A sequent calculus with entanglement

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We develop a predicative sequent calculus in the framework of basic logic [5], a logical platform underlying extensional logics, including quantum logics [2]. In basic logic, connectives and their rules come from suitable metalinguistic links, by putting and solving definitory equations. This method ensures a cut-free system [3].

Extending the method of basic logic to the combination of links, we can easily define "ternary connectives", given by syntax-independent couples of connectives. This has advantages in proof search and has a stright solution in the propositional case. In the predicative case, such extension is possible only under the assumption of considering couples of independent variables, to avoid inconsistency. In order to treat couples of dependent variables, it is necessary to conceive a new metalinguistic link and then to define a new kind of quantifier. It can be interpreted as describing an entanglement link between particles, once an interpretation of quantum superposition by means of quantifiers is given [1]. Then our setting is suited to interpret quantum parallelism in proof-theoretical terms.

Here we study the whole sequent calculus which considers this new link. The main fact of our calculus is that the entanglement link requires a paraconsistent setting and seems to be alternative to logical implication. This resembles an alternative between entanglement and causality. When variables are substituted by closed terms, the entanglement link disappears, and this suggests that variables are necessary to obtain such interpretation. We think that this is due to the holistic meaning that can be attributed to a link containing a variable, which can be fully expressed only with such particular way to treat first-order variables. On the contrary, propositional links lead to compositional objects. Then, in our calculus, we obtain a positive and constructive notion of entanglement rather than the usual algebraic characterization in Hilbert spaces, that is obtained by the negation of a compositional object.

References

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