Abstract projector and its components in a spin model and its interpretation in a pre-logical environment suited for psychoanalysis

Giulia Battilotti*
Miloš Borozan
Rosapia Lauro Grotto

IQSA
Bruxelles July 22-27 2024

July, 23 2024

Contents

Representing the result of a quantum measurement in logic

Let us consider:

- the set D of the results of the measurement of a quantum particle $\mathcal{A}$ w.r.t. a given observable
- the closed terms of the language t denoting each result, namely each t denotes a state with the associated positive probability of being measured
- the formulae $A(t)$ for "the particle A is measured in state s with probability $P(s)$"

If Γ is a set of hypothesis for the measurement (the preparation), we obtain the logical consequences

$$\Gamma \vdash A(t) \quad \text{for all } t \in D$$

This defines the ω-quantifier $\forall_\omega$ on the set of formulae $A(t)$, gathering the parametrized objects $A(t)$:

$$\Gamma \vdash (\forall_\omega x \in D) A(x) \quad \equiv \quad \Gamma \vdash A(t) \text{ for all } t \in D$$

Then the formula $(\forall_\omega x \in D_d) A(x)$ represents the mixed state after measurement [actually in most cases it coincides with a conjunction $A(t_1) \& \ldots \& A(t_n)$].

First abstraction: representing pure states in logic

As a first level of abstraction, let us drop the closed terms t and introduce a free variable of the language:

$$z$$

for the elements of the domain D, in order to represent the measurement of $\mathcal{A}$ by the generic formula $A(z)$. Then we write the logical consequence

$$\Gamma(-z), z \in D \vdash A(z)$$

(where $\Gamma(-z)$ indicates that the set of hypothesis does not depend on z).

Since the quantifier $\forall$ can be defined by putting:

$$\Gamma(-z) \vdash (\forall x \in D)A(x) \quad \equiv \quad \Gamma(-z), z \in D \vdash A(z)$$

we obtain

$$\Gamma(-z) \vdash (\forall x \in D)A(x)$$

Then the quantified formula $(\forall x \in D)A(x)$ represents the pure state.

defined/finite and undefined/infinite

The mixed state after measurement is a defined object characterized by closed terms, being defined by external parameters.

The pure state prior to measurement is described by an internal variable in the logical language, ranging on a domain D, and hence is defined as long as D is defined.

We keep that D is defined if and only if

$$z \in D \text{ means that } z = t \text{ for some } t$$

that is not the case, prior to measurement.

Moreover, we keep that, if it is not the case, the elements of D cannot be counted and then D is infinite.

Then we have represented the pure quantum state as an undefined infinite object.

Abstract projector: general form

Let us consider spin observables only. Then the observable is σ_d, characterized by a direction d, and the measurement on $\mathcal{A}$ yields the domain D_d

By abstracting both the above definitions (for mixed and pure states) with respect to d, and dropping the domains D_d and the closed terms and variables for its elements, we get the unique form

$$\Box\Gamma \vdash \Box A \;\equiv\; \Box\Gamma \vdash A$$

that defines the modal operator $\Box$ (S4). It satisfies

$$\Box\Box A = \Box A$$

then $\Box$ is an abstract projector:

$\Box$ can attribute a sharp yet undefined state to the particle

$\Box$ can have both an infinite/undefined and a finite/defined interpretation: namely, underneath, it can depend on an internal variable or it can gather externally parametrized objects.

Abstract projector: analysis

The generic spin observable is associated to a self-adjoint matrix $\hat{O}$ that is decomposed as follows

$$\hat{O} = \alpha I + \beta_x \sigma_X + \beta_y \sigma_Y + \beta_z \sigma_Z$$

where I is the identity matrix, $\sigma_X, \sigma_Y, \sigma_Z$ are the Pauli matrices, $\alpha, \beta_x, \beta_y, \beta_z$ are real coefficients.

Usual assumption: $\hat{O} = \sigma_d$, namely a direction d for the measurement is characterized: this means that some coefficient β is not 0. Then one characterizes a couple of eigenvectors w.r.t. the direction d. Finite interpretation.

Odd case: $\hat{O} = I$. It has infinite-many eigenstates, no external characterization, d is undefined. Infinite interpretation of the abstract projector $\square$.

Abstract projector: finite interpretation

As for the finite interpretations, $\Box$ is the abstract version of $\forall_\omega$ that gathers all the results depending on an external parameter.

Let us assume then $d = d(t)$ where t is now a temporal parameter making the direction d of the spin measurement of the fixed particle $\mathcal{A}$ evolve, from the initial one (Heisenberg picture).

In Schroedinger picture, the same results are given by considering the application of the initial observable σ_d to the evolution of the state of the particle.

Then we obtain operators $\Box_d$ for every d, defined by putting:

$$\Box\Gamma \vdash \Box_d A \;\equiv\; \Box\Gamma, \sigma_d \vdash A(t) \text{ for all } t \in T$$

The Pauli matrices $\sigma_X, \sigma_Y, \sigma_Z$ give the basic finite components for the finite interpretation of the abstract projector, as we shall see in more detail.

Abstract projector: infinite interpretation

As for the infinite interpretation, $\Box$ is like an abstract quantifier defined by means of an internal variable ranging on the fixed domain T.

The parameter t is imported as a variable, that means: the above definition is abstracted on d, and the following definition of $\Box$ can be put:

$$\Box\Gamma \vdash \Box A \;\equiv\; \Box\Gamma, z \in T \vdash A(z)$$

Then $\Box A$ is interpreted as follows:

$$\Box A \equiv (\forall x \in T)A(x)$$

Then $\Box A$ is the unique sharp yet undefined state of the particle $\mathcal{A}$, obtained as a sum of all possible measurement contexts.

Other abstract components

Other elements can be abstracted, since they are independent of the direction d chosen for the measurement:

- The mixed state given by the couple of eigenstates of any observable σ_d, $\frac{1}{2}P_\downarrow + \frac{1}{2}P_\uparrow$
- The singlet state $\frac{1}{\sqrt{2}}|\uparrow\downarrow> - \frac{1}{\sqrt{2}}|\downarrow\uparrow>$

Then we interpret it by other logical constants:

- the logical constant $\perp$ (false) describes the abstract mixed state, namely the "totally non-integrated/separated element"
- the logical constant 1 (true) describes the abstract singlet state, namely the "totally integrated/non-separable object"

In logic, $\perp$ and 1 are *dual* constants.

Representation in Freud - The Interpretation of Dreams 1900

The original description of the Unconscious Representations dates back to the Freudian essay *On aphasia* (Freud, 1891) where they are described as open sets named *thing-presentations*.

The open nature of the Unconscious Representations was described also in *The Interpretation of Dreams* (Freud, 1900) where Representations are defined by the following core properties:

- Displacement
- Condensation
- Absence of negation and contradiction
- Absence of serial/temporal order
- Substitution of the external reality with the psychic one

Matte Blanco's Infinite Sets - Matte Blanco 1975

Investigating the original Freudian logical view, Matte Blanco described the Unconscious Representations as *Infinite Sets* (Matte Blanco, *The Unconscious as Infinite Sets* 1975).
Matte Blanco's infinite sets are derived from the two founding principles:

- generalization: The Unconscious does not deal with individual elements, but only with classes to which they belong;
- symmetry: the Unconscious treats the asymmetrical relations as if they were symmetrical.

The word symmetry refers to the sameness, identity between two things and their fundamental indistinguishability.

Since the relation of contradiction is nevertheless a relation, the Ucs treats opposites as identical, therefore producing the *Symmetric Mode* of thinking, as the formal description of the structural Unconscious.

The need of infinite singletons

Following *The Unconscious as Infinite Sets*, the guideline for finding of a logical embedding of the Symmetric Mode into the usual logical setting is provided by the assumptions:

- every set is infinite
- every relation is symmetric

However: relations are all symmetric only in singletons! For: if $a, b \in U$, and $a \neq b$, one can put $a < b$.

Solution : *infinite singletons*

Infinite singletons and quantum states

An infinite singleton is achieved as a non-extensional domain D whose elements cannot be distinguished one w.r.t. another, namely:

$$z \in D \text{ is not equivalent to } z = t \text{ for some } t \text{ at the formal level}$$

even when the extensional equality characterizing D as a set is recognized at the metalevel.

The elements of an infinite singleton are not separable: they are (like) one and only. But its elements cannot be counted: they are infinite.

Notice that when a singleton is infinite there is no name for the object it contains!

We assume that an infinite singleton corresponds to the domain D associated to the quantifier describing the state of a particle prior to measurement, as seen above.

Need for a finitization - Freud On Aphasia 1891

Freud's original requirement in *On Aphasia* is that Unconscious Representations (*Thing Presentations*) can access consciousness only when linked to words (*Word Presentations*), which are necessarily finite Representations.

According to Freud, Word Presentations are closed sets as they correspond to memory traces of the spoken words.

As such, they are acquired from the external reality, they are not generated at a psychic level.

"*that's why the idea of the object does not appear to us as closed, and indeed hardly as closable, while the word concept appears to us as something that is closed though capable of extension*" (Freud, 1891, p. 80).

Need for a finitization - Matte Blanco 1988

... there is in human beings and in the world a mode of being that expresses itself in the distinctions between things, hence in their division, and another mode which treats any object of knowledge as if it were non-divided: the heterogenic and the indivisible modes.
(Matte Blanco, Thinking, Feeling, and Being, 1988. p. 64).

In the quantum model: finitization by measurement

The infinite singleton D contained in the representation of the quantum state $(\forall x \in D)A(x)$ collapses into a finite set by measurement, since the outcomes are characterized.

In particular, when the observable corresponds to the preparation, the result is "sharp" and described by a unique well characterizable element:

$$z \in D \equiv z = u$$

then D is the finite singleton $\{u\}$.

Then the representation is closed. The term u is the *Word Presentation of the state*.

The third component - Freud The Ego and the Id 1923

In the First Topics Freud has described the symmetric nature of Unconscious Representations.

In the Second Topics Freud exploits the normative dimension of reality in order to limit the spreading of the mode of thinking of the Unconscious (successively characterized as symmetric by Matte Blanco).

This is obtained with the interiorization of the limiting functions of the external authorities in the form of the Super Ego, representing the Law in front of the drives and the unconscious desires.

Bion's theory of thought - Second Thoughts 1967

Following the original Freudian intuition, characterization can only be achieved by a contact with the external reality. And, therefore, by taking time into account.

According to Bion, in the development of thought, we first reach the abstract level of *preconception*, that has to be tested w.r.t. reality.

Then, in the contact with reality, three different cases can be given:

- The realization corresponds to the preconception: positive case, a concept can be derived.
- The realization does not correspond to the preconception: negative case, the representation must be rejected.
- The experience of reality is unbearable, therefore the possibility to create a representation is destroyed ("attack to the link").

Abstract projector as a Container

In Bion, the abstract level of preconception is associated to a mental Container.

The abstract projector, in its infinite interpretation, is an abstract quantifier to which we attribute a domain T:

$$\Box A = (\forall x \in T) A(x)$$

Then, in Bionian terms, $\Box A$ can be seen as a mental container, as it includes all the possible results of the contact with reality.

The container allows to learn from the experience of the object, here represented by the quantum particle $\mathcal{A}$.

We see how the three cases of test of the preconception considered by Bion correspond, in the measurement of the state of the particle, to the three finite interpretations of the modality $\Box$ with observable σ_d, where $d = z$, $d = x$, $d = y$, say z the direction of the preparation.

The positive finite case

The positive case corresponds to the acceptance of the representation of reality one has achieved by means of infinite singletons.

In the quantum model, we find it when the observable is the same as the preparation, namely its matrix is the Pauli matrix σ_Z, that corresponds to an abstract projector (finite version!) since it is the linear combination of the two projectors on the eigenstates.

Then the domain T is reduced to finite singleton $\{p\}$, where p represents the unique abstract "positive direction" of the spin, onto which the state is projected.

Then $(\forall x \in T)A(x)$, namely $\Box A$, is equivalent to $A(p)$. The modality $\Box$ has no open/infinite/symmetric character any more. When a Word Presentation u of the object is reached, it is asserted: $A(u)$.

The negative finite case

The negative case corresponds to the rejection of the representation of reality one has achieved, that means its repression, in Freud.

In the quantum model, it is found when the observable is described by the other real Pauli matrix, σ_X, so that one defines a modal operator $\Box_x$ putting

$$\Box\Gamma \vdash \Box_x A \equiv \Box\Gamma, \sigma_x \vdash A(t) \text{ for all } t \in T$$

$\Box_x$ is the abstract antiprojector, since σ_X is the real linear combination of the two antiprojectors.

It is witnessed by a negative element n representing "the other" with respect to the witness of the abstract projector p.

Then, $(\forall x \in T)A(x)$, is reduced to $A(n)$, that is equivalent to the negation of $\Box A$. When a Word Presentation u of the object is reached, $A(u)$ is negated.

One can also prove $\Box A, \Box_x A \vdash \perp$ (non contradiction).

Role of the negation so achieved - Freud Negation 1925

The above results correspond to finding negation and non-contradiction as characterizing features of the secondary process (namely the conscious process dealing with finitizations), together with time and with the contact with external reality.

They are consistent also with the Freudian conception of negation as the intellectual counterpart of repression.

For, one can model repression by quantum states by assuming that an object consciously represented by the mind (word presentation) corresponds to an eigenstate of σ_Z. Repression means that the conscious representation is forgotten and substituted by an unconscious thing-presentation (which includes its opposite) and hence by the superposition of the eigenstates in the quantum model. The last is an eigenstate of σ_X. Then, in order to find out the object, the observable σ_x must be applied.

In the original computational basis, this means the creation of the abstract antiprojector and hence of negation.

The irreal case

The irreal case is a more primitive case, closer to the original open/indefinite view of $\Box$, since it represents its opposite: it corresponds to the failure of the contact with reality entailing a failure of the process of representation.

It is achieved by the non-real Pauli matrix σ_Y: like σ_X, it can be read as a superposition of the two anti-projectors, but with imaginary coefficients. Then we associate it with a non-real negation, a negation occurring prior to measurement. No real representation is achievable.

Following Bion, this means that the preconception is abolished. This implies that no infinite singleton is created and hence the domain T is empty (from an extensional point of view).

In the Unconscious, at the intensional level, such an attitude produces "bizarre elements", namely the impossible elements of the empty set, prior to its extensional interpretation. This interpretation meets Bion's theory about the rational and irrational elements in our thoughts.

Interpretation of the irreal case in logic

By assuming σ_Y, the formula $(\forall x \in T)A(x)$ is finitized as $A(e)$, where e is another odd element of T which can contrast the representation.

Then necessity, given by the abstract projector $\Box$, is converted into an impossibility. In the usual logical interpretation, impossibility means to apply necessity, $\Box$, to a negation.

This implies the existence an "inner negative attitude" that contrasts the representation (as it was also hypothized by Freud). It is not the same as the conscious negation that has been defined due to the rejection of a reality that had been previously represented, anyway!

At the conscious level, such a negation disappears. For, one conceives the modality "possible" and identifies the impossibility with the (conscious) negation of possibility.

Hence: decomposition of judgements

Then, in the splitting of the generic spin observable σ_d represented by the self-adjoint operator $\hat{O}$:

$$\hat{O} = \alpha I + \beta_z \sigma_Z + \beta_x \sigma_X + \beta_y \sigma_Y$$

one can read the imprinting of the contact of reality (experience) in our minds, which can form our thoughts:

- the "beta" coefficients give the level of positive contact, of repression and of rejection of representation following the experience; they represent the finite component of our thinking.
- the "alpha" coefficient includes the unconscious possibilities given by the original container, that can allow to rediscuss the other parts; it represents the infinite component.

In total, all our judgements are formed like that.

Concluding remarks

- quantum theory provides a suitable foundation to develop formalized models of thought
- in our approach, the model of consciousness builds on the model of the Unconscious
- the approach offers an open conceptual platform for the integration of models from different disciplines studying the human mind, and opens a new path for the consideration of the role of human thought, in the era of A.I.

We wish to thank Stuart Hameroff
for his valuable suggestion to consider Bi-logic as a quantum logic

Thanks

Thank you for your attention

Giulia Battilotti - Dept. of Mathematics –University of Padua –Italy
giulia@math.unipd.it
Miloš Borozan - CETAPS UR 3832 Lab. –University of Rouen-Normandy –France
milos.borozan@univ-rouen.fr
Rosapia Lauro Grotto - Dept. of Health Sciences –University of Florence –Italy
rosapia.laurogrotto@unifi.it