Psychoanalytic theory and logic

Giulia Battilotti*1

Miloš Borozan² Rosapia Lauro Grotto³

¹Department of Mathematics University of Padua

²CETAPS UR 3832 Laboratory University of Rouen-Normandie

³Department of Health Sciences University of Florence

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- 3 Quantum model and modal operators
- Interpretation of the modalities in psychoanalysis

Federigo Enriques - Problemi della Scienza 1906

F. Enriques, Problemi della Scienza, Cap. III "Problemi della logica", paragrafo 3 "Logica simbolica e logica psicologica", p. 164:

"Riconosciamo, ad ogni modo, che la Logica può riguardarsi come un insieme di norme, le quali *debbono* osservarsi, *se si vuole* la coerenza del pensiero. Ma ciò può anche essere espresso dicendo, che: fra i varii procedimenti mentali, se ne distinguono alcuni, in cui vengono volontariamente soddisfatte certe condizioni di coerenza, i quali si denominano appunto procedimenti logici.

In questo senso la Logica può riguardarsi come una parte della Psicologia." Corsivo nel testo.

"Anyway, we recognize that Logic can be regarded as a set of norms, which *must* be observed *to the aim of* coherence of thinking. On the other hand, this can also be expressed by saying that: among the different mental procedures, some are characterized, in which certain coherence conditions, that indeed are termed logical procedures, are willingly satisfied. *In this sense, Logic can be regarded as a part of Psychology.*" Italics in the text, and

- Our research develops a formalized approach to the theoretical foundations of psychoanalysis in logic.
- It characterizes a pre-logical setting, interpreting and integrating theoretical views developed in psychoanalysis by Freud, Klein, Matte Blanco, Bion and other authors.
- The pre-logical elements can be read as a basis for logic.

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Coherence conditions: Freud - On Aphasia 1891

... among the different mental procedures, some are characterized, in which certain coherence conditions, that indeed are termed logical procedures...

Coherence conditions are required in order to characterize mental objects. According to Freud, mental objects are first characterized by

thing-presentations

a concept at the basis of his theorization (Freud, On Aphasia, 1891).

Thing presentations are non-verbal open representations of objects operated by the Unconscious.

Thing presentations can access consciousness only when closed by words. Then, *word presentations* are the closed representation of objects managed by consciousness. They require pre-existing thing-presentations.

Open representations: infinite singletons

How to represent, formally, a thing-presentation?

Coherence can be investigated on the basis of the idea of variable on a domain.

To represent thing-presentations we consider quantified formulae on non-extensional domains termed *infinite singletons*.

Infinite singletons are characterized intensionally rather than extensionally. One says that V is a singleton if and only if it satisfies the equivalence

$$(\forall x \in V)A(x) \equiv (\exists x \in V)A(x)$$

for every formula A, whereas the equivalence:

$$z \in V$$
 if and only if $z = u$

for some closed term u, is not assumed. This means:

The Unconscious cannot characterize

Matte Blanco - The Unconscious as infinite sets 1975

The idea of infinite singleton is derived from Matte Blanco's logical characterization of the Structural Unconscious, proposed in *The Unconscious as Infinite Sets*, 1975.

In the Unconscious:

- All relations are symmetric
- All sets are infinite

If two different elements in a set are characterized, an order can be put.

Then the Unconscious operates on infinite singletons.

In formal terms: it operates on variables.

Whereas constants are adopted for word-presentations, when objects are defined, in the conscious process of thinking.

When representations are closed, infinite sets *unfold* into finite sets and singletons.

Freud - The interpretation of Dreams 1900

How does the Unconscious operate?

Freud characterized the process of the Unconscious (Primary Process) in The Interpretation of Dreams. The Primary Process has the following features:

- Displacement;
- Condensation;
- Absence of contradiction;
- Substitution of the external reality with the internal one;
- Timelessness.

Moreover, Matte Blanco characterized the mode of the Unconscious (Symmetric Mode) by two principles:

- The Symmetry Principle
- The Generalization Principle

Displacement and condensation are the mental procedures possible only in presence of the strong coherence allowed by the Unconscious.

Displacement: it means the displacement of a property from an element to "another".

It occurs in any infinite singleton. For, if $z \in U$ and A(z), then $(\exists x \in U)A(x)$, but then $(\forall x \in U)A(x)$, by definition of infinite singleton.

Condensation: any two "different" objects can condense into a unique one. Any two infinite singletons U and V cannot be distinguished in the symmetric mode. For, $z \in U$ and $z \notin V$ is impossible, since the Unconscious can establish a membership, but it cannot exclude one (generalization principle). Therefore, any two infinite singletons are forced to condense together.

When the Symmetric Mode finds no obstacle at all, one gets a unique object. Coherence is too strong.

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The characterization of an external reality yields to the closure of the representations and to finite sets, describable by constants.

Objects are defined and separated. Coherence stops.

We need to preserve coherence avoiding its spreading.

In Freud, the obstacle to the spreading of symmetry is found in the moving from the First to the Second Topic, with the introduction of a normative instance moderating the encounter of the psychic dimension with the external reality.

In logic, normativity is described by means of modal operators.

In the following, we develop this view by considering a quantum model, since the quantum world is a natural model to discuss coherence.

The Bloch Sphere

Let us consider the one-qubit spin model (Bloch Sphere):

- two opposites: e.g. the north and south poles | ↑>, | ↓> (the eigenvectors of σ_z)
- the opposites can condense together:

one wave, e.g. :

$$\frac{1}{\sqrt{2}}|\uparrow>+\frac{1}{\sqrt{2}}|\downarrow>=|\uparrow_x>$$

(eigenvector of σ_x)

- the wave is characterized by its phase: $+1 = e^{i0}$
- the observables σ_x and σ_z are incompatible
- the wave is not characterizable in terms of "objective property"



Infinite singletons and quantum states

We can characterize the state of a quantum particle with respect to a given spin observable σ_d (direction d) by the formula

 $(\forall x \in D_d)A(x)$

where D_d is the set of outcomes of the spin measurement (with the associated probabilities) along the direction d.

 D_d is an infinite singleton prior to measurement.

Whereas the mixed state after measurement is characterized by the formula

 $(\forall_{\omega} t \in D_d)A(t)$

The following sequent describes quantum measurement:

$$(\forall x \in D_d)A(x) \vdash (\forall_\omega t \in D_d)A(t)$$

It is derivable by substitution of the variable by the closed terms denoting the elements of D_d .

Both the quantum measurement and the sequent are irreversible

Evaluation at the Unconscious level

Assuming Freudian theory, any mental function, including judgement, is first operated by the Unconscious. Then, in our view, the whole Bloch sphere, not only one direction, is considered by the function of judgement.

Asserting $(\forall x \in D_d)A(x)$, with hypothesis Γ , is the result of putting the equation

$$\Gamma(-z) \vdash (\forall x \in D_d)A(x) \equiv \Gamma(-z), z \in D_d \vdash A(z)$$

(where $\Gamma(-z)$ is closed w.r.t. the variable z)

Asserting $(\forall_{\omega} t \in D_d)A(t)$ is the result of putting the equation

$$\Gamma \vdash (orall_{\omega} t \in D_d) A(t) \ \equiv \ \Gamma \vdash A(t) ext{ for all } t \in D_d$$

where t is a parameter for the elements of D_d .

Considering the whole Bloch sphere means that:

We need to consider all directions d at the same time.

Introducing the modal operator by abstraction

By abstracting both the above definitions of \forall and \forall_{ω} with respect to d, and dropping the domains D_d and the closed terms and variables for its elements, we get the unique form

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\Box \Gamma \vdash \Box A \equiv \Box \Gamma \vdash A
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that defines the modal operator \Box of S4. Then \Box satisfies

 $\Box \Box A = \Box A$

In quantum terms, \Box can be interpreted as an *abstract projector*.

It can attribute a sharp yet undefined state to the particle

It can have both an infinite/undefined and a finite/defined interpretation: namely, underneath, it can depend on an internal variable or it can gather externally parametrized objects.

Modal operator: decomposition

The generic spin observable σ_d is associated to a self-adjoint matrix \hat{O} , that is decomposed as follows

 $\hat{O} = \alpha I + \beta_x \sigma_X + \beta_y \sigma_Y + \beta_z \sigma_Z$

where I is the identity matrix, $\sigma_X, \sigma_Y, \sigma_Z$ are the Pauli matrices, $\alpha, \beta_x, \beta_y, \beta_z$ are real coefficients.

Finite interpretations: $\hat{O} = \sigma_d$, namely a direction *d* is characterized, and so the couple of eigenvectors w.r.t. the direction *d* is characterized.

Infinite interpretation: $\hat{O} = I$. It has infinite-many eigenstates, no external characterization, d is undefined.

In the finite interpretations, \Box is the abstract version of \forall_{ω} that gathers all the results depending on an external parameter.

In the infinite interpretation, \Box is like an abstract quantifier defined by means of an internal variable ranging on a domain.

Modal operator: finite and infinite interpretations

Let us assume d = d(t) where t is a temporal parameter making the direction d of the spin measurement of the fixed particle A evolve, from the initial one. Equivalently, one considers the application of a fixed initial observable σ_d to the evolution of the state of the particle.

Then we obtain operators \Box_d for every direction d (in particular: z, x, y), defined by putting:

$$\Box \Gamma \vdash \Box_d A \equiv \Box \Gamma, \sigma_d \vdash A(t)$$
 for all $t \in T$

If we put *I* instead of σ_d in the above, all eigenvectors are possible together, then the parameter *t* is imported as a variable. So the following definition of \Box can be put:

$$\Box \Gamma \vdash \Box A \equiv \Box \Gamma, z \in T \vdash A(z)$$

Then $\Box A$ is interpreted as follows:

$$\Box A \equiv (\forall x \in T) A(x)$$

Other elements can be abstracted, since they are independent of the direction d chosen for the measurement:

• The mixed state $\frac{1}{2}P_{\downarrow} + \frac{1}{2}P_{\uparrow}$, given by the couple of eigenstates of any observable σ_d

• The singlet state
$$rac{1}{\sqrt{2}}|\uparrow\downarrow>-rac{1}{\sqrt{2}}|\downarrow\uparrow>$$

Then we interpret them as logical constants:

- ▲, describing the abstract mixed state, namely the "totally non-integrated/separated element"
- 1 (true), describing the abstract singlet state, namely the "totally integrated/non-separable object"

According to Klein, thoughts are derived in the integration of previously non integrated elements (switching from the Paranoid-Schizoid to the Depressive position)

According to Bion, in the development of thought, we first reach the abstract level of *preconception*, that is associated to a mental Container. It includes all the possible results of the contact with reality.

Then, in the contact with reality, three different cases can be given:

- The realization corresponds to the preconception: positive case, a concept can be derived.
- The realization does not correspond to the preconception: negative case, the representation must be rejected.
- The experience of reality is unbearable, therefore the possibility to create a representation is destroyed ("attack to the link").

In the splitting of the generic spin observable σ_d represented as a self-adjoint operator

$$\hat{O} = \alpha I + \beta_z \sigma_Z + \beta_x \sigma_X + \beta_y \sigma_Y$$

one can read the imprinting of the contact of reality (experience) in our minds, which can form our thoughts.

We interpret it as a decomposition of judgements into infinite and finite components:

- the *I* component, that is infinite, includes the unconscious possibilities given by the original Container and allow to rediscuss the other parts;
- the $\sigma_Z, \sigma_X, \sigma_Y$ components represent the finite component of our thinking, they give the level of positive contact, of repression and of rejection of representation following the experience, as we see below.

The positive case corresponds to the acceptance of the representation of reality one has achieved by means of infinite singletons.

In the quantum model, we find it when the observable is the same as the preparation, namely its matrix is the Pauli matrix σ_Z , that corresponds to an abstract projector (finite version!) since it is the linear combination of the two projectors on the eigenstates.

Then the domain T is reduced to finite singleton $\{p\}$, where p represents the unique abstract "positive direction" of the spin, onto which the state is projected.

Then $(\forall x \in T)A(x)$, namely $\Box A$, is equivalent to A(p). The modality \Box has no open/infinite/symmetric character any more. When a Word Presentation u of the object is reached, it is asserted: A(u).

The negative case corresponds to a modality \Box_n , for the rejection of the representation of reality one has achieved, that means its repression, in Freud.

In the quantum model, \Box_n is found when the observable is described by the other real Pauli matrix, σ_X , and is interpreted as the *abstract antiprojector* (since σ_X is the real linear combination of the two antiprojectors).

Then, $(\forall x \in T)A(x)$, is reduced to A(n), where *n* is the negative witness, that represents "the other" with respect to the positive witness *p* of the abstract projector.

One can derive non contradiction: $\Box A$, $\Box_n A \vdash \bot$.

So one has the usual coherence condition of logic, non contradiction.

(Battilotti, Borozan, Lauro Grotto, Entropy 2022)

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Adopting \Box_n , when a word presentation of an object is reached, it is negated: $\neg A(u)$. This is the Freudian conception of negation as the intellectual counterpart of repression.

For, in the quantum model, an object consciously represented by the mind (word presentation) corresponds to an eigenstate of σ_Z . Repression means that the conscious representation is forgotten and substituted by an unconscious thing-presentation (which includes its opposite) and hence by the superposition of the eigenstates in the quantum model. The last is an eigenstate of σ_X . Then, in order to find out the object, the observable σ_x must be applied.

In the original computational basis, this means the creation of the abstract antiprojector, that is, the negation operator.

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The irreal case is a more primitive case, closer to the original open/indefinite view of \Box , since it represents its opposite: it corresponds to the failure of the contact with reality entailing a failure of the process of representation.

It is achieved by the non-real Pauli matrix σ_Y : like σ_X , it can be read as a superposition of the two anti-projectors, but with imaginary coefficients. Then we associate it with a non-real negation, for which no real representation, neither asserted nor negated, is achievable.

Following Bion, this means that the preconception is abolished. This implies that no infinite singleton is created and hence the domain T is emptied (from an extensional point of view).

In the Unconscious, at the intensional level, such an attitude produces Bion's *no-thing*, namely the impossible elements of the empty set, prior to its extensional interpretation.

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Interpretation of the irreal case in logic

By assuming σ_Y , the formula $(\forall x \in T)A(x)$ is finitized as A(e), where e is another odd element of T, witnessing the contrast to the representation: since thing-presentation is impossible, word presentation is impossible as well and no real finite element is characterized.

Let us suppose that the missing finite elements are re-swallowed as a variable on a different domain N, and hence define impossibility, \square , instead of necessity, \square :

 $\Box \Gamma \vdash \not \Box A$ if and only if $\Gamma, z \in N \vdash A(z)$

In the usual logical interpretation, impossibility means to apply necessity, \Box , to a negation. Such a negation is an "inner negative attitude" (hypothized by Freud as well), not the same as the conscious negation due to the repression of the previously represented reality.

At the conscious level, such a negation disappears: in its place, one conceives the dual modality \Diamond and identifies impossibility with the (conscious) negation of possibility. (Battilotti, Borozan, Lauro Grotto, Entropy 2023)

Intuitionistic logic can be obtained adding the modality to classical propositional logic.

The present proposal is a first attempt to derive logic starting from the infinite modal pre-logic of qubits rather than from the finite propositional logic of bits.

It is in accordance with Matte Blanco's idea that the primary mode of sets, that is of the objects of the mind, is infinite and not finite.

The approach offers an open conceptual platform for the integration of models from different disciplines studying the human mind, for the consideration of the role of human thought, in the era of A.I.