Quantum computation and cognition: a logical approach

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Abstract

We show a logical calculus where massive quantum parallelism is expressed. This is possible within an inconsistent logical framework, analogous to the logic of the unconscious of I. Matte Blanco. We hypothize that inconsistency is necessary to intelligence, since it gives strong computational and then cognitive advantages to our mind.

Alan Turing wrote:

"... if a machine is expected to be infallible, it cannot be also intelligent. There are several theorems [Goedel's incompleteness theorems] which say almost exactly that. But these theorems say nothing about how much intelligence may be displayed if a machine makes no pretence at infallibility."

Quantum Computational Speed Up

As it is well known, quantum computers would allow an enormous speed up of computation. Quantum computational speed up is due to the so called massive quantum parallelism, that is the parallel computation created by the peculiar quantum features of information, namely quantum superposition and quantum entanglement. Quantum superposition is the co-existence, in the same particle, of different states, one of which will be measured.
A quantum unit of information (qubit) is represented by the vector $|q\rangle$

$$|q\rangle = a|0\rangle \oplus b|1\rangle$$

where a, b are complex numbers (probability amplitudes) s.t. $|a|^2+|b|^2 = 1$ and $|0\rangle$, $|1\rangle$ are two orthogonal unitary vectors. $|q\rangle$ is then the superposition of the two states $|0\rangle$, $|1\rangle$, where $|0\rangle$ will be measured with probability $|a|^2$ and $|1\rangle$ with probability $|b|^2$.
Two qubits are maximally entangled when they are represented by one of the following four states (Bell's states)

$$|q_1q_2\rangle = 1/\sqrt{2}|00\rangle \pm 1/\sqrt{2}|11\rangle \qquad |q_1q_2\rangle = 1/\sqrt{2}|01\rangle \pm 1/\sqrt{2}|10\rangle$$

This means that the two states $|0\rangle$ and $|1\rangle$ are equally probable after measurement, and that the measurement of one of the two is determined by the measurement of the other.

Consciousness as a quantum phenomenon

Do quantum phenomena contribute to make our mind? Several theories have been proposed. In 1989 Roger Penrose suggested that consciousness involved a type of quantum computation involving quantum state reduction in the brain, implying that processes utilized quantum information - superpositions of multiple possibilities (i.e. quantum bits or qubits). Penrose further proposed that the specific type of quantum state reduction was caused by quantum gravity due to an objective treshold - a critical separation in spacetime geometry underlying the superpositions, and subsequently defined a treshold for such "objective reduction". In 1994 Penrose and Hameroff proposed that the quantum information in our brain (our qubits!) was located in tubulines, in turn component of microtubules in the brain's neurons. The quantum computation in microtubules was described as being "orchestrated" by feedback through synapses, and the model thus called "orchestrated objective reduction", or Orch OR.
Following such theory, our mind avails of quantum computation, but cannot be aware of it, since it is unconscious. This could be an explanation of the reason for which, up to now, we has been able to grasp a so poor idea of computation, which cannot give back results comparable with the natural ones.

The logic of the unconscious: Matte Blanco's infinite sets

Which features of the unconscious could reveal its quantum computational origin? What could such origin imply in cognitive terms? We have tried to answer in logical terms. In fact, logic can link semantics with syntax, that is, it can focus what our mind retains true in a certain computational framework, that can be a classical or a quantum framework. To this aim, we have considered bi-logic, proposed by the chilean psycoanalyst I. Matte Blanco, that is the unique systematic study concerning the logic of the unconscious. Following Matte Blanco, the "symmetrical mode" underlies our unconscious reasoning. In it, every relation is considered symmetrical. The main logical consequences are:

- Paraconsistency: contradiction is not forbidden. On the contrary, two opposite facts can be retained true at the same time.
- Meaningless negation.
- No true implication: reversibility of logical consequence due to the symmetrical treatment of any relation, including consequence.
- Treatment of any set as an infinite set.

The last point is furtherly precised as follows: *The only unit for the (symmetrical) unconscious is the class, or set, in which all individuals belonging to it are included. The unconscious cannot, therefore, deal with parts, except by treating them as classes or sets.* As a consequence, the unconscious cannot recognize individuals, and our sound treatment of first order logical variables ranging on sets of individuals is not suited to its reasoning.

Basic logic: from metalinguistic links between assertions to logical rules

Following basic logic, logical connectives are the result of importing into the formal language some pre-existing meta-linguistic links between assertions. Such links, for the propositional case, are "and" and "yields". The metalinguistic link "yields", is the consequence relation between two logical judgements, that is it puts two assertions together in a *sequential* way. It is translated into the connective of implication $\rightarrow$. If assertions are represented by *sequents*, its rules are obtained solving the following definitional equation:

$$\Gamma \vdash A \rightarrow B \qquad \equiv \qquad \Gamma, A \vdash B.$$

This is the equation for the true intuitionistic implication. In basic logic, implication is weaker. This goes well with the inconsistent framework a la Matte Blanco we are searching. The metalinguistic link "and" links two logical judgements at the same level, that is considered "in parallel". Two kinds of connectives correspond to "and": the additive and the multiplicative. In sequents, we can represent them at the right or at the left of the sequent sign. Representing at the right, we solve the definitional equations:

$$\Gamma \vdash A\&B \qquad \equiv \qquad \Gamma \vdash A \quad \Gamma \vdash B$$

$$\Gamma \vdash A \odot B \qquad \equiv \qquad \Gamma \vdash A, B$$

from which the rules of the additive conjunction $\&$ and the multiplicative disjunction, here denoted $\odot$.
For the predicative case, the metalinguistic link "forall", at the right, gives the equation for the universal quantifier $\forall$:

$$\Gamma \vdash (\forall x \in D)A(x) \qquad \equiv \qquad \Gamma, z \in D \vdash A(z)$$

(where Γ does not depend on the free variable z!)
If D is any two-elements domain, e.g. $D = \{|0\rangle, |1\rangle\}$, the equation for the binary connective $\&$ follows from the equation for $\forall$. Moreover the quantifier can be interpreted as an infinitary additive connective.

A logical approach to quantum parallelism

So, one obtains two kinds of parallelism in logical derivations: the additive and the multiplicative. The multiplicative connectives are usually considered to model parallel processes. We have proposed that additive connectives can give a logical representation of quantum superposition, responsible of quantum parallelism. The qubit $|q\rangle$ is represented by the proposition $A\&A^{\perp}$. If we consider the two kinds of "and" together, we obtain a complex link. The question is: Is there a logical connective for a complex link? A possible answer is the following: we have a connective when the link is effectively given in a unique way. A temptative interpretation of such effectiveness is the following: it occurs when the syntactical order is irrelevant, that is when the point is the whole so created, not the particular nesting of the two simple links forming the complex one. This is distributivity:

$$A \odot (B\&C) = (A \odot B)\&(A \odot C)$$

In a predicative setting, we can interpret superposition by the quantifier $\forall$. What about its relation with the multiplicative $\odot$? The sequent

$$(\forall x \in D)A(x) \odot (\forall x \in D)B(x) \vdash (\forall x \in D)(A(x) \odot B(x)$$

is derivable. The opposite sequent

$$(\forall x \in D)(A(x) \odot B(x) \vdash (\forall x \in D)A(x) \odot (\forall x \in D)B(x)$$

is clearly false, and plenty of counterexamples can be found, interpreting $\odot$ as a normal disjunction. We see what one should allow in an attempt to derive it. In the first derivation, we see that it is derivable dropping the conditions on the free variable in $\sharp$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A(z) \vdash A(z) \qquad B(z) \vdash B(z)}{A(z) \odot B(z) \vdash A(z), B(z)}\,\odot r}{(\forall x \in D)(A(x) \odot B(x) \vdash A(z), B(z)}\,\forall r}{(\forall x \in D)(A(x) \odot B(x)) \vdash (\forall x \in D)A(x), B(z)}\,\forall f\sharp}{(\forall x \in D)(A(x) \odot B(x)) \vdash (\forall x \in D)A(x), (\forall x \in D)B(x)}\,\forall f}{(\forall x \in D)(A(x) \odot B(x)) \vdash (\forall x \in D)A(x) \odot (\forall x \in D)B(x)}\,\odot f$$

In the second derivation, we see that dropping the condition can be "hidden" by a sort of parallel formation rule for $\forall$:

$$\dfrac{\dfrac{\dfrac{\dfrac{A(z) \vdash A(z) \qquad B(z) \vdash B(z)}{A(z) \odot B(z) \vdash A(z), B(z)}\,\odot r}{(\forall x \in D)(A(x) \odot B(x) \vdash A(z), B(z)}\,\forall r}{(\forall x \in D)(A(x) \odot B(x)) \vdash (\forall x \in D)A(x), (\forall x \in D)B(x)}\,\forall f\|}{(\forall x \in D)(A(x) \odot B(x)) \vdash (\forall x \in D)A(x) \odot (\forall x \in D)B(x)}\,\odot f$$

Such derivation creates the logical expression of Bell's states, considering $D = \{|0\rangle, |1\rangle\}$. In fact the Bell's pair $|q_1q_2\rangle = 1/\sqrt{2}|00\rangle \pm 1/\sqrt{2}|11\rangle$ is here the proposition $(\forall x \in D)(A(x) \odot B(x))$, the derived sequent $(\forall x \in D)(A(x) \odot B(x)) \vdash (\forall x \in D)A(x) \odot (\forall x \in D)B(x)$ states that it is all the information stored in the two-qubit register given by q_1 and q_2, that is represented by $(\forall x \in D)A(x) \odot (\forall x \in D)B(x)$.
In the last derivation, we show that the derivations above would be possible without any variation on rules, if inconsistent axioms of the form $B(z) \vdash B(y)$, where y, z are distinct first order variables ranging on the domain D, were assumed.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A(z) \vdash A(z) \qquad B(z) \vdash B(y)}{A(z) \odot B(z) \vdash A(z), B(z)}\,\odot r}{(\forall x \in D)(A(x) \odot B(x) \vdash A(z), B(y)}\,\forall r}{(\forall x \in D)(A(x) \odot B(x)) \vdash (\forall x \in D)A(x), B(y)}\,\forall f}{(\forall x \in D)(A(x) \odot B(x)) \vdash (\forall x \in D)A(x), (\forall x \in D)B(x)}\,\forall f}{(\forall x \in D)(A(x) \odot B(x)) \vdash (\forall x \in D)A(x) \odot (\forall x \in D)B(x)}\,\odot f$$

We argue a computational equivalence with the logical setting described by Matte Blanco, where the elements of a domain cannot be determined and hence axioms of the form $B(z) \vdash B(y)$ are assumed.

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