

# A discussion of Bi-logic and Freud's representation theory in formal logic

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## Body and mind in Freud: drives and their representation

The originality of Freudian thinking lies in conceiving the connection between body and mind not in terms of parallelism neither of dualism

At the border between body and mind Freud puts **drives**, that emerge when satisfaction of needs is enriched by pleasure, therefore **transforming instinct into desire**

Freud defines drives as "a measure of the work imposed to the psychic apparatus due to its connection with the body"

Hence drives have a **quantitative order**.

The key point is the concept of **representation** of drives which is to be intended

in a political sense: ambassador as the representative of the state, a lawyer as a legal representative....

something damping the original strength... in order to negotiate.

The **representation of drives** is composed of:

- the feeling of pleasure or displeasure (close to the somatic level)
- internal mental objects that allow hallucinations of desire (the first kind of thinking, governed by the primary process)

Representation allows the discharge of drives: the quantitative order of body excitations can be translated into the **qualitative order of psyche**.

Freud distinguished two kinds of representation:

- **thing-presentation** (Objektvorstellung or Sachvorstellung)
- **word-presentation** (Wortvorstellung)

**Thing-presentations** are essentially pre- or non- verbal images of objects, they are associated with the **primary process**, and are not necessarily connected with reality.

In Freud's words: *"the appearance of a "thing" the properties of which are conveyed to us by our senses, originates only from the fact that in enumerating the sensory impressions perceived from an object, we allow for the possibility of a large series of new impressions being added to the chain of associations (J.S. Mill). This is why the idea of the object does not appear to us as closed, and indeed hardly closable..."*

**Word-presentations** involve the linking of a conscious idea to a verbal stimulus, are associated with the **secondary process**, and are oriented towards reality.

*"...the word acquires its significance through its associations with the idea (concept) of the object, at least if we restrict our considerations to nouns.[...] the word concept appears to us as something that is closed though capable of extensions."*

Therefore, on the topic of the relationship between language and thinking, Freud promoted the idea that thought precedes language: thought is initially unconscious and concerned with the sense impressions left by objects, when it later becomes conscious, it does so only by linking with the mental representations of the words. In the development of psychoanalytic theory, he kept the assumption that thing representations are always to be connected to word-presentations in order to allow access to conscious processing.

Freud attributes an indefinite/open character to thing presentation and a definite/closed character to word presentation.

We observe that thing presentation is open since it is endogenic, created on the mnestic traces of senses, whereas word presentation is exogenic, therefore is closed and equipped with a prescriptive character.

## Matte Blanco's Bi-Logic Model

The Chilean psychoanalyst I. Matte Blanco (1907-1995) wanted to further develop the Freudian psychoanalytic theory. He developed his own view of the human mind with the help of notions from the field of logic.

He proposed a system, the so called Bi-Logic Model, which describes the human thinking as underlined by a mixture of two modes - the conscious and unconscious ones, corresponding to the Freudian Primary and Secondary Processes.

the **asymmetric and heterogenic mode**, follows the rules of classical reasoning

the **symmetric and homogenic mode**, can be described as a logical system operating on the basis of two fundamental principles.

## The generalization principle

*The system Ucs treats an individual thing (person, object, concept) as if it were a member or element of a set or class which contains other members; it treats this class as a subclass of a more general class, and this more general class as a subclass or subset of a still more general class, and so on. (Matte Blanco, 1975, p.38)*

## The symmetry principle

*The system Ucs treats the converse of any relation as identical with the relation. In other words, it treats asymmetrical relations as if they were symmetrical. (Matte Blanco, 1975, p.38).*

As a consequence, any subset is idempotent with the whole set:  
then any set is infinite!

**The homogenic/symmetric mode is characterized by infinite sets**  
(Matte Blanco, The Unconscious as Infinite Sets, 1975).

**The heterogenic mode, in Matte Blanco, is identified with the bivalent mode: two truth values.**

In his second book, entitled *Thinking, Feeling and Being*, Matte Blanco explores the mutual relationships between the asymmetric and symmetric logical modes: "...the heterogenic mode is the realm of the logical. The symmetric mode is the realm of the illogical. The Freudian Unconscious is the realm of bi-logical structures and, as such, the realm of antinomies."

However, he concludes *If, instead, the question could be seen in the light of a unitary super-logic, which is not yet available (...), the conclusion just mentioned might no longer be true.* (Matte Blanco, 1988, p.66).

Matte Blanco had no time to investigate about the unitary super-logic. We think that an approach to Bi-logic in formal terms allows a better clarification of its role in the framework of Freudian theory and offers an opportunity to approach the issue of the unitary super-logic.



## Towards a formalization

In our proposal, the elements of the formal logical language and the logical systems considered, in order to interpret the Freudian theory formally, are the following:

- First order language, with open and closed terms, and predicate logic, for the theory of representation
- Modal operators which can express prescription, and modal systems, to consider the prescriptive aspects
- Linear modal operators (exponentials), and linear logic, in order to consider the quantitative aspects of the theory (related to the degree of investment and the economic point of view).

We have developed the first point, relying on Matte Blanco's proposal.

Then, we can propose how to obtain a modality from the formalization of the theory of representation, providing some elements of interpretation.

We have some first hints about the use of linear logic. Since, in the literature, a correspondence between modal operators and exponentials has been proved, we find that developing an interpretation in linear logic could reveal very intriguing aspects in order to discuss the relationship between the economic-quantitative aspects of the theory and the other aspects.

## Bi-logic in formal terms

In order to tell Matte Blanco's theory in formal terms, we need to characterize the symmetric mode. Hence we need that sets satisfy the two requirements:

- every binary relation defined on them is symmetric. This is true only for singletons (for, otherwise, two different elements can always be put in some order).
- they are infinite

### How can a singleton be infinite?

The notion of finite/infinite is level-sensitive: one can consider a set which is finite at the meta-level and cannot be proved such at the formal, object level.

In fact, the counting process implies the identification of the items. Then, even if a set  $U = \{u_1, \dots, u_n\}$  is recognized as finite at the metalevel, the same may not be possible at the object level, when the formal system does not include the equivalence between the membership relation  $z \in U$  and the disjunction  $z = \bar{u}_1 \vee \dots \vee \bar{u}_n$ , where the  $\bar{u}_i$  are the closed term of the language to denote the elements  $u_i$ .

The same in the particular case of singletons: even if we recognize that a set  $U$  is a singleton  $\{u\}$  at the metalevel, since, given any element, we can identify such an element with  $u$ , the same may not be true inside the formal system.

Then we can have an infinite singleton

Notice that this means: no word is available for the thing we have in mind: thing-presentation only!

Avoiding if the usual extensional characterization, we characterize a set  $U$  as a singleton putting the equivalence between existential and universal quantification on it, for every predicate  $A$ :

$$(\forall x \in U)A(x) \equiv (\exists x \in U)A(x)$$

That is, we can introduce a unique "symmetric quantifier"  $\mathcal{S}$  when the domain is a singleton:

$$(\mathcal{S}x \in U)A(x)$$

It allows the thing presentation of the mental object  $U$ .

Then, when the element of the singleton  $U$  is characterized inside the system, by the equivalence

$$z \in U \equiv z = \bar{u}$$

quantification can be avoided, since the quantified formulae  $(\forall x \in U)A(x)$  and  $(\exists x \in U)A(x)$  are both equivalent to the closed predicate

$$A(\bar{u})$$

that allows its word presentation.

A closed predicate simply attributes a defined property to an object, for example "the apple is red".

One can see that the symmetric quantifier is created whenever the equivalence

$$z \in U, A(y) \equiv A(z), y \notin U$$

holds in the formal system. It contains the negated form of membership:  $y \notin U$ .

By the generalization principle, the symmetric mode cannot exclude a membership. Then the negated form  $y \notin U$  is equivalent to the positive one  $y \in U$ : they are "condensed together". This means that a proposition and its negation are to be treated as a unique object. Then negation is not allowed and the non contradiction principle disappears.

Moreover, one can furtherly formalize the situation in which a particular infinite singleton allows to displace an object from a property to another, or conversely.

Then, with infinite singletons, we find the main logical features of the primary process: absence of negation and contradiction, condensation, displacement.

When the singleton is finite, namely when the thing is characterized by a word,  $y \in U$  is  $y = \bar{u}$  and  $y \notin U$  is  $y \neq \bar{u}$ . In order to render the last as close to membership as possible, one could be tempted to find an object "opposite to  $u$ ", let us denote it  $\bar{u}^\perp$ .

This is what happens in the quantum model, for the spin case.

The same does not happen in predicate logic. Word presentation is not bivalent: it can characterize plenty of different objects at the same time!

However, it happens with the values of judgements, truth values: true or false, the bivalent mode.

Then bivalence can be seen as a sort of natural collapse of symmetry.



## Introducing modalities

In logic, one can see that the universal and existential quantifier and their logical rules, if one abstracts from the domain  $U$  of quantification, give rise to the modal operators:

$\Box$  (necessity) comes from  $\forall$

$\Diamond$  (possibility) comes from  $\exists$

and to the rules of the modal system S4.

For example, the axiom  $(\forall x \in U)A(x) \vdash A(t)$  ( $t$  any, open or closed, term of the language) becomes the modal axiom  $\Box A \vdash A$ .

Dually, the axiom  $A(t) \vdash (\exists x \in U)A(x)$  becomes the modal axiom  $A \vdash \Diamond A$ .

What in case of the symmetric quantifier, when  $U$  is a singleton?

We have a symmetric predicate  $(\mathcal{S}x \in U)A(x)$ .

Abstracting from the domain  $U$  and introducing a symmetric modal symbol  $\nabla$ , we find a formula  $\nabla A$ .

Since  $\mathcal{S}$  is  $\forall$  and  $\exists$  at the same time:

-  $\nabla A$  is equivalent to  $A$ .

It is equivalent to the open predicate  $A(z)$  ( $z$  a variable for elements of  $U$ ), in case of open thing-presentation.

It is equivalent to the closed predicate  $A(\bar{u})$  in case of closed word-presentation.

-  $\nabla A$  contains possibility and necessity together.

As a consequence, in the symmetric mode, prescription, if any, is not yet separated from description, necessity is not separated from possibility.

In particular, without word-presentation: we should have an open object that can get a prescriptive content, at the same time!

In logic, one can discuss the existence of such strange mental objects, for example the notion of logical consequence itself could be an example (it provides an abstract schema for the idea of *necessary* consequence).

However, logical objects are conscious mental objects. In the symmetric mode, our idea is to model drives, once the quantitative aspect can be discussed.

This implies to extend the definition of the symmetric quantifier, in order to see that any formula consists actually of "clusters" of formulae, in the symmetric mode.

WORK IN PROGRESS

Facts are quite clear when singletons are characterized by words, namely, given  $U$ ,  $z \in U$  means  $z = \bar{u}$ , for some word  $\bar{u}$ , inside the system.

In this hypothesis,  $(\mathcal{S}x \in U)A(x)$  is equivalent to  $A(\bar{u})$ , that is a closed predicate, and hence the abstraction  $\nabla A$  itself is closed.

Then, in the hypothesis of word presentation, one finds out that forming a closed predicate, such as  $A(\bar{u})$ , has a normative value, that comes before its descriptive value.

For example, when we say "the apple is red", such a description of an apple assumes that we are adopting some social agreement about "redness", and this is because, for example, someone else taught us what "red" means. Then, in turn, our description can have a prescriptive value for someone else.

## Thinking, feeling and being - How to link the two modes?

The need to go beyond the characterization of the two modes is clearly expressed by Matte Blanco, particularly in "Thinking, feeling and being".

The idea is already present in Freud, since, in his idea, affects are forced into the representational domain.

We have seen that the transition from symmetric to bivalent is provided by an **identification**:  $z \in U$  becomes  $z = \bar{u}$ .

By means of the modality we obtain a kind of "abstract identification". Namely a way of giving the singleton a "value" even when we cannot declare it.

Then the modality can describe propositions in the middle, between infinite and finite mental objects.

In logic, Kurt Gödel defined the modality in order to introduce an "infinite" provability predicate (w.r.t. the provability by finite methods) with the aim of avoiding incompleteness. Then, he showed that one can define intuitionistic logic by adding the necessity operator to classical propositional logic, and that intuitionistic logic is infinite-valued.

Then, by means the modal operator, one can add an infinite content to bivalent logic.

An intriguing question is:

Can the modal operator "create" logic from the symmetric mode, keeping its infinite content by means of the modality?

In this setting, normativity, represented by the necessity operator, would get an intermediate status between the infinite and finite mode.

There is consistency between the logical features of the necessity operator introduced in this way, and the character of the super-ego, described by Freud in "The ego and the id". **According to Freud, the super-ego, formed before the characterization of the parental figures, has the form of an abstract authority.**

Concerning its role between the ego and the id, Freud stresses that

*...Thus the super-ego is always close to the id and can act as its representative vis-a-vis the ego. It reaches deep down into the id and for that reason is farther from consciousness than the ego is.*

*...the super-ego knew more than the ego about the unconscious id.*

*Quantum mechanics forbids statements about what really exists - statements about the object. Its statements deal only with the object-subject relation. Although this holds, after all, for any description of nature, it evidently holds in a much more radical and far reaching sense in quantum mechanics.*

- Erwin Schroedinger, 1931 letter to Arnold Sommerfeld; quoted in C. A. Fuchs et al., An Introduction to QBism.



*[Some] factors crucial to the perception of scientific work [are]... that the task of science is described in full if we limit it to showing how, because of our unique organization, the world must inevitably appear to us; that the eventual results of science, precisely because of the manner of their acquisition, are conditioned not only by our organization but also by what influenced that organization, and lastly that the problem of a world constitution that takes no account of the mental apparatus by which we perceive it is an empty abstraction, of no practical interest.*

- Sigmund Freud, *The Future of an Illusion*, 1927; quoted in C. A. Fuchs et al., *An Introduction to QBism*.

S. Freud, *Zur Auffassung der Aphasien. Eine kritische Studie* (1891)

S. Freud, *Das Ich und das Es* (1923)

I. Matte Blanco, *The Unconscious as Infinite Sets* (1975)

I. Matte Blanco, *Thinking, Feeling and Being* (1988)

Thank you