Energy-preserving methods and B-series

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Abstract

Given the system of ordinary differential equations (ODEs)
\[
\dot{y} = f(y), \quad y(0) = y_0 \in \mathbb{R}^n,
\]
assume \( H : \mathbb{R}^n \to \mathbb{R} \) is an invariant or the energy of the system, and that we have \( f(y) = S \nabla H(y) \) with \( S \) a \( n \times n \), skew-symmetric matrix\(^1\).

To derive energy-preserving numerical integration methods for these problems one can use discrete gradient techniques, [3], [4]. Such methods rely on appropriate approximations of \( \nabla H(y) \) and \( S \) (in the case \( S \) depends on \( y \)), and in general cannot be expanded in a B-series. One surprising exception is the average vector field (AVF) method, namely
\[
y_{n+1} - y_n = h \int_0^1 f((1 - \xi)y_n + \xi y_{n+1}) \, d\xi, \quad y_n \approx y(t_n), \quad n = 0, 1, \ldots
\]
Under the assumption that \( S \) is a constant matrix, this is both a B-series method and a discrete gradient method, as recently observed by Quispel and McLaren [5].

Faou, Hairer and Pham, [2], and Chartier, Faou and Murua, [1], characterized precisely all energy preserving B-series for Hamiltonian problems (\( \dot{y} = S \nabla H(y) \), with \( S \) the canonical symplectic matrix).

The recent work on (1) brings new possibilities for the construction of concrete energy-preserving, B-series methods.

After a brief introduction of skew-gradient systems and B-series, we will discuss possible extensions of (1) within the class of B-series methods. To illustrate the performance of the methods some numerical experiments on partial differential equations will be presented.

\(^1\)This implies that the energy \( H \) is preserved along the solution of the ODE, that is
\[
\frac{dH(y(t))}{dt} = \nabla H(y)^T \cdot f(y) = \nabla H(y)^T \cdot S \nabla H(y) = 0.
\]
References


