LAPLACE BELTRAMI OPERATOR IN THE BARAN METRIC AND PLURIPOTENTIAL EQUILIBRIUM MEASURE: THE BALL, THE SIMPLEX AND THE SPHERE.

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ABSTRACT. The Baran metric δ_E is a Finsler metric on the interior of $E \subset \mathbb{R}^n$ arising from Pluripotential Theory. We consider the few instances, namely E being the ball, the simplex, or the sphere, where δ_E is known to be Riemaniann and we prove that in such cases the eigenfunctions of the associated Laplace Beltrami operator (with no boundary conditions) are the orthogonal polynomials with respect to the pluripotential equilibrium measure μ_E of E. Moreover, we notice that in all the considered examples (int E, δ_B) is an Einstein manifold. Finally we conjecture that these relationships may hold in a wider generality.

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