A modal interpretation of quantum spins and its application to Freudian theory

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The characterisation of objects

Classical physics was developed assuming the sharp distinguishability of objects

Quantum physics discovered that it cannot be assumed

In the meantime, the mathematical language for the "exact sciences" had been developed

So its implicit assumptions were widely applied to quantum physics as well, in particular those concerning how to distinguish and characterise objects.

Can we say how an object we need to describe is uniquely characterized by the language we are adopting?

Closed and open terms

The traditional mathematical language is based on first order language: the *objects* are denoted by the *terms* of the language, that can be *closed* or *open*, namely made of *constants* only, or including *variables* too

The notion of term confines the distinction between closed and open, namely between constants and variables, at the metalevel

This rules out the possibility of discussing the characterization of the objects inside the formal apparatus

The limitations of first order language have been discussed in the mathematical foundations, however, in practice, the formal apparatus has been accepted without discussion, for its convenience.

How can assertions on quantum states be related to constants and variables of first order language?

Infinite and finite sets associated to quantum measurements

Let us consider a particle, an observable (discrete) and the corresponding measurement. One obtains a set D of n outcomes.

The n outcomes are represented by closed terms of the language t_i, $i = 1, \ldots, n$. Each term t_i denotes an outcome of the measurement with its associated probability.

If the membership predicate $z \in D$ (z variable of the language) is equivalent to the disjunction $z = t_1 \vee \cdots \vee z = t_n$ inside the formal system, the formal system can distiguish and hence count n elements, otherwise it cannot.

D is finite at the metalevel and infinite at the object level.

Then we have a finite (external) and an infinite (internal) interpretation for D

Assertions on quantum states via equations

Correspondingly, we obtain an *infinite* and a *finite* assertion for the state of the particle.

In order to formalise assertions on quantum states, we shall apply the general technique of putting convenient *equations*: each equation can import a metalinguistic link into the formal language as a connective.

Γ is a set of assumptions for the measurement (describing the preparation), $\vdash$ is a sign for consequence and $A(t_i)$ is the formula

"the particle in found in the i-th state with its associated probability"

Then we have n assertions:

$$\Gamma \vdash A(t_i)$$

We can:

- group the n assertions at the metalevel (where D is finite) and say

$$\Gamma \vdash A(t_i) \textit{ forall } t_i \in D$$

- introduce a variable z of the language and import the assumptions $t_i \in D$ as a formal premise $z \in D$. D is infinite now, since closed terms allowing to distinguish its elements are not available and consider the overall assertion

$$\Gamma, z \in D \vdash A(z)$$

By putting the equation

$$\Gamma \vdash (\forall_\omega x \in D)A(x) \quad \equiv \quad \Gamma \vdash A(t_i) \textit{ forall } t_i \in D$$

we represent the overall result of the measurement, that is a mixed state, by the "finite quantifier" $\forall_\omega$ - that coincides with the conjunction of n formulae

$$A(t_1)\& \ldots \& A(t_n)$$

By putting the equation

$$\Gamma \vdash (\forall x \in D)A(x) \quad \equiv \quad \Gamma, z \in D \vdash A(z)$$

we represent the pure state prior to measurement, by means of the quantifier $\forall$:

$$(\forall x \in D)A(x)$$

Then we have a finite representation deriving from closed terms for the mixed state after measurement and an infinite representation for the pure state prior to measurement

Spin 1/2 and infinite singletons

Let us assume to have a spin 1/2 particle prepared along the direction z and consider the spin observables σ_d for any direction d.

One can see that the domain D determined by σ_d can be interpreted as an "infinite singleton", namely it satisfies

$$(\forall x \in D)A(x) \equiv (\exists x \in D)A(x)$$

for every A.

In particular, if d is z, one has a standard, extensional singleton: $D = \{\uparrow\}$ or $D = \{\downarrow\}$.

In general, infinite singletons are not extensional, namely there is no closed term u such that
$z \in D$ if and only if $z = u$.
(no word to describe the element of D)

Generalising the assertions on the spin

In a measurement of the spin with respect to a generic direction d, where D_d is the set of outcomes, one derives the corresponding finite representations for the mixed state putting:

$$\Gamma \vdash (\forall_\omega x \in D_d)A(x) \quad \equiv \quad \Gamma \vdash A(t_i) \textit{ forall } t_i \in D_d$$

while the infinite representations for the pure state are described putting:

$$\Gamma, z \in D_d \vdash (\forall x \in D_d)A(x) \quad \equiv \quad \Gamma, z \in D_d \vdash A(z)$$

The premise Γ and the conclusions $(\forall_\omega x \in D_d)A(x)$ and $(\forall x \in D_d)A(x)$ do not contain the variable z free. They are *closed* with respect to that variable!

A very general equation and its solution

Then let us "sum up" all equations *forall d*, and hence put the unique equation, that generalizes the finite quantifier $\forall_\omega$ and the usual (infinite) quantifier $\forall$ at the same time, by means of the operator $\Box$:

$$\Box\Gamma \vdash \Box A \quad \equiv \quad \Box\Gamma \vdash A$$

where $\Box\Gamma = \Gamma$ since Γ is is closed with respect to any variable ranging on the *generic* domain D_d.

Directions and probabilities have disappeared.

The equation is solved by the modal necessity operator $\Box$ of the modal system $S4$ (originally proposed in (Gödel 1933) as a way out from incompleteness). Then

$$\Box\Box A = \Box A$$

$\Box$ is interpretable as the general projector. Notice that it is unique.

Infinite and finite solution

Since $\Box$ generalises the quantifier $\forall$, let us assume that there is an infinite singleton T such that

$$\Box A = (\forall x \in T) A(x)$$

where T does not depend on any measurement but only on the preparation. (T contains all the unrecognized unique outcomes of the measurements for each direction d. Notice: incompatible observables, e.g. σ_z and σ_x, all give a contribution to T).

In the finite case, let us assume that there is a closed term $\updownarrow$ such that

$$\Box A = A(\updownarrow)$$

that is $\Box$ is the projector $P_{\updownarrow}$ (notation mutuated from Hilbert spaces even if we are not in a Hilbert space any more).

The "positive term" $\updownarrow$ is a witness of the necessity of A and also of its truth, since $\Box A \to A$ holds in S4.

Temporal parameter

Then we find that truth is never given by a particular measurement. As in Qbism, probability one is not truth! Truth is not the simple result of experience. It it appears at a further level of abstraction, when probabilities disappear.

To reconcile truth with experience, we see how $\updownarrow$ is derived as a particular solution of the equation introducing $\Box$, when an initial condition is introduced.

We can think that the set of all directions d is ordered and that an initial direction is characterized. The other directions are reached during a temporal evolution of the observable.

Equivalently, the observable is fixed at $d = d_0$ and the initial state has a unitary evolution with temporal parameter (Schroedinger picture).

Separation of negation from assertion

We are interested in two cases: $d_0 = z$ and $d_0 = x$.

- With initial condition $d_0 = z$: we characterize again the general projector $\Box$, identified with its finite form $P_{\updownarrow}$, since $\sigma_z = P_{\uparrow} - P_{\downarrow}$ represents the superposition of the two projectors on the eigenstates of the observable.
- With initial condition $d_0 = x$: since $\sigma_x = Q_{\uparrow} + Q_{\downarrow}$ represents the superpositions of the two antiprojectors $Q_{\uparrow}$ and $Q_{\downarrow}$, one characterizes the general antiprojector $\Box_n$ as a different particular solution, witnessed by the "negative term" $\updownarrow_n$

we have:

$$\Box_n(A) = A(\updownarrow_n)$$

where

$$z =\updownarrow_n \equiv \; z \neq \updownarrow$$

Modal uncertainty

Unlike the two opposite eigenvectors $\uparrow$ and $\downarrow$, the positive term $\updownarrow$ and the negative term $\updownarrow_n$ are not bearable together, since superposition and probabilities have disappeared.

The sequent

$$\Box A, \Box_n A \vdash \perp$$

(modal uncertainty)

can be read equivalently:

- as an instance of the uncertainty, if the incompatibility of the observables σ_z and σ_x is extended to their "sum over time"
- as an instance of the non contradiction law: $z = \updownarrow, z \neq \updownarrow_n \vdash \perp$ (law of identity)

Notice: in order to prove the equivalence, one needs to consider both the infinite and the finite interpretation of the modality $\Box$.

Modal negation

Then $\Box_n$ is a primitive negative modality. Differently from $\Box$, it has no infinite aspect since it derives from a particular solution of the equation, with parameters, only.

Summing up: Introducing the temporal parameter allows to consider the finite aspect of the positive modality and to find the negative modality, that is finite. Then one has non contradiction.

Hence a negation connective $\sim$ on modal formulae is definable putting

$$\sim \Box A \equiv \Box_n A$$

An introduction to the Unconscious Mode in Freud

We now see how the features we have considered in the representation of quantum states correspond to the features of a logical model for the Unconscious.

The *Unconscious Mode* was first described in *The Interpretation of Dreams* (Freud, 1900) where Representations are treated according to the following mechanisms:

- Displacement
- Condensation
- Absence of negation and contradiction
- Absence of serial/temporal order

Reformulation of the Unconscious Mode in Bi-logic

The Chilean psychoanalyst Ignacio Matte Blanco, in 1975, proposed a reformulation of Freudian theory in logical terms, by describing the so-called *Symmetric Mode*, which is characterized by two principles:

- the Principle of Symmetry
- the Principle of Generalization

the Principle of Symmetry

According to him, as proposed in the seminal work *The Unconscious as Infinite Sets*, the Unconscious Mode

treats the asymmetrical relations as if they were symmetrical (Matte Blanco, 1975, p. 38).

The word *symmetry* refers to the sameness, identity between two things and their fundamental indistinguishability.
Thus, since the relation of contradiction is nevertheless a relation, the Ucs treats opposites as identical.

the Principle of Generalization

The Unconscious Mode does not deal with individual elements, but only with classes to which they belong.

To provide an example, for the child the mother is not a single, individual person, it is rather a sum of all of the attributes of all members of its defining class – the class of mothers.
Therefore, the individual thing is made identical to the class it belongs to.

Based on Dedekind's observation that if a set is equivalent to its part, the set itself is necessarily infinite (Dedekind, 1901), Matte Blanco's formal explanation accounts for the infinite, all-or-nothing character of Ucs processes.

Back to infinite singletons

Following *The Unconscious as Infinite Sets*, the guide-line to find a logical embedding of the Symmetric Mode into the usual logical setting is provided by the assumptions:

- every set is infinite
- every relation is symmetric

However: relations are all symmetric only in singletons! For: if $a, b \in U$, and $a \neq b$, one can put $a < b$.

Solution : Infinite singletons

this means also: *no name for the object.*

Representations in Freud

This corresponds to Freud's original requirement on Representations, contained in *On Aphasia*:

Uncouscious Representations (*thing presentation*) can access consciousness only when linked to words (*word presentation*), that is, to finite representations (Freud 1891).

Freud's requirement was proposed short after the introduction of first order language.

Freud himself describes thing presentations as *open* and word presentations as *closed*.

We see further correspondences between the Freudian characterization of the processes of the Unconscious (primary process) w.r.t. conscious processes (secondary process)and the representation of quantum spins.

- Without access to consciousness, the processes of the mind are characterized by condensation and displacement. One can see that infinite singletons allow such processes (Battilotti, Borozan, Lauro Grotto 2021).
- Access to consciousness, in terms of finitization, is guaranteed by time. We have seen this fact above: time is a parameter which allows to separate negation from assertion. Negation is only finite and, with finitization, non contradiction can emerge.

Negation in Freud

In such a view finitization goes together with negation. All this is consistent with the observations of Freud concerning negation, contained in his paper Negation (1925).

In Freud's idea, negation represents the end of the infinite mode, since it is considered a *de-fusion*:

The general wish to negate, the negativism which is displayed by some psychotics, is probably to be regarded as a sign of defusion of instincts that has taken place through the withdrawal of the libidinal components.
(Freud, 1925, p. 235)

Freud's example

Following Freud, negation is *the intellectual counterpart of repression*.

His example is the following: a patient tells her dream, in which an unspecified person is contained, to the psychotherapist. When asked:

"Who was that person?"

the patient answers

"It is not clear, but for sure she was not my mother"

The psychotherapist can conclude that the person was the patient's mother.

Freud explains that the patient had repressed that fact and hence she denies it rather than admitting: negation is the symptom of repression.

In our terms, we can formalise the whole process as follows:

The patient has an original conscious information about her mother, characterised by the sharp state $z =$ mother.

Then she has repressed this. Hence that information is contained only in the Unconscious, in the superposed state mother + not-mother, since condensation with the opposite occurs in the Unconscious. This means that a Hadamard gate has been applied to the original information. The new state is processed in the dream.

When asked to characterize the person, the patient should correctly apply the observable σ_z, and she actually does.

Had the patient not repressed, the answer would have been the eigenvector "my mother". Since a Hadamard has been applied to the eigenvector, "I don't know" is the first correct answer, obtained by the application of σ_z.

However, the patient is asked to characterize the person. Then she adopts the observable for which the actual state of her information is an eigenvector, that is σ_x. This means that she is performing a judgement by means of the negative-dominated equation whose solution is the negative modality $\Box_n$, that creates the negation connective. Then her second correct answer, the word presentation of her mind's content, is: "not my mother".

The performance of the function of judgement is not made possible until the creation of the symbol of negation has endowed thinking with a first measure of freedom from the consequences of repression, and, with it, from the compulsion of the pleasure principle
(Freud, Negation, 1925)

Thank you for your attention!

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