RELATIVE CAPACITY ON ALGEBRAIC SUB-VARIETIES OF $\mathbb{C}^n$ AND APPLICATION TO THE BERNSTEIN MARKOV PROPERTY

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Abstract. Let $\mathcal{A}$ be a m dimensional algebraic sub-variety of $\mathbb{C}^n$. For any open bounded hyperconvex $\Omega \subset \mathcal{A}$ and any compact set $K \subset \Omega$ we denote by $\mathrm{Cap}_{\mathcal{A}}(K, \Omega)$ the relative (Monge-Ampere) capacity of K in Ω; see [2, 8]. Once the Rudin Coordinates $(z, w) \in \mathbb{C}^m \times \mathbb{C}^{n-m}$ for $\mathcal{A}$ are chosen as in [5], we consider the pseudo-balls $\Omega(z_0, r) := \{(z, w) \in \mathcal{A} : |z - z_0| \leq r\}$ and re-define the Chebyshev constant $T(K, \mathcal{A})$ taking the normalization of polynomials in its usual definition on the pseudo-ball $\Omega := \Omega(0, 1)$ in place of the unit ball. We prove that for any $r < 1$ there exist $0 < c_1, c_2 < +\infty$ such that for any non pluripolar compact set $K \subset \Omega(0, r)$ we have

$$\left(\frac{c_1}{\mathrm{Cap}_{\mathcal{A}}(K, \Omega)}\right)^{1/m} \leq -\log T(K, \mathcal{A}) \leq \frac{c_2}{\mathrm{Cap}_{\mathcal{A}}(K, \Omega)}.$$

This *Comparison Theorem* was originally proved in the flat case (i.e., $\mathcal{A} = \mathbb{C}^n$) in [1].

This estimate can be used to recover the main result of [4] in this slightly modified setting. If $\{K_j\}$ is a sequence of compact subsets of the regular non-pluripolar compact set $K \subset \Omega(0, r)$, $r < 1$ and $\Omega \subset \mathcal{A}_{\mathrm{reg}}$, then the condition

$$\lim_j \mathop{\mathrm{Cap}}_{\mathcal{A}}(K_j, \Omega) = \mathop{\mathrm{Cap}}_{\mathcal{A}}(K, \Omega)$$

is equivalent (among other properties) to the local uniform convergence of the plurisubharmonic extremal functions $V_{K_j}(z, \mathcal{A})$ to $V_K(z, \mathcal{A})$ as they are defined in [6].

As an application, we prove a sufficient mass density condition for the Bernstein Markov property on algebraic sets in $\mathbb{C}^n$ and a weighted Bernstein Markov property for unbounded sets in $\mathbb{C}$ using the techniques from [7, 3].

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