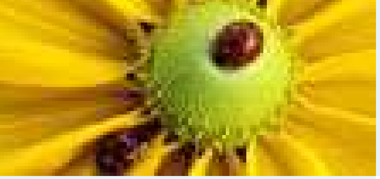
Some considerations on logic and consciousness

Giulia Battilotti
Dept. of pure and applied mathematics
University of Padua



I

A *logical judgement* states that a certain proposition is true or false.

In formal logic, we have:

- primitive propositions $A, B, C \ldots$,
- compound propositions, formed by means of logical connectives:
  propositional connectives: $A\&B, A \vee B, A \rightarrow B, \neg A$
  quantifiers: $(\forall x \in D)A(x), (\exists x \in D)A(x)$

II

Then:

Logic can *derive* true conclusions from true assumptions, considering *sound* rules of inference.

III

The problem is:

How can our mind reach the conviction that a proposition is true?

How can our mind consider that a rule of inference is sound?

IV

From the point of view of the mind, in mathematical logic, a century ago:

- F. Enriques made the proposal of studying logic as "psychological logic",
- L. E. J. Brouwer considered the foundation of mathematics as the product of one’s mind. Intuition is at the basis of mathematical truth.

V

The development of our logical ability goes together with the development of self-consciousness.

How can logical judgements be influenced by consciousness?

How can the soundness of inference rules depend on consciousness?

VI

Matte Blanco (The unconscious as infinite sets)

We have (at least) two logical modes, one for the conscious thinking and the other for the unconscious thinking.

Features of the unconscious mode:

- The opposite truth values coexist: no negation
- No implication, every relation is symmetric ("symmetric mode")
- The part is equivalent to the whole thing ("infinite sets", "indivisible mode")

VII

Fact: the "truth" of the unconscious is the absurdity.

Hypothesis: the unconscious adopts a different computational strategy, w.r.t. the sound strategy of usual logical rules of inference.
So we could have (at least) a different logical calculus, not sound, due to a computational advantage.

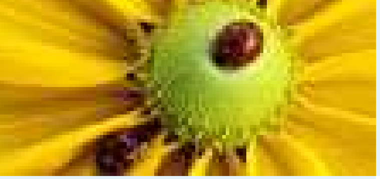
VIII

Such a calculus should be linked to a different treatment of first-order variables.

Variables are a way to import the "infinitary side" of the logical thinking into a formal setting.

The conquest of the notion of mathematical variable is from the adolescence: then the variable becomes an object of our thinking as a part of our object-language.

We can think that the logical processes described by classical predicate calculi can interpret the notion of variable of which we are or can become aware.

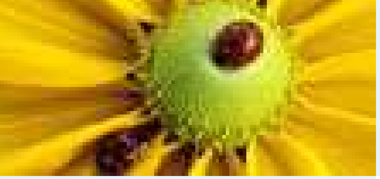


IX

Perhaps there are "hidden variables" of which we never become aware.

We adopt them in our unconscious thinking.

How could we describe (and guess) all this?



X

Sequent calculus is a logical calculus which processes objects called *sequents*:

$$A_1, \ldots, A_n \vdash B_1, \ldots, B_m$$

(abbreviated $\Gamma \vdash \Delta$)
where $\vdash$ represents a consequence relation.

The logical rules of inference are then rules which transform sequents into other sequents.

X

In logical calculi the variable is treated via the quantifiers $\forall$ (universal quantifier: for all) and $\exists$ (existential quantifier: there exists)

The equation defining the quantifier $\forall$ is:
$\Gamma \vdash (\forall x \in D)A(x)$ means "*for all* $z \in D$, $\Gamma \vdash A(z)$"
(where Γ cannot depend on z itself).

The variable is like a glue joining judgements which depend on it. Such a glue works despite the coherence of the judgements $A(z)$ to be glued.
Let’s try to widen the action of the variable as a glue.

XI

Let us consider two propositions $A(z)$ and $B(z)$ both depending on the same variable z ranging on a domain D.

We consider the judgement
"*for all* $z \in D$, $\Gamma \vdash A(z), B(z)$"
which says that from the premises Γ one obtains the two alternatives $A(z)$ and $B(z)$.

We can represent it by logical connectives only in the following way: $\Gamma \vdash (\forall x \in D)(A(x) \vee B(x))$
not in the other one: $\Gamma \vdash (\forall x \in D)A(x) \vee (\forall x \in D)B(x)$.

For, this would give raise to the following false statement:

$$(\forall x \in D)(A(x) \vee B(x)) \vdash (\forall x \in D)A(x) \vee (\forall x \in D)B(x)$$

XII

Nevertheless, we "prove" statements of the form

$$(\forall x \in D)(A(x) \vee B(x)) \vdash (\forall x \in D)A(x) \vee (\forall x \in D)B(x)$$

and their symmetric, with the existential quantifier

$$(\exists x \in D)A(x)\&(\exists x \in D)B(x) \vdash (\exists x \in D)(A(x)\&B(x))$$

in several occasions of our life.

XIII

Computational advantage: the equality

$$(\forall x \in D)(A(x) \vee B(x)) = (\forall x \in D)A(x) \vee (\forall x \in D)B(x)$$

(where A and B are glued by a common variable) defines a "simple" object.
In order to save consistency, one should consider the two variables in A and B as different independent variables on possibly different domains, obtaining the object defined by the equality:

$$(\forall x \in D)(\forall y \in D')((A(x) \vee B(y)) = (\forall x \in D)A(x) \vee (\forall y \in D')B(y)$$

where the complexity increases exponentially in the number of independent variables.

XIII

We so define a new connective $\bowtie$

XV

A fenomenological interpretation of the implication requires that it derives from a cognitive schema of PATH (method=path in greek).

The path has two directions. Confusing the two directions (as in children) means to give up implication in favour of symmetric associative schemata.

The "normative" interpretation of the implication (implication as a rule/ function as a law) requires that one is able to understand rules.