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Interpreting Matte Blanco's Bi-logic by sequents

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Bi-logic

Bi-logic was described by the Chilean psychoanalyst Ignacio Matte Blanco. It has two modes:

- *Asymmetric mode*: proper only of the conscious reasoning.
  It can deal with non-symmetric relations
  It can separate objects
  It permits "normal" - sound - logic: two distinct truth values
- *Symmetric mode*: it is the mode of the unconscious.
  It has symmetric relations only
  It gathers, identifies objects
  It creates different links between judgements and has an unsound logical behaviour.

By symmetry: *any part is treated as the whole thing*
So, any subset and the whole set are idempotent, and then:

- The "objects" of the unconscious are *infinite sets*.
- The unconscious can make a set larger and larger: *generalization*
- *condensation* $\longrightarrow$: the opposites coexist - no mutual contradiction - no negation
- *no temporal processes*[a] $\longrightarrow$: no algorithmic/step-by-step processes - no logical consequence
- *displacement* $\longrightarrow$: different hidden symmetric links between judgements $\longrightarrow$ correlations?

Total symmetrization characterizes the *indivisible mode*, where "the endless number of things tend to become, mysteriously, only one thing".

[a] MB suggests to speak about "manifestations" rather than "processes" of the unconscious.

Finite and infinite sets at meta-level and object-level: infinite singletons

Logic requires to distinguish between a *meta-level* (the logic one is using) and an *object-level* (the logic one is studying).

Assume that D is any set. By the logical rules on $\exists$ and $=$, one proves the equivalence

$$z \in D \equiv (\exists x \in D) z = x$$

However, even if we recognize that $D = \{t_1, \dots, t_n\}$ is finite at the metalevel, the consequence

$$z \in D \vdash z = t_1 \vee \dots \vee z = t_n$$

is not provable by the rules on the finite disjunction $\vee$

Then characterizing the set D as finite or infinite depends on the level:

If the equivalence holds in a logic, it is possible to count the elements of D in that logic: D finite in that logic.
If the equivalence does not hold, it is not possible to count the elements of D: D is infinite in that logic.

In particular: *Singletons* are usually conceived as sets V for which there is an element u of V such that, if z is any element of V, then z coincides with u. We could write: $z \in V \vdash z = u$ where u is a closed term of the logical language denoting the same element (extensionality). This would render singletons finite in our logic. However:

Singletons are not splitted by a disjunction: they are similar to infinite sets in this. Singletons have a borderline behaviour in logic!

One can characterize singletons inside a logic, without assuming extensionality, as domains of quantifiers, requiring, for every formula A: $(\forall x \in V) A(x) \equiv (\exists x \in V) A(x)$

Equivalently[a], one takes a duality d on propositions and puts:

$$z \in V, A(y) \vdash A(z), (y \in V)^d$$

that characterizes singletons, possibly infinite.
By substitution z/u, we would prove $V = \{u\}$. Then nontrivial infinite singletons are present only "in the realm of variables", namely where substituting by closed terms is not allowed (see the spin model of quantum mechanics).

[a] We apply the equations which define connectives in basic logic.

Abstract

We show how a logical model based on sequents, developed in the framework of basic logic and originally introduced for quantum states, can explain the "infinite sets" of Matte Blanco. Then we model the symmetric mode of Bi-logic, and suggest an approach to correlations and to the problem of contextual reasoning, linked to the structural rules of sequent calculus.

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Infinite sets and quantum states

Let us consider a quantum particle $\mathcal{A}$. If $A(z)$ is the proposition "the particle $\mathcal{A}$ is found in state $s(z)$ with probability $p\{Z = s(z)\}$", where Z is the random variable given by the measurement w.r.t a fixed observable, and

$$D_Z = \{(s(z), p\{Z = s(z)\})\}$$

is the set of the outcomes $s(z)$, with their probabilities, *we attribute the state to the quantum particle $\mathcal{A}$, w.r.t. the given observable, by means of the predicative formula*

$$(\forall x \in D_Z) A(x)$$

If we recognize the outcomes $t_1, \dots, t_n$ after measurement, $D_Z = \{t_1, \dots, t_n\}$. *The proposition $A(t_1) \& \dots \& A(t_n)$ describes the mixed state*, obtained after measurement.

The consequence

$$(\forall x \in D_Z) A(x) \vdash A(t_1) \& \dots \& A(t_n)$$

describes the collapse from the pure state to the mixed state after measurement. This is proved by substituting the free variable z by the closed terms t_i. Substitution is like "the collapse of the variable": *Substitution represents measurement.*

The equivalence $(\forall x \in D) A(x) \equiv A(t_1) \& \dots \& A(t_n)$ is not true: the pure state does not coincide with the mixed state! One can see that the equivalence is provable if and only if the equivalence $z \in D \equiv z = t_1 \vee \dots \vee z = t_n$ is provable for every A.

Quantum superposition corresponds to the infinite character of the set D_Z.[a]

[a] One can see that defining an equality predicate which renders D_Z finite at the object level, is equivalent to dropping the phase factors in the representation of the state as a vector in the Hilbert space.

Infinite singletons and symmetry in the spin model

We measure the spin of a particle w.r.t. the z axis: The sets associated to the sharp states $\uparrow$ and $\downarrow$ are two singletons. The formulae quantified on them are then equivalent to propositional formulae, say $A_\uparrow$ and $A_\downarrow$. We put a duality $\perp$ switching $\uparrow$ and $\downarrow$. It translates the Pauli matrix σ_X (namely the NOT gate) into logic. We extend it to all formulae and obtain a negation (Girard's negation), that behaves as usual with respect to the consequence: $A \vdash B$ if and only if $B^\perp \vdash A^\perp$.

The dual states $+$ and $-$ are switched by the Pauli matrix σ_Z and are eigenvectors for σ_X. Translating all this into logic, we see that:
The sets associated to $+$ and $-$ contain the *two opposite pieces of information $\uparrow$ and $\downarrow$*.
The corresponding predicative formulae *are fixed point for negation $\perp$*.
Then such formulae satisfy $A \vdash B$ if and only if $B \vdash A$: *no direction for logical consequence*.

Moreover, we have a new duality $\top$ from σ_Z. Changing the measurement context and measuring the spin w.r.t. x would produce an objective property for $+$ and $-$, that would be represented by singletons. However, different spin observables are incompatible and so the sets associated to $+$ and $-$ are infinite singletons, due to the duality $\top$ from σ_Z.

This is the logical framework expected by bi-logic: an asymmetric mode where negation is meaningful, a symmetric mode where negation is meaningless, due to infinite singletons.

Infinite singletons, correlations and structural rules, via basic logic

From the axioms $z \in V, A(y) \vdash A(z), (y \in V)^d$, one proves

$$(\forall x \in V) A_i(x) * A_i(x) = (\forall x \in V) A_i(x) * (\forall x \in V) A_j(x)$$

($*$ is a disjunction, namely the multiplicative disjunction in linear logic). It is sound if and only if only if V is a singleton.

For infinite singletons the equality corresponds to correlations. We consider infinite singletons together with *infinite singletons of indices of formulae*: The correlation takes place since the same first-order variable is displaced elsewhere, considering another index.

Consider a family of formulae $A_i(z)$, $i \in I$, where z is a common free first order variable and I is an infinite singleton of indices: $A_i(z)$ and $A_j(z)$ are correlated when i and j are in I. We represent the correlation in the object language, translating it into a connective $\bowtie$, that extends the multiplicative disjunction $*$. We have:

$$(\forall x \in V) A_i(x) \bowtie A_j(x) = (\forall x \in V) A_i(x) \bowtie (\forall x \in V) A_j(x)$$

In the quantum model, we have represented Bell states adopting such a technique.

Infinite singletons of indices allow the displacement of first order variables on "identical" formulae. This could be a logical approach to the representation of psychoanalytic displacement, as considered by Matte Blanco. In Matte Blanco, displacement takes place since two subclasses are both identified with a larger class (generalization) and then treated as identical. In logic, this is a kind of second-order justification, that we translate into the identification of two indices once they are in the same infinite singleton.

The amount of information contained in $A_i(z)$ is considered the same amount contained in the pair $A_i(z)$, $A_j(z)$, under the hypothesis of correlation. We write $A_i(z),_\sim A_j(z)$ for the correlation when $i \sim j$, namely they are in the same infinite singleton of indices I. Then we write the equivalence by sequents:

$$\Gamma, z \in V, i \sim j \vdash A_i(z) \quad \equiv \quad \Gamma, z \in V \vdash A_i(z),_\sim A_j(z)$$

Has displacement a counterpart in our conscious reasoning, namely is it the symmetric counterpart of some different asymmetric link?
A judgement where the propositions are correlated is not suited to be processed in a context-free way. On the contrary, sequent calculus is context free. This yields, in particular, the definability of implication. One could consider implication as an asymmetric correlation between two certainties, and hence a sort of natural collapse of correlations, once infinite singletons disappear. We recall that implication is the standard way to model the input-output orientation of processes in logic. On the contrary, recently introduced models are showing that quantum processes do not follow such an orientation, because of the quantum correlations.

If we drop the correlation, and then the indices, the left to right direction of the above equivalence resembles the structural rule of "weakening" in sequent calculus; the converse direction is exactly the structural rule "contraction": They are used only in a context-free kind of reasoning, and rejected in many logics developed for computation (e.g. linear logic). Our conception of structural rules as valid rules could be originated from an original attitude of dealing with infinite singletons and displacement of variables, that is preserved, even if logic requires that correlations disappear, in our conscious processing of judgements.

From a computational point of view, discussing all the above topics requires to discuss the role of contexts in sequents. Basic logic, that was developed as a common platform for sequent calculi of extensional logics, discussing such a role, supplies a suitable tool to this aim.

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