Studiare i seguenti limiti:

1. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y}{x^2}$;

2. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^4}{x^4+y^2}$;

3. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy(2y^2+x^3)}{x^4+y^2}$;

4. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1-\cos(xy)}{\log(1+x^2+y^2)}$;

5. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3-y^5}{(x^2+y^2)^2}\operatorname{sen} xy$;

6. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{\sqrt{x^2+y^2}}-2}{x^2+y^2}$;

7. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3+y^5}{x^2+y^5}$;

8. $\displaystyle\lim_{(x,y)\to(1,1)} \frac{\log(x^2+y^2+2xy-3)}{\operatorname{sen}(x+y-2)}$;

9. $\displaystyle\lim_{(x,y)\to(0,0)} xe^{-\frac{y}{x}}$;

10. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{|y|^\alpha\cos x}{\sqrt{x^2+y^2}}$ al variare di $\alpha\in\mathbf{R}$;

11. $\displaystyle\lim_{(x,y)\to(1,0)} \frac{y^2\log x}{(x-1)^2+y^2}$;

12. $\displaystyle\lim_{(x,y)\to(x_o,y_o)} \frac{(x-x_o)(y-y_o)}{(x-x_o)^2+(y-y_o)^2}$;

13. $\displaystyle\lim_{(x,y)\to(x_o,y_o)} \frac{|x-x_o|^\alpha|y-y_o|^\beta}{|x-x_o|^\gamma+|y-y_o|^\delta}$ al variare di $\alpha,\beta,\gamma,\delta\in\mathbf{R}$;

14. $\displaystyle\lim_{(x,y)\to(x_o,y_o)} \frac{(x-x_o)(y-y_o)\log(x-x_o+1)}{(x-x_o)^2+(y-y_o)^2}$;

15. $\displaystyle\lim_{(x,y)\to(1,2)} \frac{xy-2x-y+2}{x^2-2x+y^2-4y+5}$;

16. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\operatorname{sen}(x^3+xy^2)}{\log(x^6+x^2y^4+2x^4y^2)}$;

17. $\displaystyle\lim_{\substack{|(x,y)|\to+\infty \\ (x,y)\in A}} \frac{y}{x^2}$ dove $A = \{(x,y)\in \mathbf{R}^2 \mid x^2 \leqslant y \leqslant 2x^2\}$;

18. $\displaystyle\lim_{\substack{|(x,y)|\to+\infty \\ (x,y)\in A}} \frac{y}{x^2}\log\left(\frac{1+x+y}{x+y}\right)$ dove $A = \{(x,y)\in \mathbf{R}^2 \mid x^2 \leqslant y \leqslant 2x^2\}$;

19. $\displaystyle\lim_{\substack{|(x,y)|\to+\infty \\ (x,y)\in A}} \frac{y}{x^2}\left[(x+y)\log\left(\frac{1+x+y}{x+y}-1\right)\right]$
dove $A = \{(x,y)\in \mathbf{R}^2 \mid x^2 \leqslant y \leqslant 2x^2\}$;

20. $\displaystyle\lim_{\substack{|(x,y)|\to+\infty \\ x\in A}} \frac{y\,e^{-(x^2+y^2)}}{x-x^3-xy^2}$ dove $A = \{(x,y)\in \mathbf{R}^2 \mid |y| \leqslant \frac{1}{x}, x\geqslant 1\}$;

21. $\displaystyle\lim_{\substack{|(x,y)|\to+\infty \\ (x,y)\in A}} \log(1+y)^x$ dove $A = \{(x,y)\in \mathbf{R}^2 \mid |y| \leqslant \frac{1}{x}, x\geqslant 1\}$;

22. $\displaystyle\lim_{\substack{|(x,y)|\to+\infty \\ (x,y)\in A}} \frac{y}{x^2}e^{-\frac{1}{x+y}}$ dove $A = \{(x,y)\in \mathbf{R}^2 \mid x^2 \leqslant y \leqslant 2x^2\}$;

23. $\displaystyle\lim_{\substack{|(x,y)|\to+\infty \\ (x,y)\in A}} (1+x)\mathrm{sen}\frac{1}{y}$ dove $A = \{(x,y)\in \mathbf{R}^2 \mid y \leqslant \sqrt{1+x^2}, x \geqslant \sqrt{1+y^2}, x \geqslant 0, y\geqslant 0\}$;

24. $\displaystyle\lim_{\substack{|(x,y)|\to+\infty \\ (x,y)\in A}} x^3ye^{-xy}$ dove $A = \{(x,y)\in \mathbf{R}^2 \mid x\geqslant 0, x \leqslant y \leqslant 2x\}$;

25. $\displaystyle\lim_{\substack{|(x,y)|\to+\infty \\ (x,y)\in A}} \left(1+\frac{1}{x}\right)^y$ dove $A = \{(x,y)\in \mathbf{R}^2 \mid x\geqslant 1\}$;

26. $\displaystyle\lim_{\substack{|(x,y)|\to+\infty \\ (x,y)\in A}} \frac{y}{x}$ dove $A = \left\{(x,y)\in \mathbf{R}^2 \mid -\frac{1}{1+x^2} \leqslant y \leqslant \frac{1}{1+x^2}\right\}$;

27. $\displaystyle\lim_{\substack{|(x,y)|\to+\infty \\ x\in A}} |y|^\alpha |x|^\beta$ dove $A = \left\{(x,y)\in \mathbf{R}^2 \mid -\frac{1}{1+x^2} \leqslant y \leqslant \frac{1}{1+x^2}\right\}$
al variare di $\alpha,\beta\in\mathbf{R}$;

28. $\displaystyle\lim_{\substack{|(x,y)|\to+\infty \\ (x,y)\in A}} \frac{\mathrm{sen}\, y\log|y|}{xy}$ dove $A = \left\{(x,y)\in \mathbf{R}^2 \mid -\frac{1}{1+x^2} \leqslant y \leqslant \frac{1}{1+x^2}\right\}$;

29. $\displaystyle\lim_{|(x,y,z)|\to+\infty} (x^4+y^2+z^2-x+3y-z)$;

30. $\displaystyle\lim_{|(x,y,z)|\to+\infty} (x^4+y^2+z^2-x^3+xyz-x+4)$;

31. $\displaystyle\lim_{|(x,y,z)|\to+\infty} (x^3+y^2+z^3-xy)$;

32. $\displaystyle\lim_{|(x,y,z)|\to+\infty} (x^4+y^4+z^4-x^3-y^3-z^3+x^2yz^2)$.

Se non specificato diversamente si studi la continuità, la derivabilità e la differenziabilità delle seguenti funzioni trovando anche l'insieme più grande nelle quali risultano di classe C^1:

1. $f(x,y) = \begin{cases} \dfrac{\operatorname{sen}(x^2+y^2)}{x^2+y^2} & (x,y) \neq 0 \\ 1 & (x,y) = (0,0) \end{cases}$

2. $f(x,y) = \begin{cases} y^2 & |y| > x^2 \\ x^2 & |y| \leqslant x^2 \end{cases}$

3. $f(x,y) = |y - x^2|$

4. $f(x,y) = \begin{cases} (x^2+y^2)\operatorname{sen}\dfrac{1}{\sqrt{x^2+y^2}} & (x,y) \neq 0 \\ 0 & (x,y) = (0,0) \end{cases}$

5. $f(x,y) = \begin{cases} \dfrac{x^3+2y^4}{x^2+y^2} & (x,y) \neq 0 \\ 0 & (x,y) = (0,0) \end{cases}$

6. $f(x,y) = \begin{cases} \dfrac{|x|^\alpha \operatorname{sen} y}{x^2+y^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$

7. $f(x,y) = |y| \operatorname{sen}(x^2+y^2)$

8. $f(x,y,z) = \begin{cases} \dfrac{z^4(x^2+y^2)^\alpha}{x^2+y^2} & (x,y,z) \neq 0 \\ 0 & (x,y,z) = (0,0) \end{cases}$

9. $f(x,y) = \begin{cases} \exp\left(-\dfrac{1}{x^2+y^2}\right) & (x,y) \neq 0 \\ 0 & (x,y) = (0,0) \end{cases}$

10. $f(x,y) = \begin{cases} \dfrac{x^n+y^m}{x^2+y^2} & (x,y) \neq 0 \\ 0 & (x,y) = (0,0) \end{cases} \qquad n,m \in \mathbf{N}$

11. $f(x,y) = \begin{cases} \dfrac{\operatorname{sen}(x^2+y^2)}{x^2+y^2} & (x,y) \neq 0 \\ 1 & (x,y) = (0,0) \end{cases}$

12. $f(x,y) = \begin{cases} \dfrac{x^2+y^2}{|x|^a+|y|^{1/2}} & (x,y) \neq 0 \\ 0 & (x,y) = (0,0) \end{cases} \qquad a > 0$

13. Stabilire se la seguente funzione è continua, derivabile, differenziabile nei punti $(t, 0)$**,** $t \in \mathbf{R}$ al variare del parametro $a > 0$

$$f(x,y) = \begin{cases} y^2 \operatorname{sen}(x+y) & y \leqslant 0 \\ \dfrac{\log(1+xy^2)}{y^a} & y > 0 \end{cases}$$

14. Stabilire se la seguente funzione è continua, derivabile, differenziabile nei punti (t, t), $t \in \mathbf{R}$

$$f(x,y) = \begin{cases} x \operatorname{sen}\left(\dfrac{1}{\log|x-y|}\right) & 0 < |x-y| < 1 \quad \text{oppure} \quad |x-y| > 1 \\ 0 & \text{altrimenti} \end{cases}$$