Analogamente,

$$\int_{-1}^{0} \frac{dx}{(x-4)\sqrt{|x|}} = \lim_{\epsilon\to 0^-} \int_{-1}^{\epsilon} \frac{dx}{(x-4)\sqrt{-x}},$$

che si tratta con il cambiamento di variabile $x = -t^2$:

$$\int \frac{dx}{(x-4)\sqrt{-x}} = \int \frac{2dt}{t^2+4} = \operatorname{arctg}\frac{t}{2} = \operatorname{arctg}\frac{\sqrt{-x}}{2}.$$

Si ha allora

$$\int_{-1}^{0} \frac{dx}{(x-4)\sqrt{|x|}} = -\operatorname{arctg}\frac{1}{2}$$

e in conclusione

$$\int_{-1}^{1} \frac{dx}{(x-4)\sqrt{|x|}} = \frac{1}{2}\ln\frac{1}{3} - \operatorname{arctg}\frac{1}{2}. \ \blacksquare$$

Esercizi

Si calcolino i seguenti integrali:

101. $\displaystyle\int_0^1 \frac{x-1}{(x+2)\sqrt{x}}\,dx$ **102.** $\displaystyle\int_0^\infty (x^2-x)\,e^{-x}\,dx$ **103.** $\displaystyle\int_0^3 \frac{\ln x}{x\sqrt{x}}\,dx$

104. $\displaystyle\int_2^\infty \frac{\ln x}{(x+1)^2}\,dx$ **105.*** $\displaystyle\int_0^\infty \frac{\ln x}{(x+1)^{3/2}}\,dx$ **106.** $\displaystyle\int_{-1}^1 \frac{\sqrt{|x|}}{x^2-2x}\,dx$

107. $\displaystyle\int_0^\pi \frac{dx}{\sqrt{1-\sin x}}$ **108.** $\displaystyle\int_0^\infty \frac{\operatorname{arctg} x}{x\sqrt{x}}\,dx$ **109.** $\displaystyle\int_0^\infty \frac{\operatorname{arctg} x - \pi/2}{\sqrt{x}}\,dx$

110. $\displaystyle\int_0^\infty x^{-2}e^{-1/x}\,dx$ **111.** $\displaystyle\int_0^1 \frac{x^3}{\sqrt{1-x^2}}\,dx$ **112.** $\displaystyle\int_0^\infty \frac{dx}{e^x+e^{-x}}$

113. $\displaystyle\int_0^\infty x^n e^{-x}\,dx$ **114.** $\displaystyle\int_0^\infty e^{-\beta x}\sin\alpha x\,dx$ **115.** $\displaystyle\int_0^\infty e^{-\beta x}\cos\alpha x\,dx.$

Dire se i seguenti integrali impropri convergono oppure no.

$$\int_5^{+\infty} \frac{x^2+1}{x^5-7x^2+3}\,dx$$

$$\int_5^{+\infty} \operatorname{arctg}\left(\frac{x^2+1}{x^5-7x^2+3}\right)dx$$

$$\int_5^{+\infty} \operatorname{arctg}\left(\frac{x^\alpha+2x}{x^\beta+3}\right)dx$$

$$\int_0^{+\infty} \frac{1}{(\operatorname{arctg} x)^\alpha}\,dx$$

$$\int_1^{+\infty} \frac{e^{\alpha x}-1}{x^\alpha}\,dx$$

(calcolarlo per $\alpha = -2$)

$$\int_0^{+\infty} \frac{\operatorname{arctg}^{2/\alpha} x}{\cos\sqrt{x}+x^2}\,dx$$

$$\int_1^{3} \frac{dx}{(x-1)^\alpha (3-x)^\beta}$$

Esercizi

Dire se esistono i seguenti integrali impropri:

116. $\displaystyle\int_0^1 \ln^2 x \,\mathrm{d}x$

117. $\displaystyle\int_0^\infty \frac{\sin x}{x^2+1}\,\mathrm{d}x$

118. $\displaystyle\int_0^1 \frac{\mathrm{d}x}{\ln(1+x)}$

119. $\displaystyle\int_{-\infty}^{-1} \frac{e^x}{x^2}\,\mathrm{d}x$

120. $\displaystyle\int_0^1 \frac{\sin\sqrt{x}}{x}\,\mathrm{d}x$

121. $\displaystyle\int_0^2 \frac{e^{\sqrt{x}}-1}{\sin x}$

122. $\displaystyle\int_0^\infty \sin^2 x\,\mathrm{d}x$

123. $\displaystyle\int_2^\infty \frac{\sin x}{\ln x}\,\mathrm{d}x$

124. $\displaystyle\int_{-1}^0 \frac{\sqrt{|x|}}{|x|-\sin x}\,\mathrm{d}x$

125. $\displaystyle\int_0^1 (x^x-1)\operatorname{cotg} x\,\mathrm{d}x$

126. $\displaystyle\int_0^1 x^{\ln x}\,\mathrm{d}x$

127. $\displaystyle\int_0^1 \sqrt{\frac{\sin(1-\sqrt{\cos x})}{x^3}}\,\mathrm{d}x$

128. $\displaystyle\int_0^1 \frac{x}{(x^x-1)^2}\,\mathrm{d}x$

129. $\displaystyle\int_{-\infty}^\infty \frac{xe^x}{(x^4+1)\sinh x}\,\mathrm{d}x$

130.* $\displaystyle\int_1^\infty \frac{(-1)^{[x]}}{x}\,\mathrm{d}x$

131. $\displaystyle\int_0^{\frac{1}{2}} \frac{\mathrm{d}x}{x^{(x^x-1)}-1}$

132. $\displaystyle\int_1^\infty \sin x \arcsin\frac{1}{x}\,\mathrm{d}x$

133. $\displaystyle\int_1^\infty \left(\operatorname{arctg} x-\pi+\frac{1}{x}\right)\mathrm{d}x.$

134. Dimostrare che, se $f(x)$ è una funzione decrescente e di classe C^1 (derivabile con derivata continua), e $\lim\limits_{x\to+\infty} f(x)=0$, allora l'integrale

$$\int_0^\infty f(x)\sin x\,\mathrm{d}x$$

è convergente.