Normal stress differences and flow-type dependence in dense suspensions

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Take-home messages

1. Orthogonal stress projections are most helpful in data analysis
2. Flow-type dependence in dense suspensions of hard spheres manifests itself in normal stress differences (not in the viscosity)
3. Hydrodynamic and contact interactions contribute together to negative values of N_1
4. Positive values of N_1 near jamming cannot be explained with a simple hard-sphere model. They must be related to the presence of boundaries or to additional persistent interactions

Classical projections

Given the Cauchy stress $\boldsymbol{\sigma}$ in a simple shear flow, tensorial projections are used to compute the material coefficients σ, N_1, and N_2:

$$\hat{\mathsf{S}} \equiv \boldsymbol{xy}, \qquad \hat{\mathsf{N}}_1 \equiv \boldsymbol{xx} - \boldsymbol{yy}, \qquad \hat{\mathsf{N}}_2 \equiv \boldsymbol{yy} - \boldsymbol{zz}$$

$$\sigma = \boldsymbol{\sigma} : \hat{\mathsf{S}}, \qquad N_1 = \boldsymbol{\sigma} : \hat{\mathsf{N}}_1, \qquad N_2 = \boldsymbol{\sigma} : \hat{\mathsf{N}}_2$$

Drawback 1: $\hat{\mathsf{N}}_1$ and $\hat{\mathsf{N}}_2$ are not orthogonal ($\hat{\mathsf{N}}_1 : \hat{\mathsf{N}}_2 = -1$) and orthogonality is important to identify *independent* effects

Solution 1: we can define $\hat{\mathsf{N}}_0 \equiv \frac{1}{2}(\boldsymbol{xx} + \boldsymbol{yy}) - \boldsymbol{zz}$, which is orthogonal to $\hat{\mathsf{N}}_1$ and $\hat{\mathsf{S}}$ and obtain the “zeroth” normal stress difference

$$N_0 = \boldsymbol{\sigma} : \hat{\mathsf{N}}_0 = N_2 + \frac{1}{2} N_1$$

Drawback 2: the definitions above depend on the choice of a basis, which is adapted to *simple shear flows only*

General projections

Solution 2: a general orthogonal basis for the stress tensor (*planar case*)

We use the eigenvectors $\boldsymbol{d}_1$, $\boldsymbol{d}_2$, and $\boldsymbol{d}_3$ of $\mathsf{D} = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$ and set

$$\hat{\mathsf{D}} \equiv \boldsymbol{d}_1\boldsymbol{d}_1 - \boldsymbol{d}_2\boldsymbol{d}_2, \quad \text{with } \mathsf{D} = \dot{\varepsilon}\hat{\mathsf{D}} \text{ and } \boldsymbol{d}_3 \text{ the vorticity direction,}$$

$$\hat{\mathsf{E}} \equiv -\frac{1}{2}\boldsymbol{d}_1\boldsymbol{d}_1 - \frac{1}{2}\boldsymbol{d}_2\boldsymbol{d}_2 + \boldsymbol{d}_3\boldsymbol{d}_3 \quad \text{and} \quad \hat{\mathsf{G}}_3 \equiv \boldsymbol{d}_1\boldsymbol{d}_2 + \boldsymbol{d}_2\boldsymbol{d}_1$$

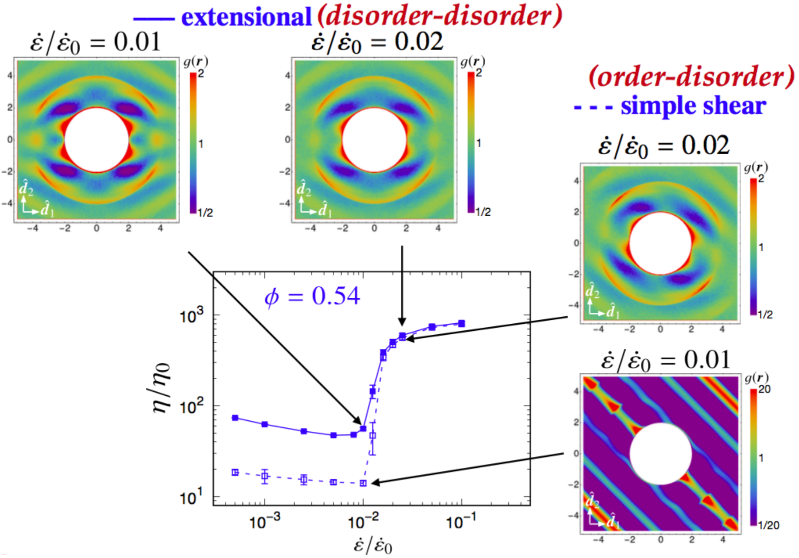
With these, $(\mathsf{I}, \hat{\mathsf{D}}, \hat{\mathsf{E}}, \hat{\mathsf{G}}_3)$ are symmetric tensors, orthogonal to each other, and we have

$$\sigma = \frac{1}{2}\boldsymbol{\sigma} : \hat{\mathsf{D}}, \qquad N_1 = -\boldsymbol{\sigma} : \hat{\mathsf{G}}_3, \qquad N_0 = -\boldsymbol{\sigma} : \hat{\mathsf{E}}$$

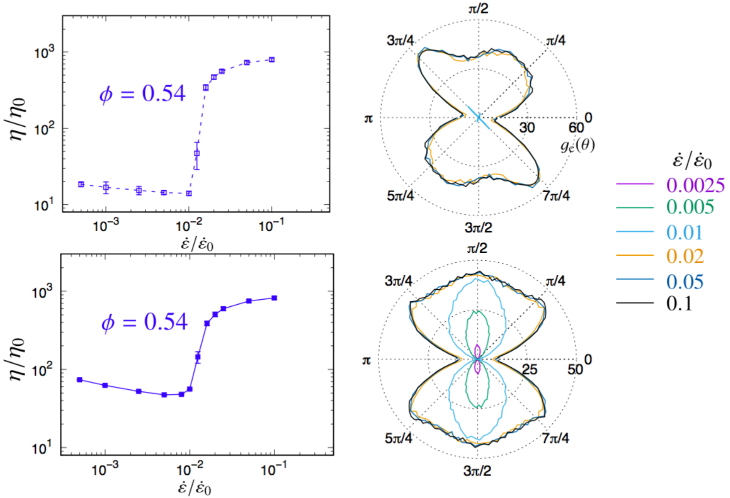
These definitions work for simple shear, extensional, and mixed flows

Details in: Giusteri G.G., Seto R., *J. Rheol.* 62(3), 713–723, 2018

Shear thickening is not an order-disorder transition



Shear thickening is a transition in contact statistics



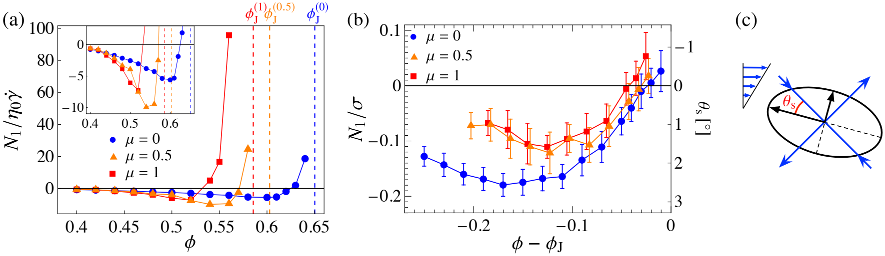
Details in: Seto R. et al., *J. Fluid Mech.* 825, R3, 2017

The reorientation angle

The ratio between the shear stress and the first normal stress difference determines the misalignment between the stress $\boldsymbol{\sigma}$ and D given by

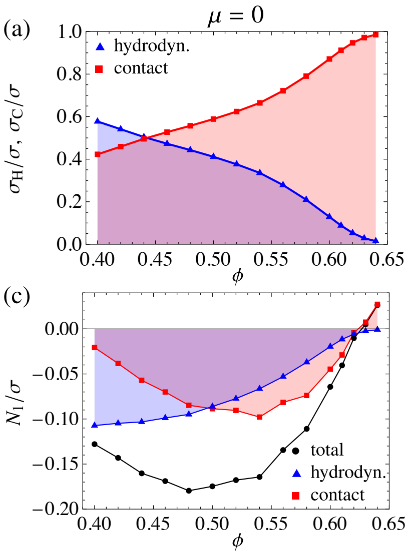
$$\theta_s \equiv \arctan\left(\frac{-N_1/\sigma}{2+\sqrt{4+(N_1/\sigma)^2}}\right)$$

This provides a cleaner way to assess the entity of the normal stress effect

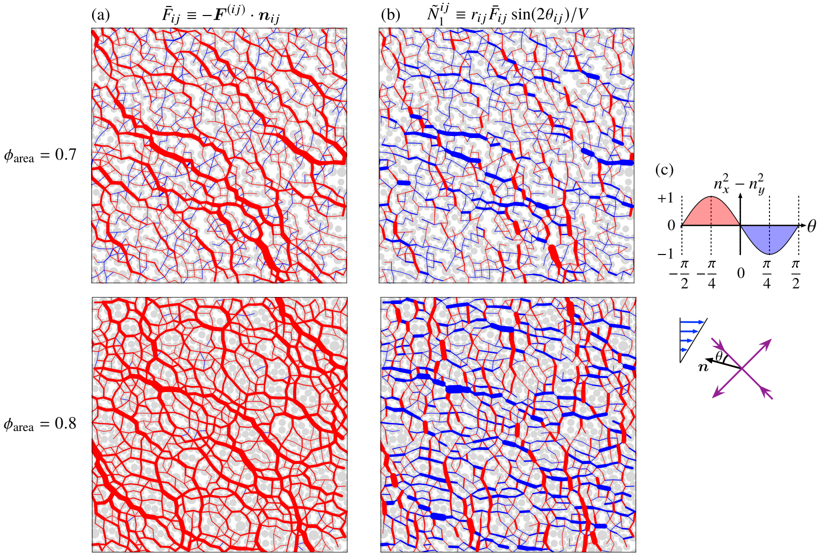


Details in: Seto R., Giusteri G.G., *J. Fluid Mech.* 857, 200–215, 2018

Both hydrodynamic and contact forces give negative N_1

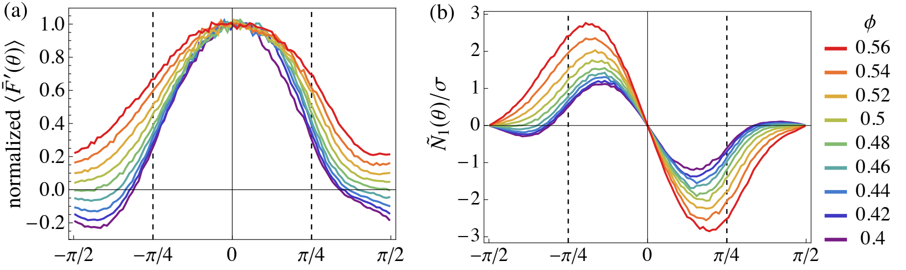


The origin of N_1 is in the normal force network



The value of N_1 is the result of cancellations

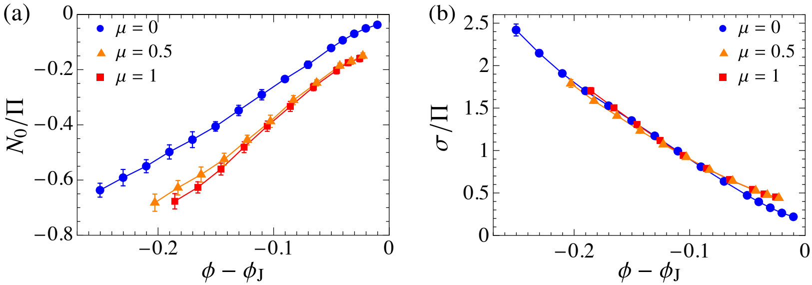
From the angular distribution of forces (in a three-dimensional simulation) we can deduce the contributions to N_1 coming from different directions



The advection due to the vorticity present in simple shear flow is responsible for the mild imbalance leading to a nonzero N_1

Anisotropy due to the planarity of the flow

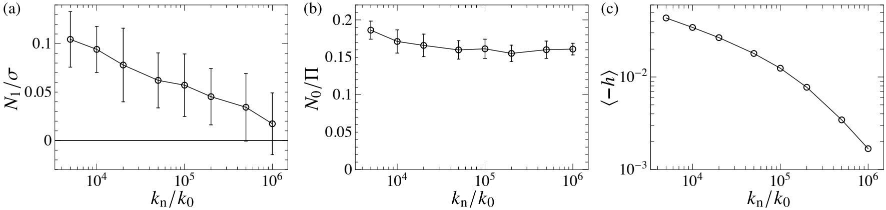
The "zeroth" normal stress difference N_0 measures a stress contribution that is isotropic in the flow plane, but anisotropic when we consider the vorticity direction.
We can quantify its relevance with the ratio over $\Pi = -\mathrm{tr}(\boldsymbol{\sigma})/3$.



Approaching jamming, the force network tends to be isotropic but some trace of the planarity of the flow remains in the frictional case.

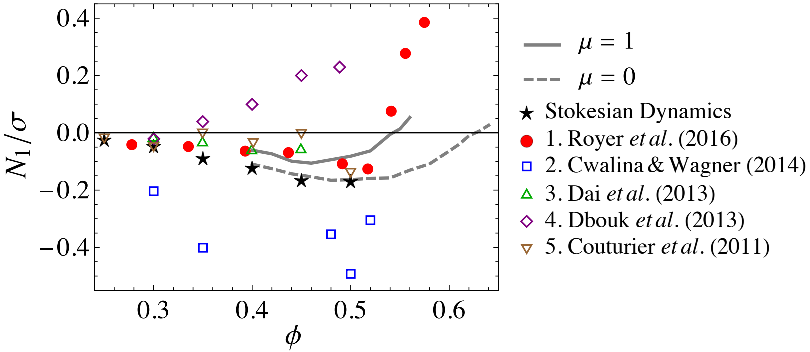
The significance of positive values of N_1

Approaching the jamming condition, the effects of vorticity in a hard sphere system should become negligible entailing a vanishing N_1. Indeed, we find that positive values of N_1 decrease as we increase the numerical stiffness k_n of the spheres used in our simulation.



This indicates that positive values of N_1 measured in experiments cannot be explained simply by a hard-sphere model.
Possible origins could be in the presence of boundaries or in the presence of additional interactions between the particles.

Comparison with experiments



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Thank you!