

MONGE AMPERE CAPACITIES ON ALGEBRAIC VARIETIES: COMPARABILITY AND CONVERGENCE RESULTS

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ABSTRACT. Pluripotential Theory is the study of plurisubharmonic functions and the Monge Ampere complex operator $MA(u) := \det[2i\partial\bar{\partial}u]_{l,j}$ in several complex variables. This theory may be understood as the \mathbb{C}^n generalization of the classical logarithmic potential theory in \mathbb{C} , both from the point of view of geometric function theory and from the one of applications.

Here we consider the pluripotential theory on a irreducible algebraic affine variety $A \subset \mathbb{C}^n$ endowed with its natural parabolic potential and we present two new results. First, we show the equivalence of two capacities for compact subsets of A , namely the Monge Ampere relative capacity and the Chebyshev capacity. Second, we use this result to prove a continuity property of the Monge Ampere operator. In particular, we show the equivalence of several modes of convergence for extremal plurisubharmonic functions.

As an application and a motivation, we combine the above results to provide a sufficient condition for the Bernstein Markov property for a measure μ compactly supported on A .

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