WAM: a Matlab package for polynomial fitting and interpolation on Weakly Admissible Meshes

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Weakly admissible meshes, polynomial approximation, discrete least squares

The notion of *Weakly Admissible Mesh* (WAM) has been introduced in the seminal paper [4] and since then it has emerged as a powerful tool in multivariate polynomial approximation. We recall that a WAM is a sequence of finite subsets of a multidimensional compact set or manifold, say $X_n \subset K \subset \mathbb{R}^d$ (or $\mathbb{C}^d$), which are *norming sets* for total-degree polynomial subspaces,

$$\|p\|_{L^\infty(K)} \leq C_n \, \|p\|_{\ell^\infty(X_n)} \; , \;\; \forall p \in \mathbb{P}_n^d(K) \; , \tag{1}$$

where both C_n and $\mathrm{card}(X_n)$ increase at most polynomially with n.

We quote among their properties that WAMs are preserved by affine transformations, can be constructed incrementally by finite union and product, and are stable under small perturbations on Markov compacts [7]. Moreover, WAMs are well-suited for uniform *least-squares approximation* [3, 4], and for *polynomial interpolation* at suitable extremal subsets extracted from them, which are approximate versions of Fekete and Leja points [1, 2].

We present here a Matlab package, named WAM, that implements construction of WAMs (with possible cardinality reduction), polynomial fitting on WAMs and polynomial interpolation on discrete extremal sets extracted from WAMs, using only basic Numerical Linear Algebra algorithms, on 2-dimensional and 3-dimensional compact sets with various geometries: convex and concave polygons, convex and star-shaped $C^{1,1}$ domains, circular sections (e.g. circular sectors, lenses and lunes), cubes, cones and pyramids, solids of rotation.

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