PLURIPOTENTIAL NUMERICS

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ABSTRACT. We introduce numerical methods for the approximation of the *extremal plurisubharmonic function* V_E^* of a compact $\mathcal{L}$-regular set $E \subset \mathbb{C}^n$ and its *transfinite diameter* $\delta(E)$.

The methods rely on the computation of a *polynomial mesh* for ∂E and numerical orthonormalization of a suitable basis of polynomials. We prove the convergence of the approximation of $\delta(E)$ and the uniform convergence of our approximation to V_E^* on all $\mathbb{C}^n$ providing an *a priori* estimate on the error. Our algorithms are based on the properties of polynomial meshes and Bernstein Markov measures.

Numerical tests are presented for some simple cases with $E \subset \mathbb{R}^2$ to illustrate the performances of the proposed methods.

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