Modelling for Constraint Programming

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4. Combining Viewpoints, Modelling Advice

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Which Viewpoint to Choose?

- Sometimes one viewpoint is clearly better, e.g. if we can't express the constraints in one
- But different perspectives often allow different expression of the constraints and different implied constraints
  - can be hard to decide which is better
- We don't need to choose one viewpoint – we can use two (or more) at once
- We need *channelling constraints* to link the variables

Combining Viewpoints: Permutation Problems

- Dual viewpoints of a permutation problem with variables $x_1, x_2, \ldots, x_n$ and $d_1, d_2, \ldots, d_n$
- Combine them using the channelling constraints $(x_i = j) \equiv (d_j = i)$
- The channelling constraints are sufficient to ensure that $x_1, x_2, \ldots, x_n$ are all different
  - but might be beneficial to have a specific allDifferent constraint as well and enforce GAC

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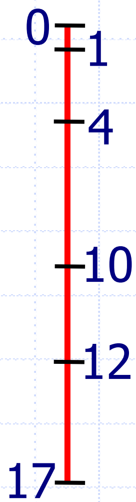
Combining Viewpoints: Integer & Set Variables

- In a nurse rostering problem, we can allocate shifts to nurses or nurses to shifts
- First viewpoint:
  - an integer variable n_{ij} for each nurse i and day j
  - its value is the shift that nurse i works on day j
- Second viewpoint:
  - a set variable S_{kj} for each shift k and day j
  - its value is the set of nurses that work shift k on day j
- Channelling constraints: $(n_{ij} = k) \equiv (i \in S_{kj})$
- Constraints on nurse availability are stated in the first viewpoint; constraints on work requirements in the 2nd (e.g. no. of nurses required for each shift)

The Golomb Ruler Problem

- A Golomb ruler with m marks consists of
  - a set of m integers $0 = x_1 < x_2 < \ldots < x_m$
  - the $m(m-1)/2$ differences $x_j - x_i$ are all different
  - Objective: find a ruler with minimum length x_m
- First viewpoint: variables $x_1, x_2, \ldots, x_m$
  - $x_j - x_i \neq x_l - x_k$ for all distinct pairs
  - $x_1 < x_2 < \ldots < x_m$
- Second viewpoint: variables d_{ij}, $1 \leq i < j \leq m$
  - allDifferent($d_{11}, d_{12}, \ldots, d_{m-1,m}$)
  - $d_{ik} = d_{ij} + d_{jk}$ for $1 \leq i < j < k \leq m$
- Channelling constraints: $d_{ij} = x_j - x_i$
  - not always binary



Constraints in Combined Viewpoints

- It is safe to combine two complete models of a problem, with channelling constraints
- But often unnecessary and inefficient
- If some constraints are more easily expressed in one viewpoint, we don't need them in both
  - e.g. nurse rostering
  - constraints on nurse availability are stated in the 'nurse' viewpoint
  - constraints on work requirements (e.g. no. of nurses required for each shift) are stated in the 'shift' viewpoint
- or if they propagate better in one viewpoint
  - e.g. $x_j - x_i \neq x_l - x_k$ v. allDifferent($d_{11}, d_{12}, \ldots, d_{m-1,m}$) in the Golomb ruler problem

Choosing the Search Variables

- We need to choose a set of variables such that an assignment to each one, satisfying the constraints, is a complete solution to the problem
- Assume we pass the search variables to the search algorithm in a list or array
  - the order defines a static variable ordering
    - though we can still use a dynamic ordering

Search Variables

- When a model combines two (or more) viewpoints of a problem, which variables should drive the search?
- Assigning values to either set of variables would be sufficient to solve the problem
  - even if we did not express the problem constraints on those variables
  - the channelling constraints ensure that we can assign values to one set of variables but define the constraints in the other viewpoint, if we want

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Search Variables – Permutation Problems

- We can use both sets of variables as search variables
  - e.g. use a dynamic variable order e.g. variable with smallest domain in either viewpoint
    - combines variable and value ordering: dual variable with smallest domain corresponds to the value occurring in fewest domains (in the other viewpoint)

Search Variables – Golomb Ruler

- Search strategy assigns values to x_1, x_2,… , x_m in that order
- NB there are only m of these variables and $m(m-1)/2$ difference variables
  - heuristic: prefer the smaller set of search variables

SONET Problem: Viewpoints

- Whether a given node is on a given ring:
  - $x_{ij} = 1$ if node i is on ring j
- Which ring(s) each node is on:
  - N_i = set of rings node i is on
- Which nodes are on each ring
  - R_j = set of nodes on ring j
- In principle, any of these viewpoints could be the basis of a complete CSP model
  - channelling constraints $(x_{ij} = 1) \equiv (i \in R_j) \equiv (j \in N_i)$
- There are also auxiliary variables
  - n_i = the number of rings each node is on ($= |N_i|$)

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Possible Choices

- Use just one set of variables, e.g. x_{ij} – the others are just for constraint propagation
- Use two (or more) sets of variables (of the same type) e.g. R_j, N_i
  - interleave them in a sensible (static) order
  - or use a dynamic ordering applied to both sets of variables
- Use an incomplete set of variables first, to reduce the search space before assigning a complete set
  - e.g. decide how many rings each node is on (search variables n_i) and then which rings each node is on (x_{ij})
    - another strategy adds assigning the objective variable first – see earlier

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Automating Modelling

- There are lots of choices to make in modelling a problem as a CSP
  - difficult even with experience
- Can it be automated?
  - some initial steps so far
    - e.g. systems that propose models given a high-level specification
    - descriptions of common patterns in modelling

Advice from the Folklore

- *Reduce the number of variables*
  - if we only use one viewpoint:
    - a model which needs fewer variables to describe the solutions to the problem is likely to be a better model
    - e.g. an integer model is probably better than a Boolean model
  - But only if the variables allow the constraints to be expressed in a way that propagates well
    - artificially reducing the number of variables by inventing a single variable to replace a pair of variables will not give a better model

Advice from the Folklore 2

- *Reduce the number of constraints*
  - rewriting a set of constraints in a more compact form is likely to be beneficial, *if* the resulting constraints can propagate efficiently
    - e.g. combine constraints with the same scope
    - use a global constraint to replace a set of constraints
  - But simply conjoining constraints for the sake of reducing their number will not give a better model if the new constraints cannot propagate efficiently

More Advice

- *Add more variables*
  - auxiliary variables to allow constraints to be expressed
  - new viewpoints allowing a different perspective on the problem
- *Add more constraints*
  - implied constraints
  - channelling constraints to link new variables
- *Check empirically*
  - that a change does reduce run-time

Conclusion

- Aim for a rich model
  - multiple viewpoints
  - auxiliary variables
  - implied constraints
- Understand the problem as well as you can
  - build that insight into the model
  - the better you can understand a problem, the better you can solve it

THE END