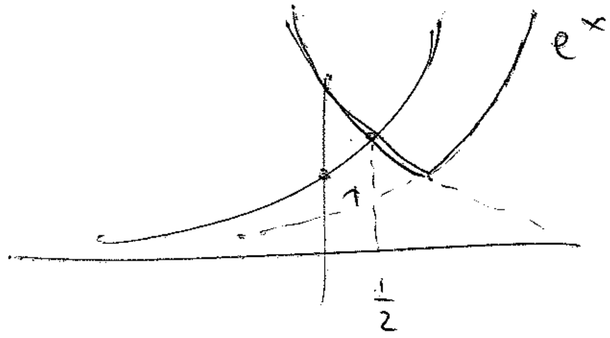
e^{x-1}



$$e^t + e^{-t} < \frac{10}{3}$$

$$e^t = x \qquad x + \frac{1}{x} < \frac{10}{3} \qquad (x > 0)$$

$$\frac{x^2+1}{x} - \frac{10}{3} < 0$$

$$\frac{3x^2+3-10x}{3x} < 0$$

$$3x^2 - 10x + 3 \geq 0$$

$$= \quad x = \frac{10 \pm \sqrt{100-36}}{6} = \begin{cases} 3 \\ \frac{1}{3} \end{cases}$$

		0		$1/3$		3	
N	+	0	+		$\cancel{-}$		+
D	−		+		+		+

$$\frac{1}{3} < x < 3$$

$\frac{1}{3} < e^t < 3$

$\log \frac{1}{3} < t < \log 3$

$\log = \log_e$

$(x+1)^{x^2-1} > 1 = (x+1)^0$

1° caso) base positiva

2° caso) distinguere due casi (anzi tre)

A $\begin{cases} 0 < x+1 < 1 \\ x^2-1 < 0 \end{cases}$

B $\begin{cases} x+1 = 1 \\ \text{non è verificata} \end{cases}$

C $\begin{cases} x+1 > 1 \\ x^2-1 > 0 \end{cases}$

A)
$$\begin{cases} -1 < x < 0 \\ x^2 < 1 \iff -1 < x < -1 \end{cases}$$

sol. $-1 < x < 0$

B) se $x = 0$ la disep. non è verificata.

c)
$$\begin{cases} x > 0 \\ x^2 > 1 \iff x < -1 , \quad x > 1 \end{cases}$$

sol $x > 1$

la disequazione è soddisfatta in

$$\{ x \in \mathbb{R} \mid -1 < x < 0 , \quad x > 1 \}$$

$$(x^2-3)^x \leq x^2-3$$

$$A\begin{cases} 0 < x^2-3 < 1 \\ x \geq 1 \end{cases}$$

$$B\begin{cases} x^2-3 = 1 \\ \text{è verificata sempre} \end{cases}$$

$$C\begin{cases} x^2-3 > 1 \\ x \leq 1 \end{cases}$$

A) $\begin{cases} 3 < x^2 < 4 \\ x \geq 1 \end{cases}$ $\quad -2 < x < -\sqrt{3},\ \sqrt{3} < x < 2$

$x \geq 1$

$1 \leq x < 2$

B) $\begin{cases} x = -2 \ ,\ x = 2 \end{cases}$

C) $\begin{cases} x^2 > 4 \\ x \leq 1 \end{cases}$ $\quad x < -2$

$$\{x \in \mathbb{R} \mid x \leq -2,\ 1 \leq x \leq 2\}$$

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$$\log(x-1)^2 - \log(x-2)^2 > 0$$

$\boxed{\log = \log_e}$

$$\log a^2 = 2\log a \quad ? \quad \text{No!}$$

$$\log a^2 = 2\log|a|$$

$$\log(x-1)^2 > \log(x-2)^2 \qquad e > 1$$

$$(x-1)^2 > (x-2)^2 \iff 2x > 3 \qquad x > 3/2$$

$$\log_a(4x^2+3x-1) - \log_a x > 0$$

$$\begin{cases} x > 0 \\ 4x^2+3x-1 > 0 \end{cases} \qquad \begin{cases} x > 0 \\ \dfrac{-3 \pm \sqrt{9+16}}{8} = \begin{smallmatrix} -1 \\ \frac{1}{4} \end{smallmatrix} \end{cases}$$

$$\frac{1}{4} < x < +\infty$$

$$\log_a\left(\frac{4x^2+3x-1}{x}\right) > 0 = \log_a 1$$

$$\frac{4x^2+3x-1}{x} > 1 \qquad \text{se } \boxed{a > 1}$$

poiché $x > 0$

$$4x^2+3x-1 > x$$

$$4x^2+2x-1 > 0$$

$$\frac{-2 \pm \sqrt{4+16}}{2 \cdot 4} = \frac{-1 \pm \sqrt{5}}{4}$$

$$\frac{1}{4} < x < +\infty \qquad \cancel{\frac{-1-\sqrt{5}}{4} < x < \frac{-1+\sqrt{5}}{4}} \qquad \boxed{x > (\sqrt{5}-1)/4}$$

$$\boxed{\cancel{\frac{1}{4} < x < \sqrt{5}-1}}$$

$$\Rightarrow \quad x > \frac{\sqrt{5}-1}{4}$$

(13)

se $a < 1$

~~16 · 7~~

64   10   5

$3 - 4x \mid \frac{18}{4} \quad - \frac{9}{8}$

$$\frac{4x^2 + 3x - 1}{x} < 1 \quad (x>0)$$

$$\Leftrightarrow \quad 4x^2 + 2x - 1 < 0$$

$$\frac{1}{4} < x < \frac{\sqrt{5}-1}{4}$$

$$\left(\frac{1}{2}\right)^{\log x} > x \qquad \text{con} \quad x > 0$$

$$\left(\frac{1}{2}\right)^{\log_e x} > \left(\frac{1}{2}\right)^{\log_{1/2} x}$$

$$\log_a t = \log_a b^{\log_b t} = \log_b t \ \log_a b$$

$$t) \quad \log_e x < \log_{1/2} x = \log_e x \ \log_{1/2} e$$

$$\underbrace{(\log_{1/2} e - 1)}_{\wedge \atop 0} \log_e x > 0$$

se $x \in (0, 1)$