Dire se esistono il massimo e il minimo delle seguenti funzioni ed eventualmente trovare i punti nei quali assumono il massimo e il minimo:

1. $f(x,y) = (x^2+y^2)e^{-x^2-y^2}$ in

$C = \{(x,y) \in \mathbf{R}^2 \,|\, 0 \leqslant x \leqslant 1\,, -1 \leqslant y \leqslant 1\}$,

$C = \{(x,y) \in \mathbf{R}^2 \,|\, 0 \leqslant x \leqslant \frac{1}{2}\,, -\frac{1}{2} \leqslant y \leqslant \frac{1}{2}\}$,

$C = \{(x,y) \in \mathbf{R}^2 \,|\, 4x^2 + \frac{64}{9}(y-\frac{5}{8})^2 = 1\}$,

$C = \{(x,y) \in \mathbf{R}^2 \,|\, x^2+y^2 \leqslant 2\}$,

$C = \{(x,y) \in \mathbf{R}^2 \,|\, 0 \leqslant y \leqslant x\}$;

2. $f(x,y) = \operatorname{arctg}\log(x^2+y^2)$ in $C = \{(x,y) \in \mathbf{R}^2 \,|\, x^2+4y^2 \geqslant 1\}$;

3. $f(x,y) = |x| - |y|$ in $C = \{(x,y) \in \mathbf{R}^2 \,|\, x^2+y^2 = 1\}$;

4. $f(x,y,z) = z^2 + xy$ in $C = \{(x,y,z) \in \mathbf{R}^3 \,|\, 0 \leqslant z \leqslant \sqrt{x^2+y^2} \leqslant 1\}$;

5. $f(x,y) = x^2 - y^2 + e^{x^2+y^2}$ in $C = \{(x,y) \in \mathbf{R}^2 \,|\, x^2+y^2 \leqslant r\}$;

6. $f(x,y) = (x^2+2y^2)e^{-2x-y}$ in $C = \{(x,y) \in \mathbf{R}^2 \,|\, x \geqslant 0, y \geqslant 0\}$;

7. $f(x,y) = \operatorname{arctg}(x^4 - y^4)$ in $C = \{(x,y) \in \mathbf{R}^2 \,|\, x^2+y^2 \leqslant 1\}$;

8. $f(x,y,z) = \operatorname{arctg}(x^2+y^2-z^2)$ in $C = \{(x,y) \in \mathbf{R}^2 \,|\, x^2+y^2+z^2 \leqslant 1\}$;