Thermodynamic of Living Systems

J. Banavar PSU
S. Hubbell Georgia
A. Rinaldo Padova

S. Azaele Princeton
S. Pigolotti Copenhagen
I.Volkov PSU
T. Zillio Alberta

UNIVERSITY OF PADOVA – ITALY
DEPARTMENT OF PHYSICS

Outline

- Kleiber's law of metabolic rate vs body mass
- Consequences of Kleiber's law
- Impact on community
- General form of scaling

Basal Metabolism , B & Body Mass, M

?

$M \sim V$

$B \sim M^{2/3}$

Basal Metabolism B & Body Mass, M

3/4

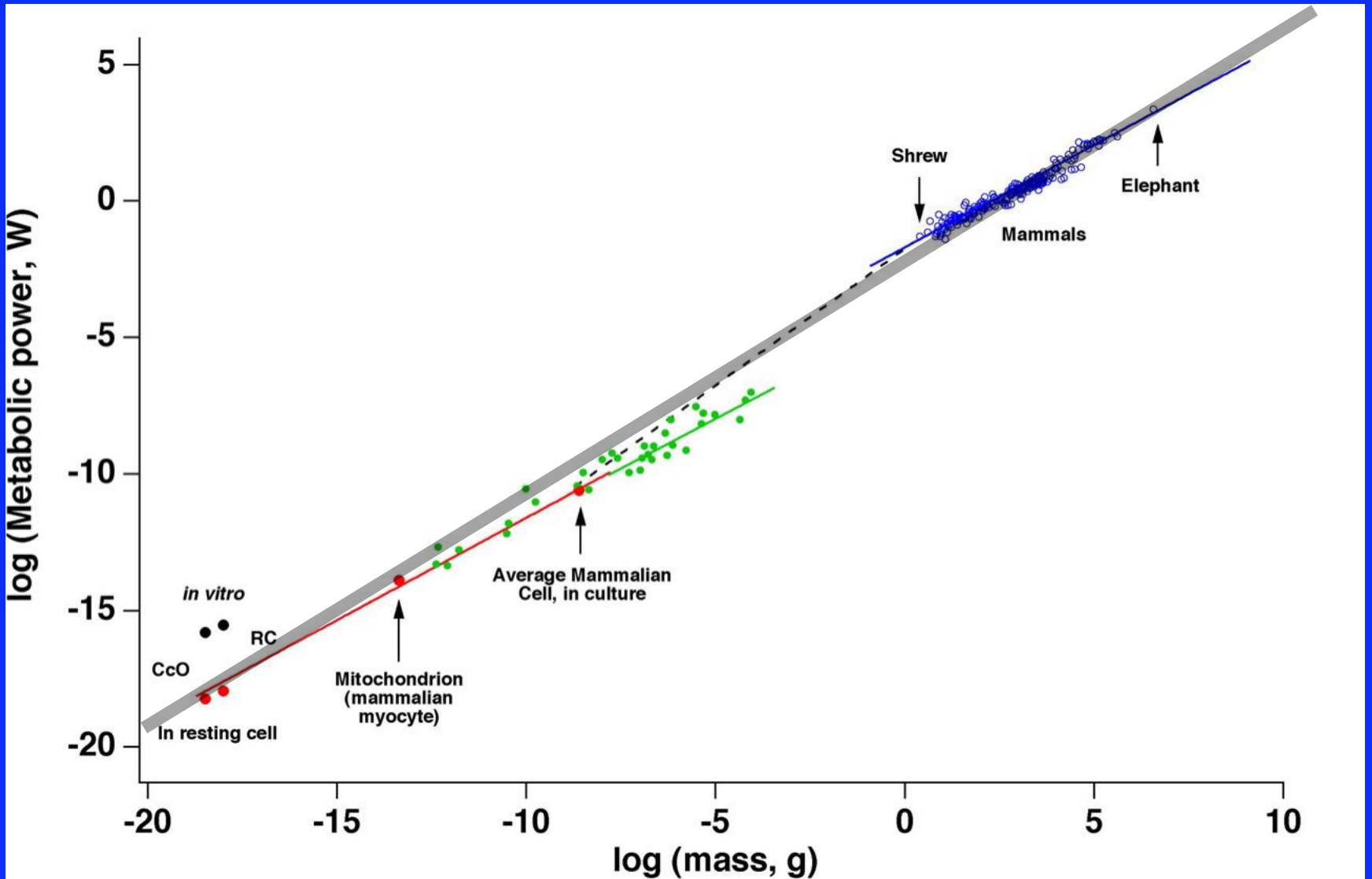
~~2/3~~

B ~ M

Kleiber's law (1940)

!

Brown&West Physics Today 2004



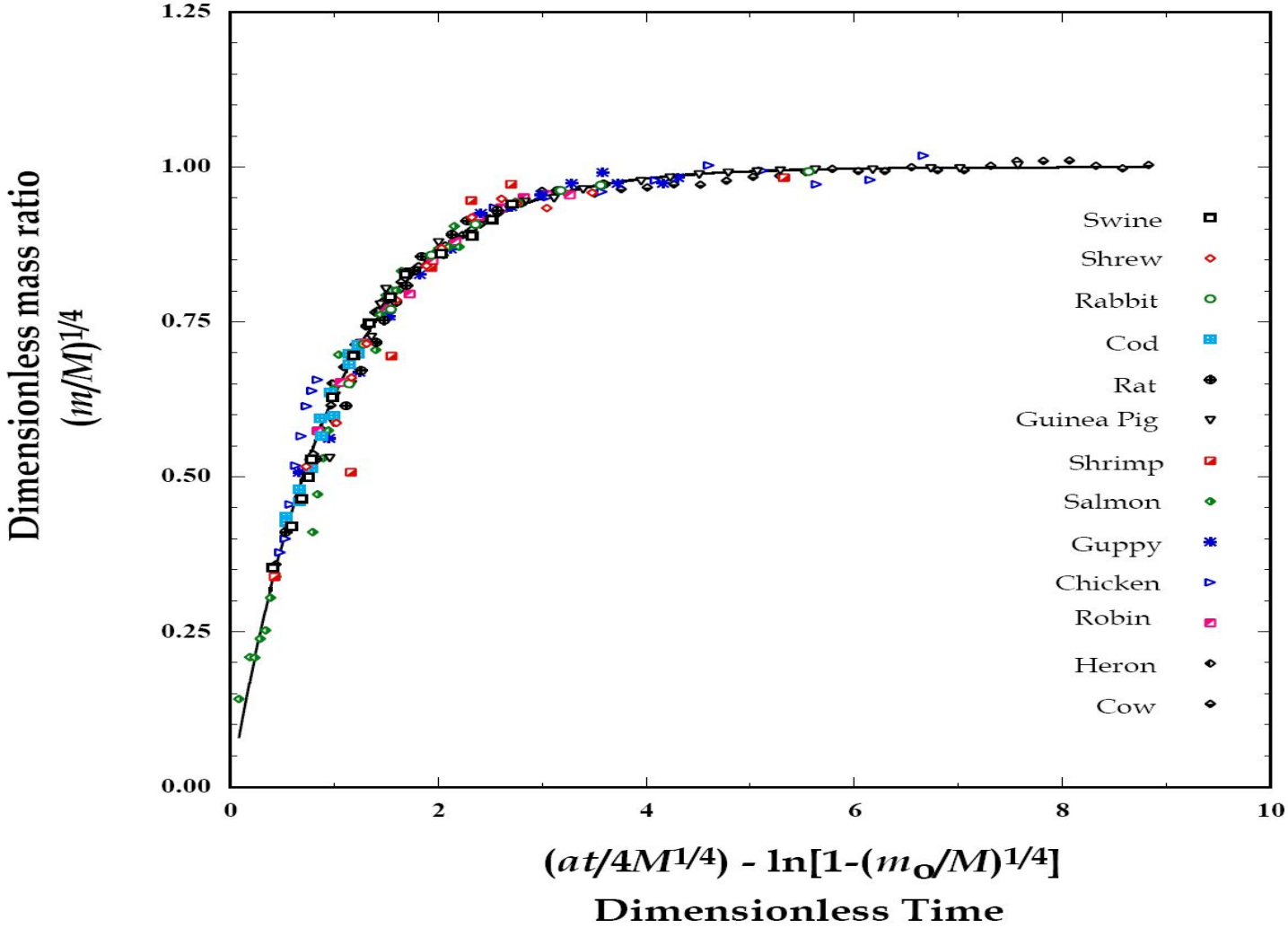
Consequences for single organism with pulsatile flow

- Life span ~ $M^{1/4}$
- Heart beat frequency ~ $M^{-1/4}$
- Aorta diameter ~ $M^{3/8}$
- Capillary density ~ $M^{-1/12}$
- ….

G. West, B. Enquist and J. Brown, Science (1997)

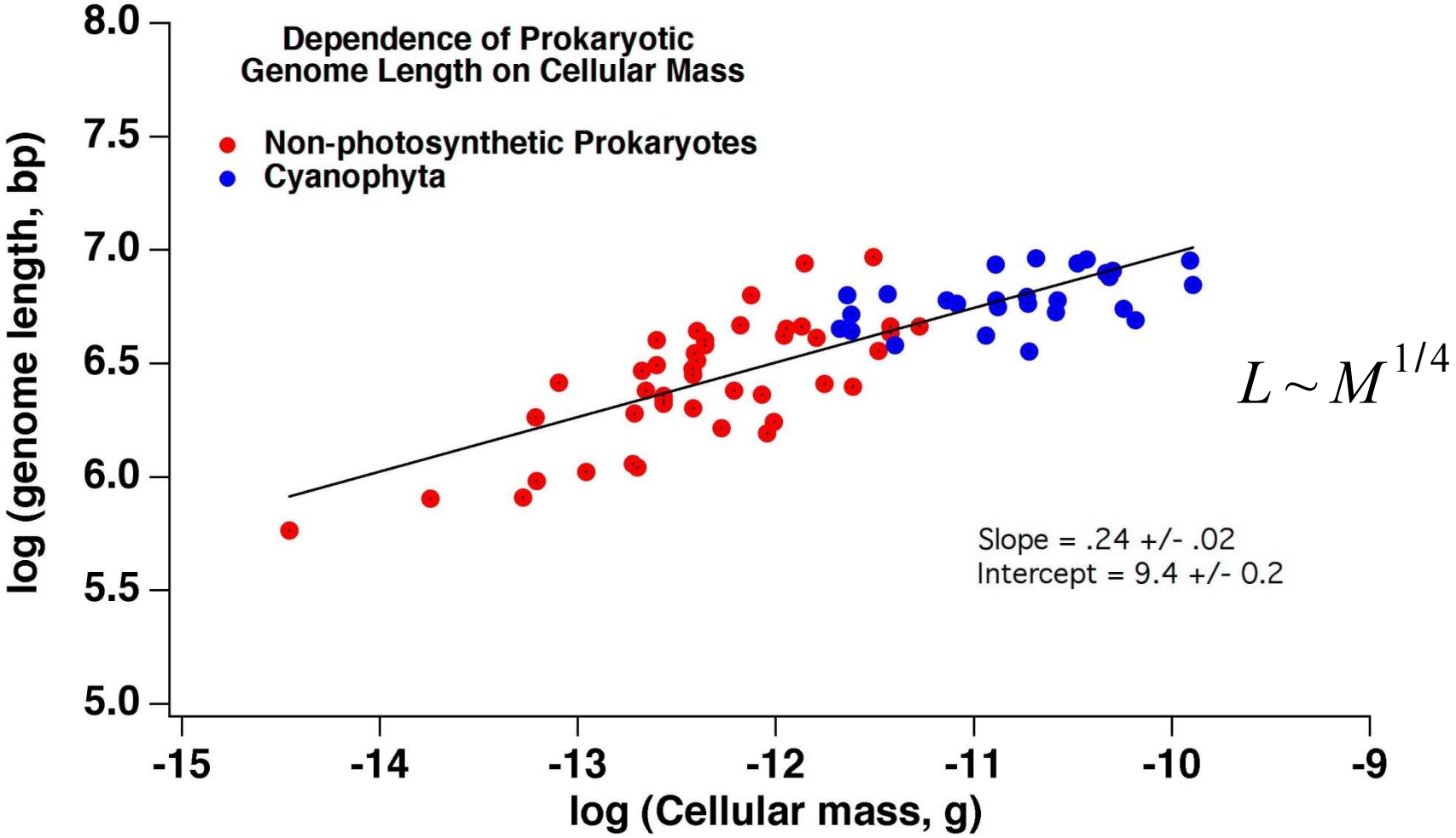
Ontogenetic growth

G.West, B. Enquist and J.Brown, Nature (2002) + comment by J. Banavar, J. Damuth A.M. and A. Rinaldo



Genome length vs cell mass

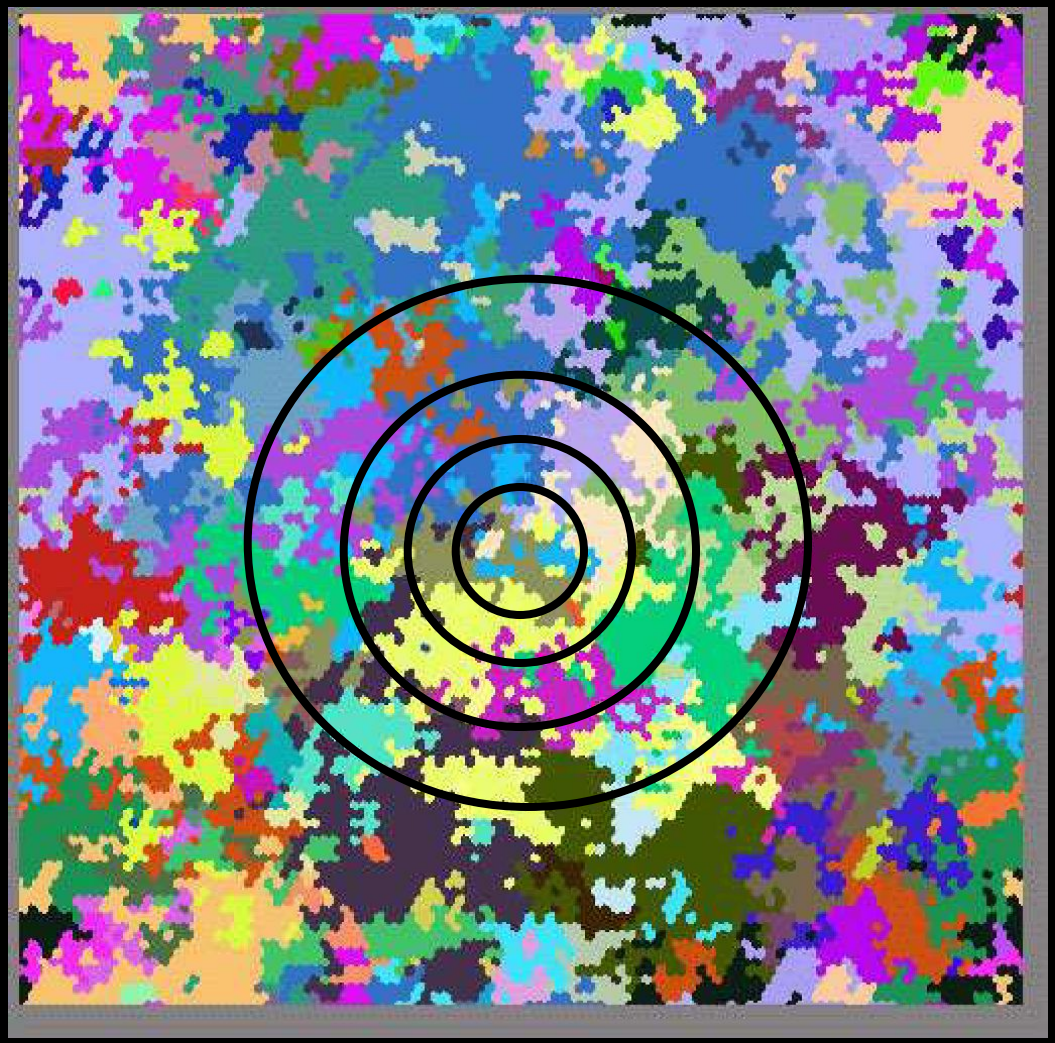
West & Brown Phys. Today (2004)



Community of organisms - forests

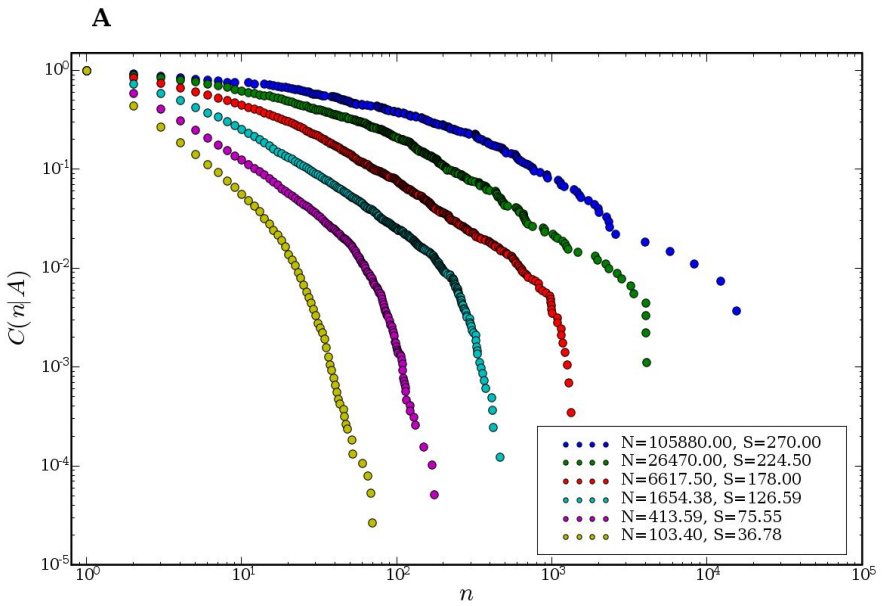
How many species in an area A

$S(A) \sim A^{z}$



Species abundance-Area collapse plot

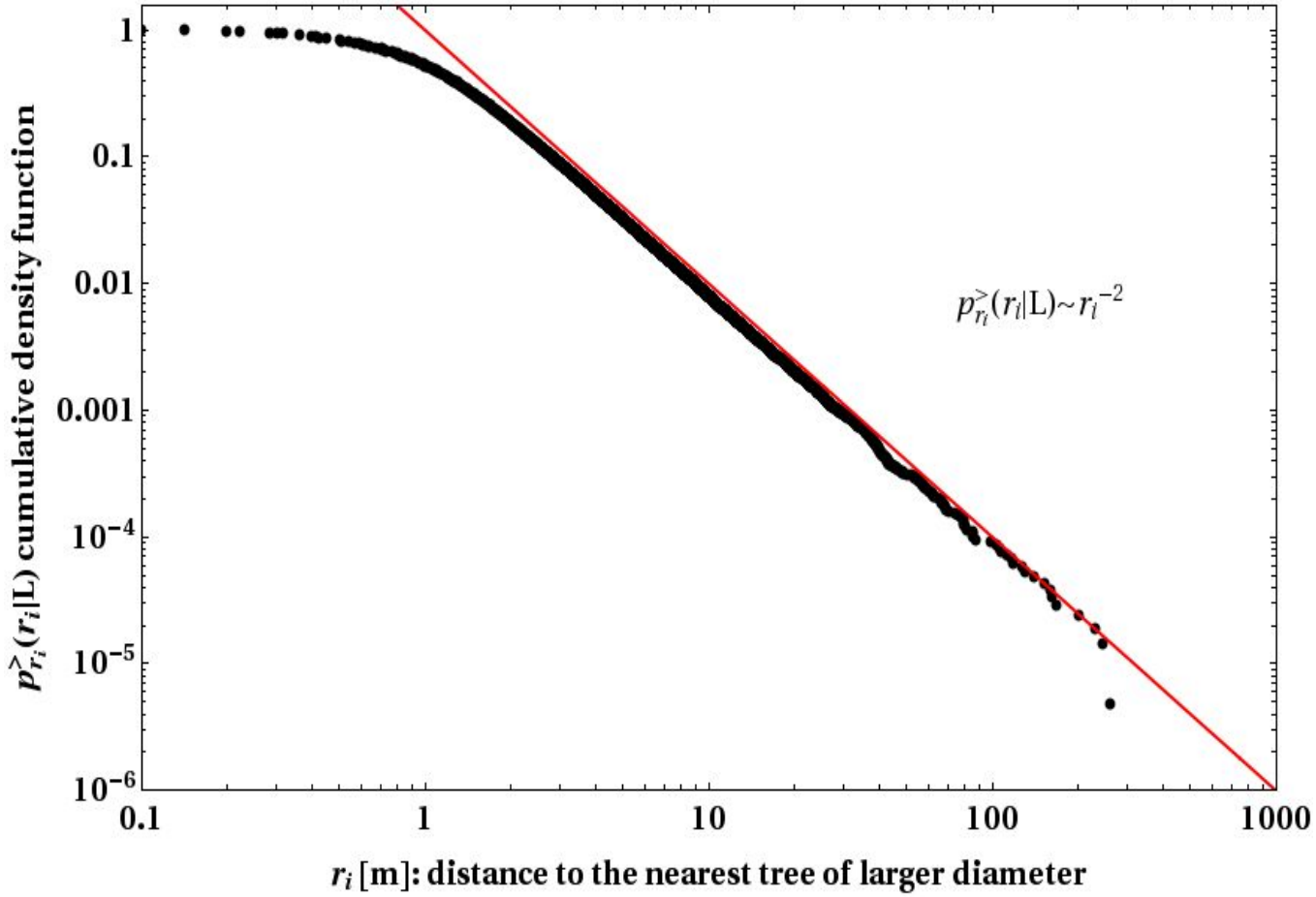
T. Zillio, J. Banavar, J. Green , J. Harte and A.M., PNAS (2007)

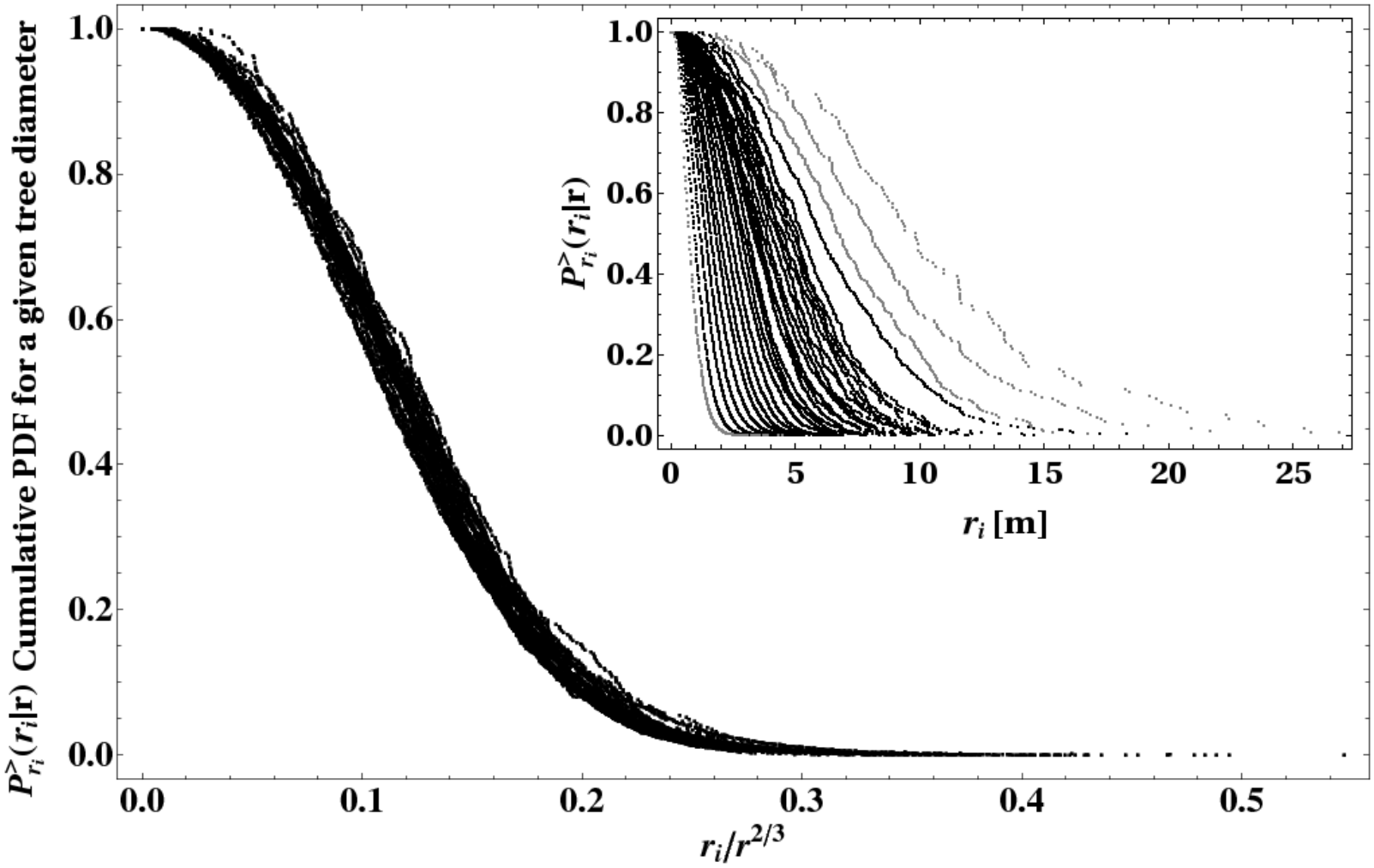


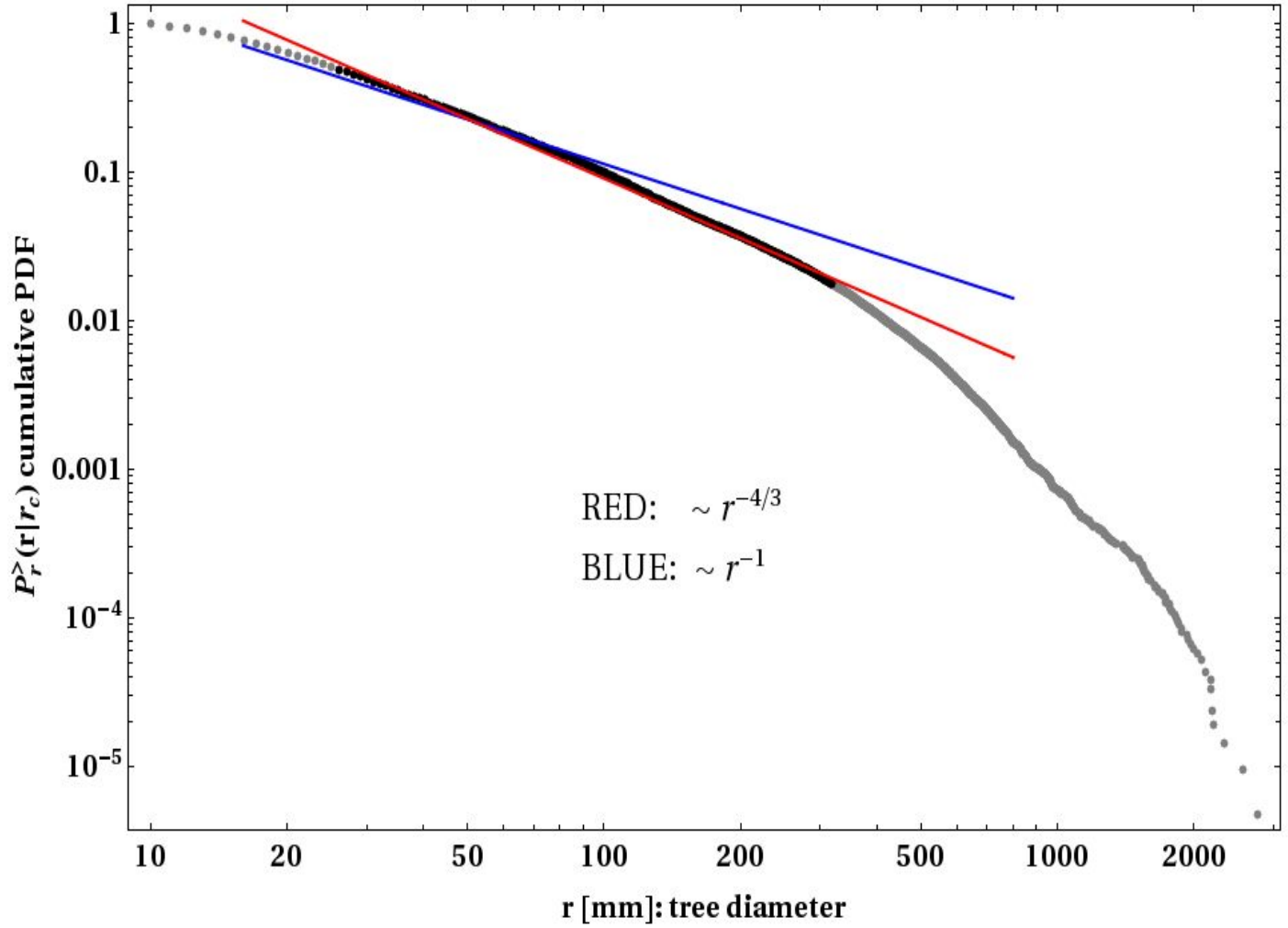
$$N \propto A$$

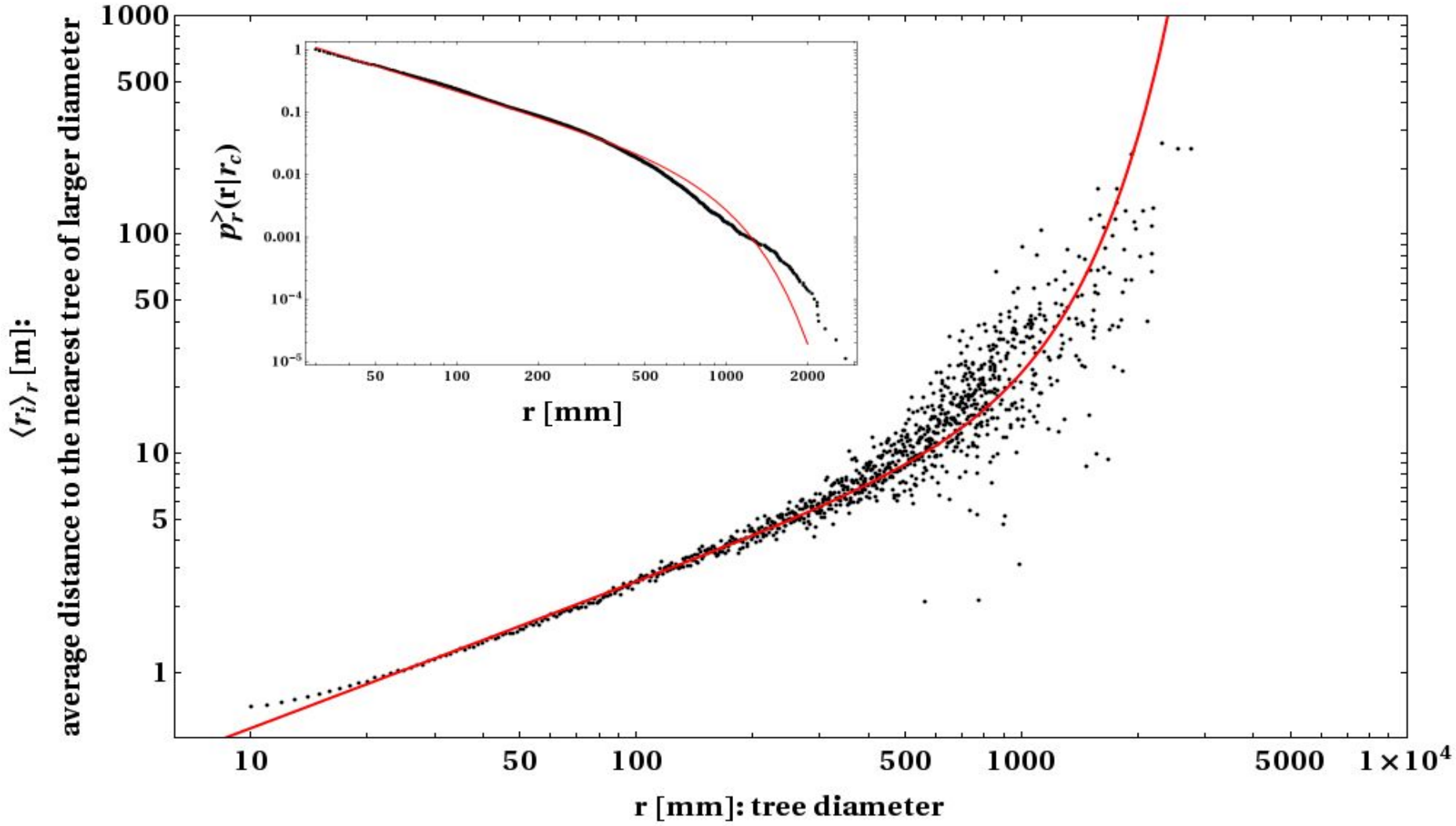
$$C(n|A) = \sum_{m \geq n} P_{RSA}(m|A)$$

If scaling holds

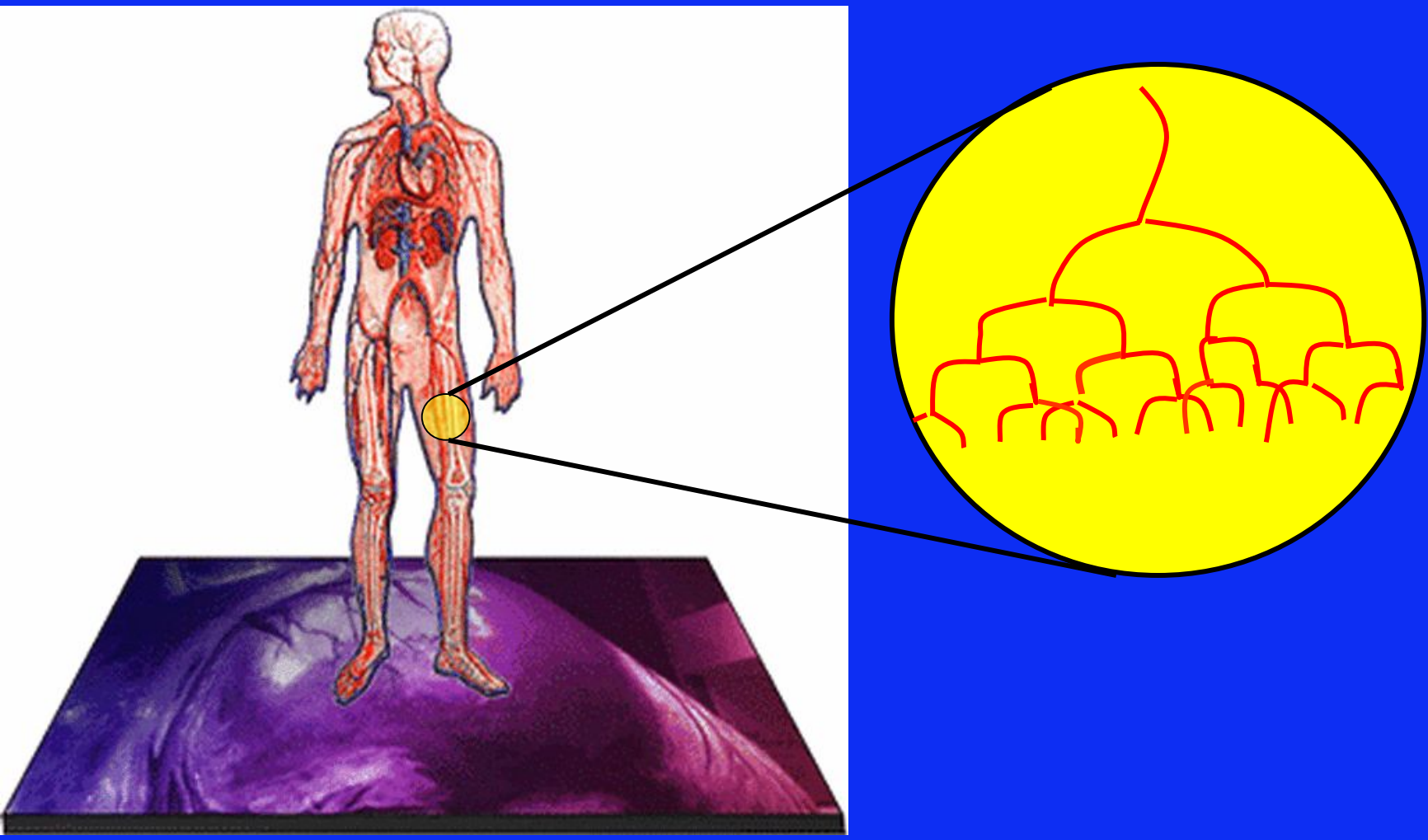

1
0.1
0.01
0.001
10^{-4}
10^{-5}
10^{-6}
0.1
1
10
100
1000
$p^{>}_{r_i}(r_i|\mathrm{L})$ cumulative density function
r_i [m]: distance to the nearest tree of larger diameter
$p^{>}_{r_i}(r_i|\mathrm{L}) \sim r_i^{-2}$


$P^{>}_{r_i}(r_i|\mathbf{r})$ Cumulative PDF for a given tree diameter
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.1
0.2
0.3
0.4
0.5
$r_i/r^{2/3}$
$P^{>}_{r_i}(r_i|\mathbf{r})$
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
r_i [m]


1
0.1
0.01
0.001
10^{-4}
10^{-5}
$P_r^{>}(r|r_c)$ cumulative PDF
RED: $\sim r^{-4/3}$
BLUE: $\sim r^{-1}$
10
20
50
100
200
500
1000
2000
r [mm]: tree diameter

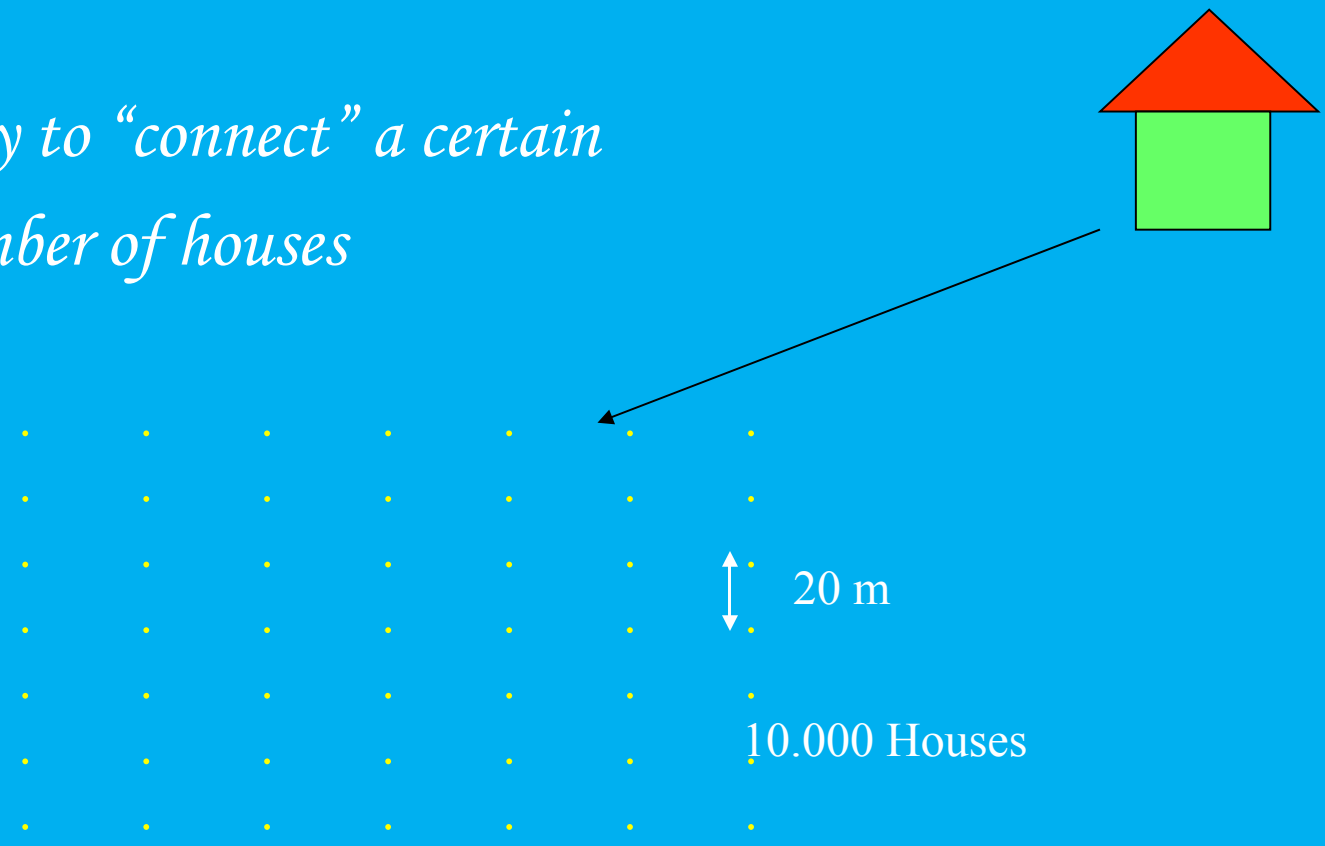

$\langle r_i \rangle_r$ [m]:
average distance to the nearest tree of larger diameter
1000
500
100
50
10
5
1
10
50
100
500
1000
5000
1×10^4
r [mm]: tree diameter
$p_r^>(r|r_c)$
1
0.1
0.01
0.001
10^-4
10^-5
50
100
200
500
1000
2000
r [mm]

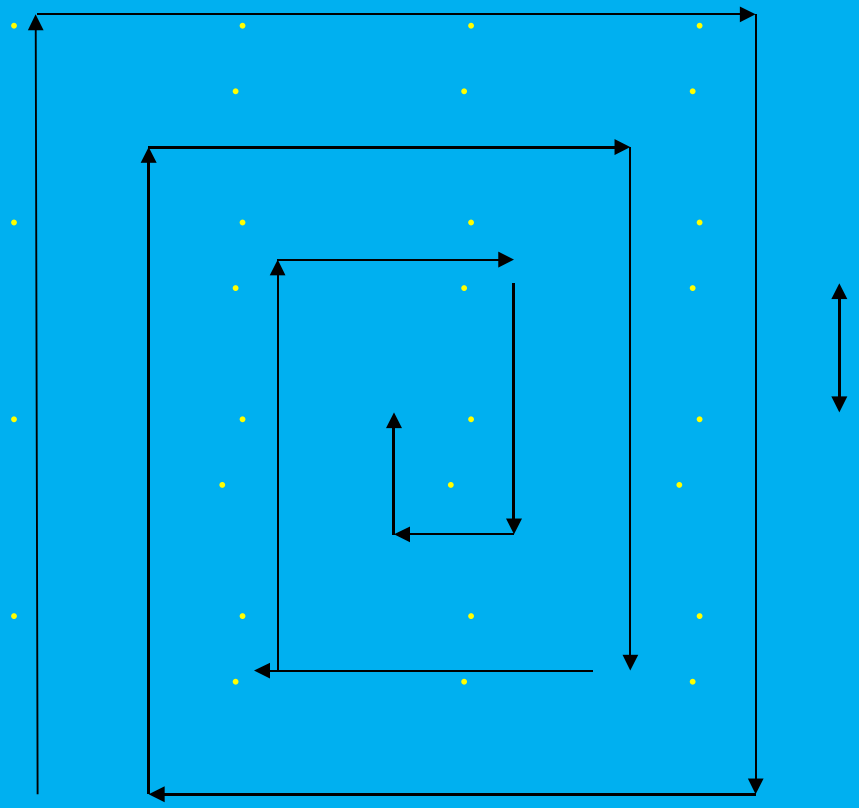
Why 3/4 ?



A simple example of Optimization Principle

The best way to "connect" a certain number of houses

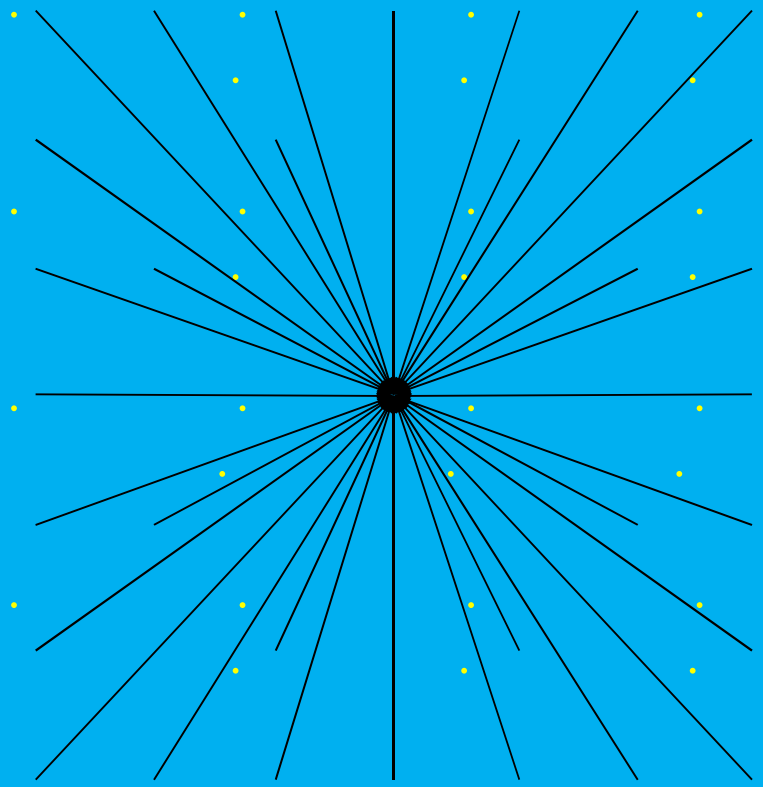




20
m

10.000
Houses

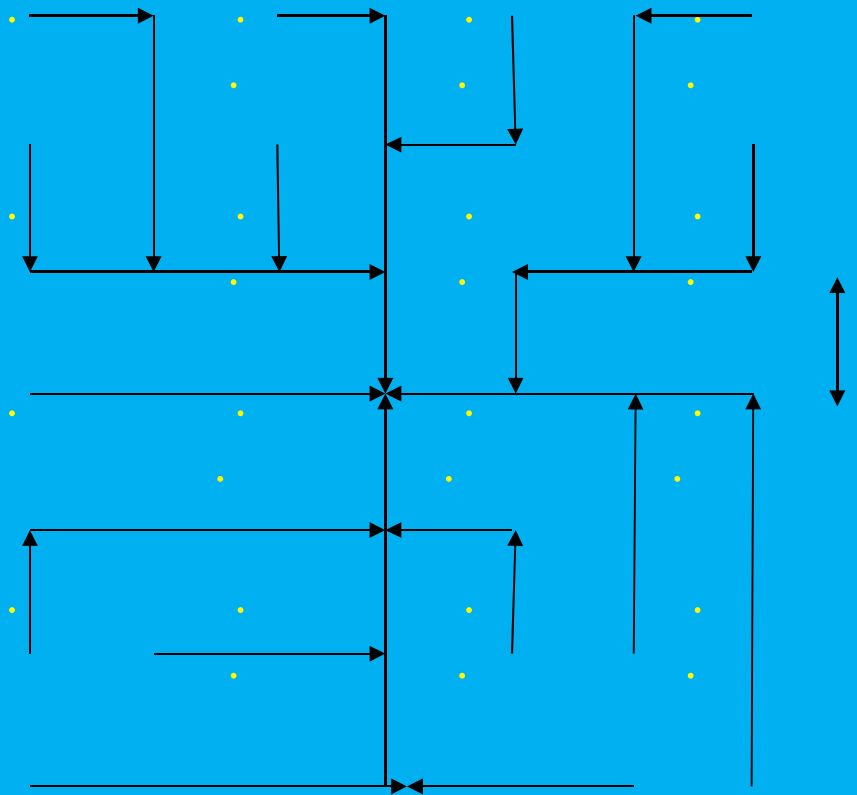
Total length of the path = 200 Km
Average distance from the center = 100 Km



20
m

10.000
Houses

Total length of the path = 7600 Km
Average distance from the center = 0.76 Km

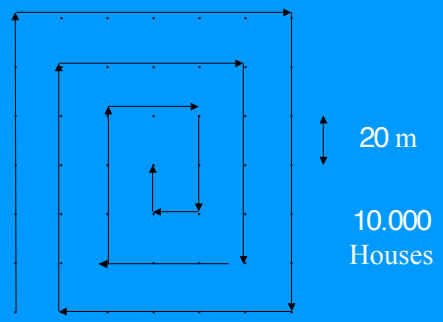


20
m

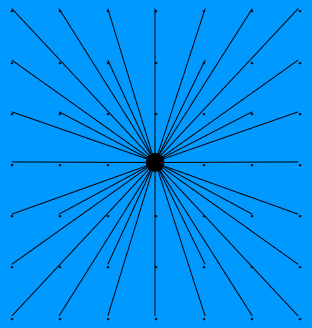
10.000
Houses

Total length of the path = 200 Km
Average distance from the center = 0.96 Km

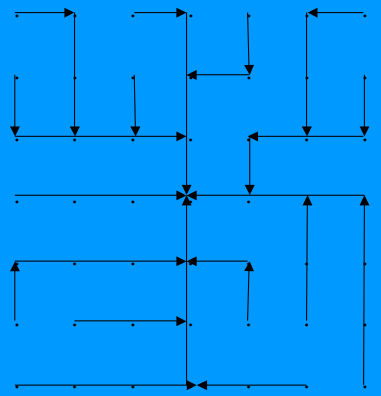
The three simplest cases



Total length of the path = 200 Km
Average distance from the center = 100 Km



Total length of the path = 7600 Km
Average distance from the center = 0.76 Km

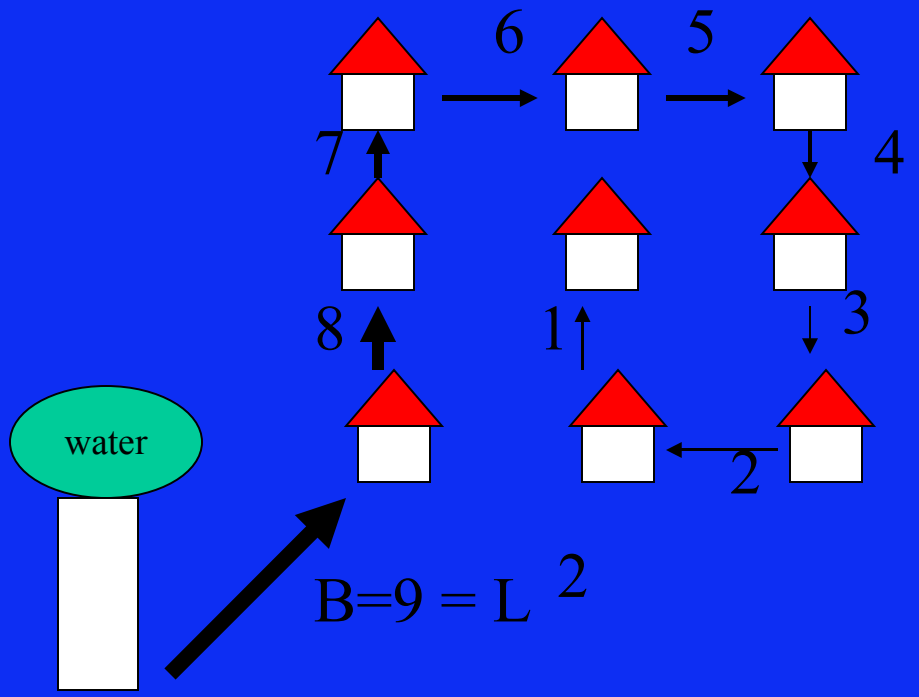


Total length of the path = 200 Km
Average distance from the center = 0.96 Km

Conclusion: The tree structures has the best attributes of the other two!

The theorem in examples

Spiral like distribution system



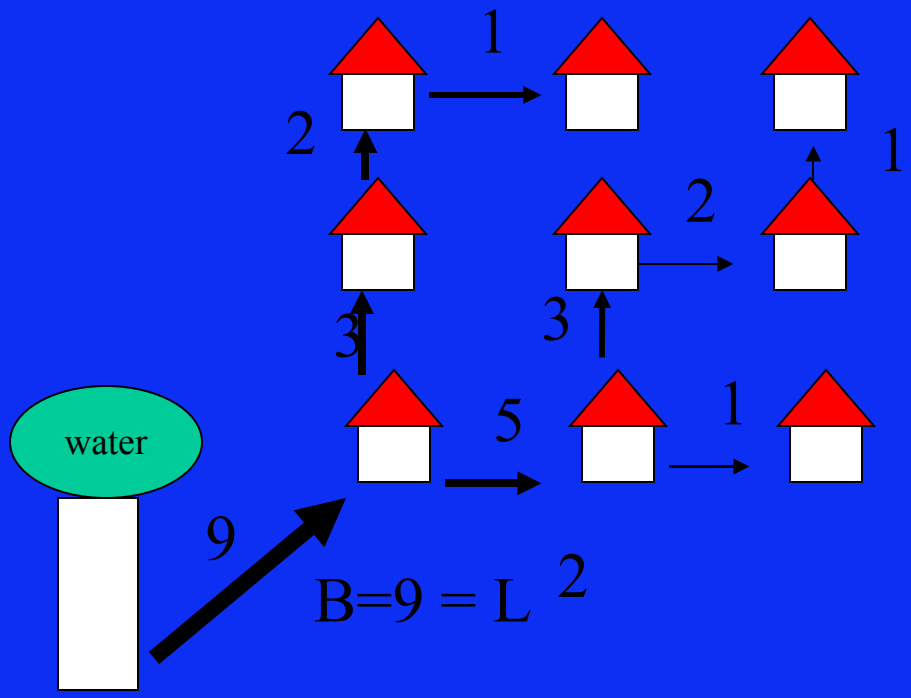
Each house uses 1 water unit per unit time

$B=9 = L^2$

M= total water present $=9+8+\ldots+1=45 \sim 1/2\ L^4$ ➔ $M \sim B^2$

The theorem in examples

Directed network distribution system

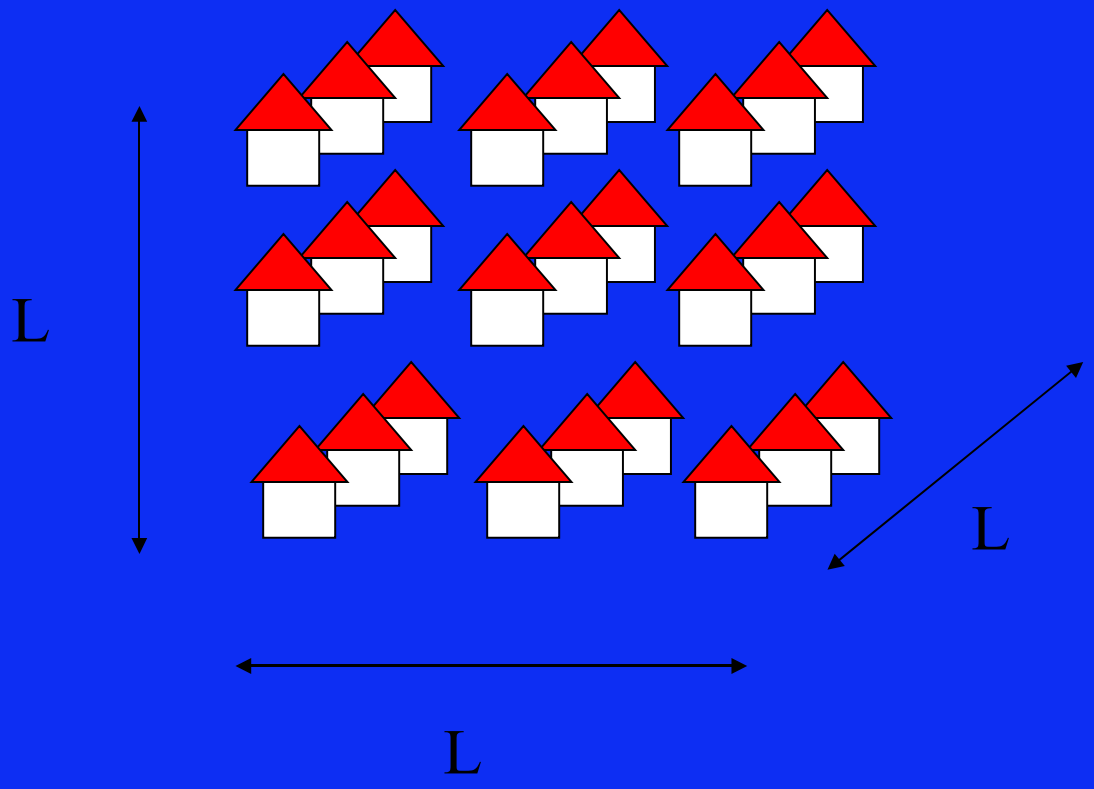


M= total water present=9+3+5…=27 = L 3

➔ M~B

3/2

3 dimension

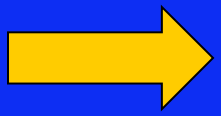


$B = L^3$ $\quad$ Min $M = L^4$ $\longrightarrow$ $M = B^{4/3}$

Optimization Principle:

For a given Metabolism, B,

Minimize the Mass, M !



$$M = B^{4/3}$$

with J. Banavar and A. Rinaldo, Nature (1999)