A model for Bi-logic

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As is well known, by the limitative results proved in the '30s by Kurt Goedel, logic (first order logic) cannot derive all mathematical truth. Namely, there are sentences of elementary arithmetic that our mind can grasp as true sentences, however they are not provable by formal logic.
This implies that the ability of our mind in processing judgements, deriving truth, goes beyond what is considered sound reasoning in logic. Then, it makes a sense to investigate about a "hidden" logic.

One can tackle the problem from the point of view of Bi-logic, proposed by Ignacio Matte Blanco to grasp all the features of the processes of judgements performed by human beings, including those due to the Unconscious [MB, MB2].

As is well known, Bi-logic has two modes: one is the asymmetric mode, proper of the conscious reasoning, which deals with non-symmetric relations, can separate objects, and permits sound logic, where two distinct truth values are present; the other is the symmetric mode, that is the mode of the unconscious. The symmetric mode has symmetric relations only, it gathers, identifies objects, creates links between judgements, different from those considered at the conscious level, and has an unsound logical behaviour. Moreover, since, by symmetry, any part is treated as the whole thing, any subset and the whole set are idempotent, and then, following Matte Blanco, the unconscious deals with infinite sets. Total symmetrization characterizes the "indivisible mode", where "the endless number of things tend to become, mysteriously, only one thing" [MB2].

The main features of the symmetric mode (condensation, absence of temporal processes, displacement) could be translated into the following logical and computational consequences:
1) condensation: the opposites coexist - no mutual contradiction - no negation;
2) absence of temporal processes: no algorithmic/step-by-step processes - no logical consequence
(MB suggests to speak about "manifestations" rather than "processes" of the unconscious);
3) displacement: there are different hidden symmetric links between judgements.

We find a formal approach to the symmetric mode, considering a model proposed in the framework of quantum computational logics [Ba2, Ba3, Ba4]. Our model exploits a common platform for extensional logics, termed basic logic [SBF]. The method adopted in basic logic permits to view logical constants as originated putting equations, that allow to import some metalinguistic links between judgements into the object language. Such a feature has been exploited to import judgements concerning quantum physics into logic, and then discuss the features of such judgements [Ba]. In basic logic, judgements are formalized by sequents, that are processed by a calculus termed sequent calculus. Then the model is suited to make an analysis of the computational features of the logic it represents.

In the model we consider random variables, represented by first order logical variables. They are exploited to characterize pure quantum states, by logical quantifiers. Propositional connectives characterize mixed states, namely the sets of states obtained after quantum measurement. Referring to our logical model, we could say that the pure quantum state, prior to measurement, is unique (whereas the result of the measurement is possibly a set of different states) since the variable acts as a glue for it.

After the representation of quantum states, we have made an analysis of the meaning of connectives in our model. We have considered negation, expressed in terms of duality, and we have shown that it can be discussed in the spin model of quantum mechanics, where, due to the uncertainty, duality is substituted by "symmetry", namely one has a new operator on judgements which cannot negate. The arising of symmetry, in the logical model, is due to the gap between the metalinguistic and the linguistic level, that permits to see that quantum states prior to measurement correspond to infinite sets, and in particular to conceive "infinite singletons". We claim that they correspond to Matte Blanco's infinite sets.

The concept of infinite singleton allows to characterize the logical aspects of the symmetric mode, corresponding to condensation, absence of time, displacement. In particular, one can develop an approach to correlations between judgements, that is originated by the representation of quantum correlations. Correlations represent a hidden part of physics and of logic as well. For, symmetry allows correlations and stops logical consequence, namely the orientation - from premises to conclusions - proper of theorems. Correlations represent a possible approach to displacement.

On the other side displacement could have a counterpart in our conscious, asymmetric, reasoning, in implication itself: One could consider implication as an asymmetric correlation between two certainties, and hence a sort of natural collapse of correlations, once infinite singletons disappear.
In the model, one can observe further consequences of correlations, for example for what concerns the "structural rules" of sequent calculus in logic. They have been discussed for a long time from a computational and from a semantical point of view. One can furtherly discuss which features of well known logical systems, for example classical logic, intuitionistic logic, linear logic, could have a symmetric origin.
Moreover one can wonder how the symmetric mode collapses into the asymmetric one, giving rise to usual logic. We make the hypothesis that normativity (formally translated into a modal logic equipped with the necessity operator) plays a decisive role in this, even if a clear formal way to see how normativity destroys symmetry is not available up to now.
It seems clear, however, that the key to model the hidden part of logic is the intrinsic kind of randomness of quantum mechanics, that can be translated into symmetry and infinite sets. This agrees with recent works in the new field of "quantum interaction", see e.g. the references below.

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